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# APPLICATION NEURO-FUZZY ECONOMIC-MATHEMATICAL MODEL TO DETERMINE THE OPTIMAL VOLUME OF INSURANCE PAYMENTS

The results obtained through the use of: classical economics and mathematical methods of forecasting the likelihood of the risk, based on the unit of mathematical statistics; fuzzy-multiple descriptions that are set adequate formalism for modeling financial systems in the face of considerable uncertainty and right for a new interpretation of the classical probability and expert evaluations; mathematical theory of fuzzy sets and fuzzy logic, which allows you to create relevant provisions regarding management decisions towards preventing crises. A special feature is taking into account expert knowledge in the subject area, which is a quantitative and qualitative information on the operation environment of insurance companies, as well as optimization of models based on operational data from indicators of financially stable insurance companies and potential bankruptcies. The proposed building a model where the priority is to identify and list the formation of a set of input parameters - independent variables based on fuzzy logic tools that are selected to construct discriminant function by testing for multicollinearity.

The construction of the membership function of input and result variables is shown for the implementation of an adequate classification regarding the level of all indicators. It is proved that the result of the model is not the division of companies on the basis of belonging to bankrupts or stable functioning, and the time remaining until bankruptcy. The model not only finds the output variable from the set of three given linguistic definitions, but also predicts the time during which a company is threatened with bankruptcy. The conceptual approach to ensuring the possibility of automatic adjustment of model parameters taking into account quantitative and qualitative factors with the provision of a number of other undeniable advantages for the simulation of financial and economic systems, which is reduced to the development of a system of response that would correspond most closely to the requirements of a particular situation, is presented. The recommended guidelines for determining the linguistic description of the bankruptcy risk of quantitative interpretation obtained from a qualitative assessment are proposed.

***Key words:*** insurance market, financial stability, financial risk, insurance payments, economic and mathematical models, analysis, neuro-fuzzy technology.

**Formulation of the problem.** In recent decades, the role continuously mentioned insurance companies goes to a new level by expanding the range of insurance products, the introduction of complex financial products and positioning itself as a reliable institutional investors.

Unstable state realities of the Ukrainian economy and the external economic environment, where today the modern insurance companies are also developing require constant adaptation to changes in the insurance business, because today is the question of financial stability of companies and the creation of new tools to ensure capacity to fulfill its obligations in the future.

The analysis of the insurance company's activity requires consideration of a wide range of insurers' functions in the formation of information security, which affects the quantitative assessment of the financial state. Particularly acute is the problem of maintaining the solvency of the insurance company.

Because provision of liabilities is a necessary component in forecasting the probability of bankruptcy in order to maintain the reputation and position of the insurance market.

Thus, the existing needs for ensuring the financial stability of the insurance company and reducing the potential for its deterioration justify the relevance of the research topic.

In view of the above, to determine the optimal volume of insurance payments in current conditions appropriate use of instruments of economic and mathematical modeling, taking into account qualitative and quantitative factors. One of them is the tool of fuzzy logic, which is currently the most promising area of research in the fieldanalysis, forecasting and modeling economic phenomena and processes.

**Analysis of recent research and publications.** The theoretical, applied and methodological aspects of the construction and use of economic and mathematical models, the study of financial flows of the financial state of insurance companies, their financial stability and crisis management are devoted to a significant number of publications by foreign authors.

Among them should be noted of E. Altman [1] W. Beaver [2] A. Kleffner [3] and national scientists A. Matviychuk [4, 5, 6], V. Vitlinskoho [7] O. Cherniak [8] O. Kozmenko [9, 10] O. Olkhovska [11] Z. Sokolovsky [12] and others.

**Formulating the goals of the article.** With the development of the insurance market more and more attention is paid to the study of the conditions of maintenance and maintenance of its stable activities.

The continuous monitoring of the environment for the operation and control of the company's development strategy that would meet the requirements of a particular situation requires the features and quality of the financial system in which the domestic insurance business functions. Because of unforeseen circumstances it is in an unstable economic development can break the financial stability of the insurer and its ability to carry out its other obligations.

Accordingly, in the conditions of uncertainty and instability of the environment of the insurance industry, the task of analyzing and assessing the financial position of companies becomes of special importance. Therefore, the creation of solvency potential in the state and the promotion of microeconomic stabilization of insurance companies is a prerequisite for ensuring the reliable functioning of the insurance market.

Thus, the purpose of the article is to conduct an analysis on the development of optimal methodological support for the formation and evaluation of financial stability of insurance companies and the approach to comprehensive financial analysis of an insurance company in the context of aggravation of crisis phenomena, using methods and tools of neuro-fuzzy economic-mathematical modeling, which allows to raise efficiency of functioning of modern insurers.

**Presentation of the main research material.** Insurance is among the most integrated forms of financial activity, and is a powerful tool for smoothing losses arising in the course of business.

Insurance companies are financial institutions providing special kind of services and implement important functions of protