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APPLIED PROBLEMS OF DYNAMICS OF PIPELINES, CONVEYING LIQUID

O.S. Limarchenko[1]
Doct.Eng.

V.O. Limarchenko[1]
PhD, Phys&Math

M. Majid[1]

A.P. Timokhin[1]

[1]*The Taras Shevchenko Kiev National University, Kiev*
Glushkova avenue, 4-e, Kiev, 03680

Nonlinear applied problems dynamics of pipelines with flowing liquid were considered on the basis of complex approach, which includes variational statement of the problem with automatic derivation of dynamical boundary conditions, mixed Euler-Lagrange description of fluid-structure interaction, method of modal decomposition, qualitative and numerical research of system behavior. We investigate system behavior of pipeline on moving foundation, for liquid flow with velocities, which are in a vicinity of critical velocities of liquid flow or exceed them, Different variants of foundation motion includes vibrations, rotation. Investigation showed the presence of alternative equilibrium state, near which pipeline oscillations occur for certain ranges of velocities of liquid flow.

Keywords: nonlinear applied problems, dynamics of pipelines with flowing liquid, complex approach, vibration, rotation of moving foundation, alternative equilibrium state.

Introduction. Pipelines, conveying liquid, are significant components of numerous structures and engineering systems. Majority of investigations of these problems are done for dynamics of pipelines on immovable foundation or for liquid flows with velocities, which do not attain their critical values [1-6]. However, a lot of applied problems are connected with necessity of considering movable foundation and great values of velocities of liquid flow. Namely these problems originate in problems of fire suppression in high-rise buildings, with operation of energy installations, for abnormal modes of pipelines behavior in the case of pipeline breaking, for different problems of drilling. There are different approaches for pipeline dynamics modeling, however they are mostly focused on problems of high-pressure pipelines, which transport gas. In this case type of problem differs considerably because relative mass of gas can be neglected, therefor its motion does not affect pipeline dynamics. In the case of similar order of linear densities of liquid and pipeline material near effects of combined motion in fluid-structure interaction systems are manifested.

1. Problem statement. We consider pipeline as elastic beam, made of homogeneous material with linear density μ . It is assumed that pipelines length exceeds its transversal dimension at least ten times. In this case onedimensional model of pipeline as beam is acceptable. Further we consider pipeline of cylindrical shape with length *l*. Pipeline bending stiffness if *EJ*, where *E* is the Young modulus and *J* is meridian moment of inertia. Liquid is supposed to be homogeneous, incompressible, ideal with density ρ . Its longitudinal motion is considered to be given.

Further we consider the most complex case, when one edge of pipeline is fixed with foundation, another one is free. In the general case foundation can perform translational or rotational motion. Let us denote by $u(x,t)$ transversal displacement of certain point of pipeline (defined by longitudinal variable *x*) at time instant *t*. Translational motion of foundation is described by longitudinal ε_x and transversal ε_z displacements. Its rotational motion is characterized by angular velocity ω about axis, which corresponds with longitudinal axis of pipeline in non-perturbed stated. The general configuration of the system is shown in Fig. 1.

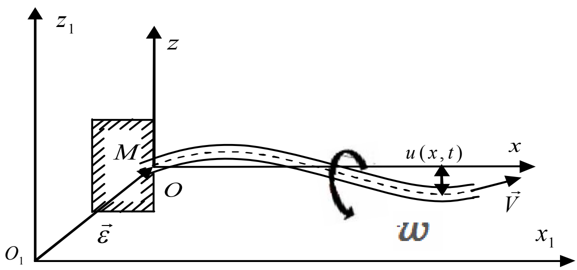

Fig. 1. System pipeline – liquid on movable foundation

For mathematical statement of the problem we make use of the Hamilton-Ostrogradskiy variational principle [1]. Difficulties of description of the considered system are defined by mixed description of system components. Liquid motion is described in the Euler variables, deformation of pipeline is described in the Lagrange variable. For determination of kinetic energy of liquid let us consider element of pipeline on system motion. For certain time instant element of pipeline is turned for angle θ relative to axis *Ox*. For determination of this angle we obtain

$$tg\theta = \frac{\partial u}{\partial x}; \quad \sin\theta = \frac{\frac{\partial u}{\partial x}}{\sqrt{1+\left(\frac{\partial u}{\partial x}\right)^2}}; \quad \cos\theta = \frac{1}{\sqrt{1+\left(\frac{\partial u}{\partial x}\right)^2}}. \tag{1}$$

Let us define velocity of liquid in absolute reference frame. Due to properties of ideal liquid only normal components of liquid will produce action on pipeline. Then, we can determine components of liquid velocity as

$$v_x = V\cos\theta + V_{n_x}^p; \quad v_z = V\sin\theta + \frac{du}{dt} + V_{n_z}^p.$$

Here the upper index p denotes belonging of the corresponding component to pipeline Normal vector to pipeline is $\vec{n} = \{-\sin\theta; \cos\theta\}$. To obtain normal components of velocity we project velocity onto axes Ox, Oz correspondingly $V_n^p = \dot{\vec{\varepsilon}} \cdot \vec{n} = -\dot{\varepsilon}_x \sin\theta + \dot{\varepsilon}_z \cos\theta$, finally we obtain

$$V_{n_x}^p = \frac{1}{1+\left(\frac{\partial u}{\partial x}\right)^2}\left(\dot{\varepsilon}_x\left(\frac{\partial u}{\partial x}\right)^2 - \dot{\varepsilon}_z\frac{\partial u}{\partial x}\right), \qquad V_{n_z}^p = \frac{1}{1+\left(\frac{\partial u}{\partial x}\right)^2}\left(-\dot{\varepsilon}_x\frac{\partial u}{\partial x} + \dot{\varepsilon}_z\right).$$

If we take into account that $\frac{du}{dt} = \frac{\partial u}{\partial t} + \frac{\partial u}{\partial x}v_x$, $\frac{\partial u}{\partial z} = 0$ we obtain expression for square of velocity of liquid

$$v^2 = V^2 + \left(\frac{\partial u}{\partial t}\right)^2 + 3V^2\left(\frac{\partial u}{\partial x}\right)^2 + 4V\frac{\partial u}{\partial t}\frac{\partial u}{\partial x} - 3V^2\left(\frac{\partial u}{\partial x}\right)^4 - 2V\frac{\partial u}{\partial t}\left(\frac{\partial u}{\partial x}\right)^3 +$$

$$+\dot{\varepsilon}_x\left[-2\frac{\partial u}{\partial t}\frac{\partial u}{\partial x} - 2V\left(\frac{\partial u}{\partial x}\right)^2 + 4\frac{\partial u}{\partial t}\left(\frac{\partial u}{\partial x}\right)^3 + 5V\left(\frac{\partial u}{\partial x}\right)^4\right] +$$

$$+\dot{\varepsilon}_z\left[2\frac{\partial u}{\partial t} + 2V\frac{\partial u}{\partial x} - 4\frac{\partial u}{\partial t}\left(\frac{\partial u}{\partial x}\right)^2 - 5V\left(\frac{\partial u}{\partial x}\right)^3\right] + \dot{\varepsilon}_x^2\left[\left(\frac{\partial u}{\partial x}\right)^2 - 3\left(\frac{\partial u}{\partial x}\right)^4\right] +$$

$$+\dot{\varepsilon}_z^2\left[1 - 3\left(\frac{\partial u}{\partial x}\right)^2 + 5\left(\frac{\partial u}{\partial x}\right)^4\right] + \dot{\varepsilon}_x\dot{\varepsilon}_z\left[-2\frac{\partial u}{\partial x} + 6\left(\frac{\partial u}{\partial x}\right)^3\right].$$

Then the Lagrange function of the system will be

$$L = \frac{1}{2}M_{\text{cuc}}\left(\dot{\varepsilon}_x^2 + \dot{\varepsilon}_z^2\right) - F_x\varepsilon_x - F_z\varepsilon_z + \frac{1}{2}\rho\int_0^l\left\{\left[V^2 + \dot{\varepsilon}_z^2\right] + \frac{\partial u}{\partial x}\left[2V\dot{\varepsilon}_z - 2\dot{\varepsilon}_x\dot{\varepsilon}_z\right] +\right.$$

$$+2\frac{\partial u}{\partial t}\dot{\varepsilon}_z + \left(\frac{\partial u}{\partial x}\right)^2\left[\frac{7}{2}V^2 - 2V\dot{\varepsilon}_x + \dot{\varepsilon}_x^2 - \frac{5}{2}\dot{\varepsilon}_z^2\right] + \left(\frac{\partial u}{\partial t}\right)^2 +$$

$$+\frac{\partial u}{\partial t}\frac{\partial u}{\partial x}\left[4V - 2\dot{\varepsilon}_x\right] + \left(\frac{\partial u}{\partial x}\right)^3\left[-4V\dot{\varepsilon}_z + 5\dot{\varepsilon}_x\dot{\varepsilon}_z\right] + \frac{\partial u}{\partial t}\left(\frac{\partial u}{\partial x}\right)^2\left[-3\dot{\varepsilon}_z\right] +$$

$$+\left(\frac{\partial u}{\partial x}\right)^4\left[-\frac{13}{8}V^2 + 4V\dot{\varepsilon}_z - \frac{5}{2}\dot{\varepsilon}_x^2 + \frac{27}{8}\dot{\varepsilon}_z^2\right] +$$

$$\left.+\frac{\partial u}{\partial t}\left(\frac{\partial u}{\partial x}\right)^3\left[3\dot{\varepsilon}_x\right] + \frac{1}{2}\left(\frac{\partial u}{\partial t}\right)^2\left(\frac{\partial u}{\partial x}\right)^2\right\}dx - \frac{1}{2}(\mu+\rho)\omega^2\int_0^l u^2 dx +$$

$$+\frac{1}{2}\mu\int_0^l\left\{\dot{\varepsilon}_x^2+\dot{\varepsilon}_z^2+2\frac{\partial u}{\partial t}\dot{\varepsilon}_z+\frac{1}{2}\left(\frac{\partial u}{\partial x}\right)^2\left[\dot{\varepsilon}_x^2+\dot{\varepsilon}_z^2\right]+\left(\frac{\partial u}{\partial t}\right)^2+\frac{\partial u}{\partial t}\left(\frac{\partial u}{\partial x}\right)^2\dot{\varepsilon}_z+\right.$$

$$\left.+\frac{1}{2}\left(\frac{\partial u}{\partial t}\right)^2\left(\frac{\partial u}{\partial x}\right)^2-\frac{1}{8}\left(\frac{\partial u}{\partial x}\right)^4\left[\dot{\varepsilon}_x^2+\dot{\varepsilon}_z^2\right]\right\}dx-\frac{1}{2}EJ\int_0^l\left(\frac{\partial^2 u}{\partial x^2}\right)^2dx-$$

$$-\frac{1}{4}EJ\int_0^l\left(\frac{\partial^2 u}{\partial x^2}\right)^2\left(\frac{\partial u}{\partial x}\right)^2dx-\frac{1}{8}EF\int_0^l\left(\frac{\partial u}{\partial x}\right)^4dx-\frac{1}{2}PF\int_0^l\left(\frac{\partial u}{\partial x}\right)^2dx.$$

This Lagrange function corresponds to nonlinear model of dynamics of pipeline with flowing liquid, which performs translational and rotational motion on movable foundation. This model takes into account centrifugal, elastic, Coriolis forces, forces of longitudinal compression [1, 6].

If we use modal decomposition of motion with respect to normal modes $A_i(x)$ with amplitude parameters $c_i(t)$ $u(x,t)=\sum_i c_i(t)A_i(x)$ we obtain the following discrete model of the system in amplitude parameters (now we restrict ourselves by the case of given motion of foundation)

$$\ddot{c}_r=-\frac{EJ}{\rho+\mu}\kappa_r^4c_r+\frac{7}{2}\frac{\rho V^2}{(\rho+\mu)N_r}\sum_i c_i\,\beta_{ir}^2+\frac{2\rho V}{(\rho+\mu)N_r}\sum_i\dot{c}_i\left(\beta_{ri}^1-\beta_{ir}^1\right)-$$

$$-\frac{PF}{(\rho+\mu)N_r}\sum_i c_i\beta_{ir}^2-\frac{2\rho\dot{V}}{(\rho+\mu)N_r}\sum_i c_i\beta_{ir}^2-\frac{1}{2N_r}\sum_{ijk}\ddot{c}_ic_jc_k\,d_{jkir}^2-$$

$$-\sum_{ijk}\dot{c}_i\dot{c}_jc_k\frac{1}{N_r}\left(d_{jkir}^2-\frac{1}{2}d_{krij}^2\right)-\frac{EJ}{(\rho+\mu)N_r}\sum_{ijk}c_ic_jc_kd_{ijkl}^6-$$

$$-\frac{EF}{2(\rho+\mu)N_r}\sum_{ijk}c_ic_jc_kd_{ijkr}^4-\frac{13}{4}\frac{\rho V^2}{(\rho+\mu)N_r}\sum_{ijk}c_ic_jc_kd_{ijkr}^4+$$

$$+(\rho+\mu)\omega^2c_r+\frac{\rho}{N_r(\rho+\mu)}\psi_r^1\left[V\dot{\varepsilon}_z-\dot{\varepsilon}_x\dot{\varepsilon}_z\right]+$$

$$+\frac{1}{N_r(\rho+\mu)}\sum_{i=1}^N c_i\beta_{ir}^2\left[-2\rho V\dot{\varepsilon}_x+\dot{\varepsilon}_x^2(\rho+\frac{1}{2}\mu)+\dot{\varepsilon}_z^2(\frac{1}{2}\mu-\frac{5}{2}\rho)\right]-$$

$$-\frac{\rho}{N_r(\rho+\mu)}\sum_{i=1}^N\dot{c}_i\left(\beta_{ri}^1-\beta_{ir}^1\right)\dot{\varepsilon}_x+\frac{3\rho}{N_r(\rho+\mu)}\sum_{i,j=1}^N c_ic_j\varphi_{ijr}^3\left[-2V\dot{\varepsilon}_z+\frac{5}{2}\dot{\varepsilon}_x\dot{\varepsilon}_z\right]+$$

$$+\frac{\mu-3\rho}{N_r(\rho+\mu)}\sum_{i,j=1}^N c_i\dot{c}_j\left(\varphi_{irj}^2-\varphi_{jir}^2\right)\dot{\varepsilon}_z+\frac{9\rho}{2N_r(\rho+\mu)}\sum_{i,j,k=1}^N c_ic_j\dot{c}_k\left(d_{ijrk}^3-d_{kjir}^3\right)\dot{\varepsilon}_x+$$

$$+\frac{2}{N_r(\rho+\mu)}\sum_{i,j,k=1}^{N} c_i c_j c_k d_{ijkr}^4\left[-4\rho V\dot{\varepsilon}_x+\dot{\varepsilon}_x^2\left(-\frac{5}{2}\rho-\frac{1}{8}\mu\right)+\dot{\varepsilon}_z^2\left(\frac{27}{8}\rho-\frac{1}{8}\mu\right)\right]-$$

$$-\frac{1}{N_r}\psi_r^o\ddot{\varepsilon}_z+\frac{\rho}{N_r(\rho+\mu)}\sum_{i=1}^{N}\dot{c}_i\beta_{ir}^1\ddot{\varepsilon}_x-$$

$$-\frac{\mu-3\rho}{2N_r(\rho+\mu)}\sum_{i,j=1}^{N} c_i c_j\varphi_{ijr}^2\ddot{\varepsilon}_z-\frac{3\rho}{2N_r(\rho+\mu)}\sum_{i,j,k=1}^{N} c_i c_j c_k d_{ijkr}^3\ddot{\varepsilon}_x.$$

This system of ordinary differential equations with denotations [1] was used for analytical investigations and numerical simulation of problems of pipelines dynamics.

2. System motion for longitudinal and translational vibrations of foundations. In the case of translational motion of foundation pipeline can perform both forced and parametric oscillations. The most interesting effects are caused by vibrations, when liquid flows at velocity 75% of the critical one. In this case vibrations of foundation create condition for manifestation of alternative position of equilibrium. Typical behavior of pipeline for transversal and longitudinal excitation of motion is shown in Fig. 2–4.

Fig. 2 shows variation in time of oscillations of pipeline free and for transversal vibration of foundation with amplitude 2 mm and frequency 20 1/s (close vicinity of resonance).

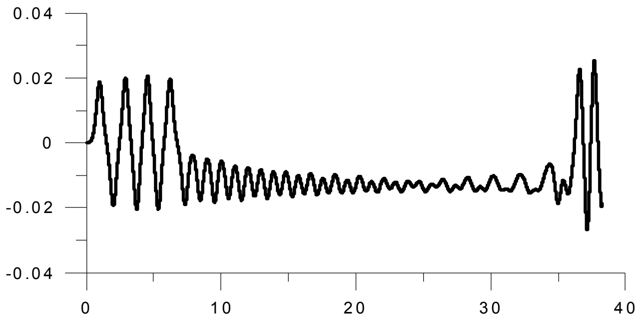

Fig. 2. Oscillations of pipeline free end for transversal vibration of foundation ($\omega = 20$)

It is seen from figure that after certain transient period of motion pipeline begin to oscillate near position of alternative equilibrium state, later due to permanent energy supply to the system oscillations increase and system passes again into oscillations with respect to initial straight shape. If we double frequency of vibration of foundation this change between three (initial, upper and lower) positions of equilibrium can happen systematically.

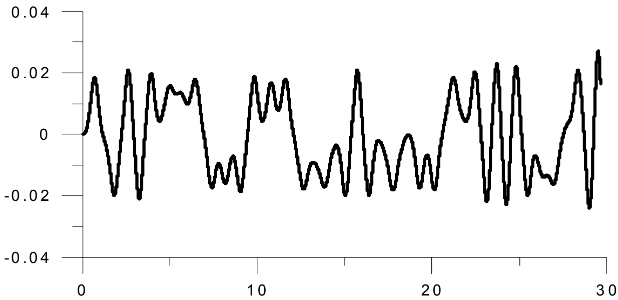

Fig. 3. Oscillations of pipeline free end for transversal vibration of foundation ($\omega = 40$)

In the case of longitudinal vibrations of foundation we observe manifestation of parametric resonance for doubled resonance frequency and for different values of initial perturbations of pipeline. Fig. 4 shows variation in time of the first and second amplitude of pipeline oscillations. In this case longitudinal vibrations of foundation occur with amplitude 2 mm, with frequency 40 1/s and with initial perturbation with respect to the second normal mode 4,7 mm. Velocity of liquid was 75% of the critical one.

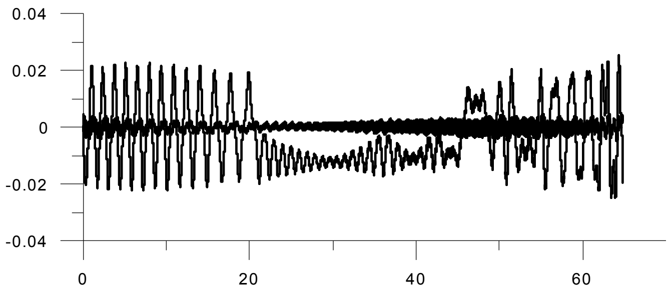

Рис. 4. Variation in time of amplitudes of the first and second normal modes

As it is seen from Fig. 4, second normal mode performs oscillations in a vicinity of initial straight shape of pipeline. However, the first normal mode performs complicated oscillations with manifestation of transition to alternative potion of equilibrium and modulation of oscillations.

3. System motion with liquid velocity, which exceeds critical values. The most complicated case of the system motion is connected with system behavior for liquid velocities, which exceed critical velocity. In this case oscillations with respect to straight shape of pipeline become unstable. However, two new stable positions of equilibrium can happen depending on character of initial excitation. If we analyze the motion equation for existence of alternative equilibrium position, we must suppose $\ddot{c}_i = 0$. This condition gives the following alternative positions (Fig. 5). In Fig. 5 x corresponds to longitudinal coordinate, vertical axis denotes amplitude, every line corresponds to velocity of liquid flow, which is given as factor relative to the value of the critical velocity.

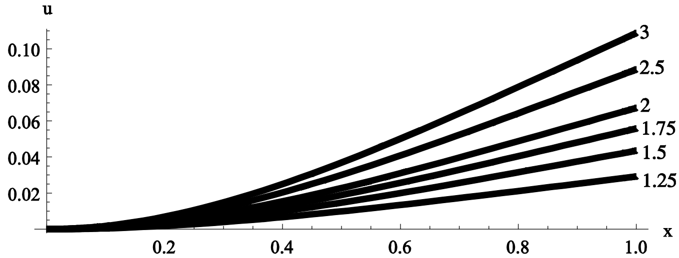

Fig. 5. Shapes of alternative positions for different values of liquid velocities

Fig. 6 shows stability chart for different values of velocities of liquid flow, which was obtained on the basis of linear model for four normal modes. We verify numerically general aspects of system behavior by means of nonlinear model of pipeline, which takes into account 12 normal modes and got good concordance with qualitative results.

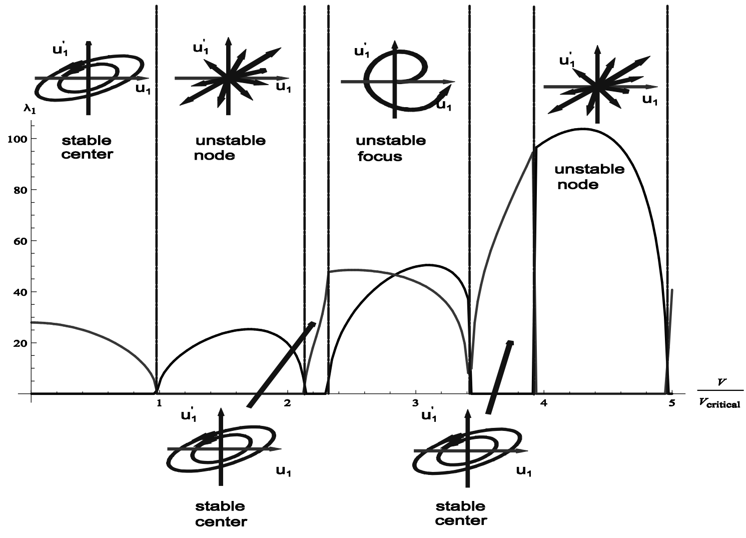

Fig. 6. Stability chart for pipeline in the case of different velocities of liquid flow

4. Dynamics of pipeline on rotating foundation. In the case of behavior of pipeline on rotating foundation rotation is reduced to supplementary term, which acts similar to centrifugal force. General structure of equations does not change. Moreover, normal modes in this case remain to be the same as in the case of pipeline without rotation with the same wave numbers k_i . At the same time frequencies p_i change in the following way

$$p_i^2 = k_i^4 a^2 - \omega^2, \quad \text{where} \quad a^2 = \frac{EJ}{\mu + \rho}.$$

For estimation of rotation effect on critical velocity of liquid flow we write down linear equation of system motion in discrete parameters for the first normal mode

$$\ddot{c}_1 = \Omega^2 c_1,$$

where Ω is frequency of pipeline oscillations. From motion equations we obtain

$$\Omega^2 = -\frac{EJ}{(\rho+\mu)}\chi_1^4 + \frac{7\rho V^2}{2(\rho+\mu)N_1}\beta_{11}^2 - \frac{PF}{(\rho+\mu)N_1}\beta_{11}^2 - \omega^2.$$

Critical velocity V_{cr}^1 can be obtained from the condition to zero of frequency

$$V_{cr}^1 = \pm\sqrt{\frac{2}{7\rho}\left(\frac{EJN_1\chi_1^4}{\beta_{11}^2} + PF - \omega^2\frac{(\rho+\mu)N_1}{\beta_{11}^2}\right)}.$$

From the expression of critical velocity it is seen that rotation causes lowering of critical velocity.

We consider numerical example of pipeline oscillations under the presence of rotation of foundations and initial kinematic excitation $c_1(0) = 0{,}01$. We consider system motion for velocity of liquid flow, which is equal to half of critical velocity in the case of absence of rotation. Rotation was changed until $\omega = 20\pi$. In the considered case critical velocity is equal to 20,94 *m/s*. We shall consider velocity of liquid flow 10 m/s. Period of oscillations of the first normal mode is T=0,1567 *s*. Frequency of oscillations will be $\Omega_1 = 2\pi / T \approx 40{,}1/s$.

The most typical modes of behavior of rotating pipeline manifest for $\omega_1 = 0$, $\omega_2 = 10\pi$, $\omega_3 = 16\pi$, $\omega_4 = 20\pi$. Variation in time of amplitude of the first normal mode is shown in Fig. 7. Here curves are numbered according to indexes of of frequencies.

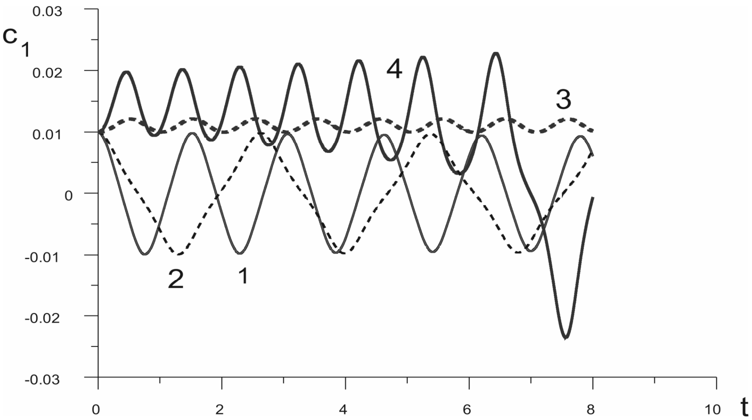

Fig. 7. Variation in time of amplitude of the first normal mode for different angular velocities

As it is seen from figure for $\omega_1 = 0$ stable mode of motion occurs in the system. On increase of angular velocity to $\omega_2 = 10\pi$ stability of processes is not violated, however, frequency of oscillations decreases. Further increase of angular velocity $\omega_3 = 16\pi$ corresponds to attainment to critical velocity. In this case oscillations are performed near alternative position of dynamical equilibrium. Here frequency of oscillations considerably increases and becomes even greater than in the case of pipeline without rotation. So, rotation in this case provides effect of stabilization. Further increase of angular frequency to $\omega_4 = 20\pi$ results in considerable growth of amplitudes of oscillations and transition of oscillations from vicinity of alternative position of equilibrium state and passage to unstable mode.

So, rotation of pipeline foundation results in origination of three modes of dynamical behavior of pipeline. Until angular velocities about $\omega = 15\pi$ steady oscillations in a vicinity of initial straight position of pipeline occur. Here on increase of angular velocity frequency of oscillations decreases, For the range $15\pi < \omega < 18\pi$ oscillations of pipeline relative to straight position can loss stability and transit to oscillations near alternative position of equilibrium, moreover, frequency of oscillations increases and becomes greater than in the case of pipeline without rotation. For oscillations of pipeline in the range $\omega > 18\pi$ oscillations are performed with increasing amplitude; loss of stability relative to alternative position of equilibrium can happen.

Conclusions. On the basis of variational approach we constructed method for investigation of complex of problems of nonlinear dynamics of pipelines on movable foundation for wide range of velocities of liquid flow, including velocities, which increase critical values. Conducted investigations showed that mobility of foundation promotes situation when system can reach effects of loss of stability of straight line shape of pipeline even for velocities of flow, which are lower than critical velocities for immovable foundation of pipeline. Both forced and parametric mechanisms manifest for translational motion of pipeline foundation. In the case of rotation of pipeline foundation variety of effects is also.

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Лимарченко О.С., Лимарченко В.О., Маджид М., Тімохін О.П.

ПРИКЛАДНІ ЗАДАЧІ ДИНАМІКИ ТРУБОПРОВОДУ З РІДИНОЮ, ЩО ТЕЧЕ

Нелінійні прикладні задачі динаміки трубопроводів з рідиною, що тече, розглядаються на сонові комплексного підходу, що включає варіаційну постановку задачі з автоматичним виведенням динамічних граничних умов, мішаний ейлерово-лагранжеве опис гідро пружної взаємодії, метод модальної декомпозиції, якісне і чисельне дослідження поведінки системи. Вивчається поведінка системи на рухомій основі для течії рідини із швидкостями, які наближаються до значень критичних швидкостей чи перевершують їх. Різні варіанти руху основи включають вібрації, обертання. Дослідження показали наявність альтернативного положення рівноваги в околі якого відбуваються коливання в деякому діапазоні швидкостей течії рідини.

Ключові слова: нелінійна прикладна задача, динаміка трубопроводу з рідиною, що тече, комплексний підхід, вібрації, обертання рухомої основи, альтернативне положення рівноваги.

Лимарченко О.С., Лимарченко В.О., Маджид М., Тимохин А.П.

ПРИКЛАДНЫЕ ЗАДАЧИ ДИНАМИКИ ТРУБОПРОВОДА С ПРОТЕКАЮЩЕЙ ЖИДКОСТЬЮ

Нелинейные прикладные задачи динамики трубопроводов с протекающей жидкостью рассматриваются на основе комплексного подхода, включающего вариационную постановку задачи с автоматическим выводом динамических граничных условий, смешанное эйлерово-лагранжевое описание гидроупругого взаимодействия, метод модальной декомпозиции, качественное и численное исследование поведения системы. Исследуется поведение системы на подвижном основании для течений жидкости со скоростями приближающихся к значениям критических скоростей или превосходящим их. Разные варианты движения основания включают вибрации, вращение. Исследования показали наличие альтернативного положения равновесия, в окрестности которого происходят колебания в некотором диапазоне скоростей течения жидкости.

Ключевые слова: нелинейная прикладная задача, динамика трубопровода с протекающей жидкостью, комплексный подход, вибрации, вращение подвижного основания, альтернативное положение равновесия.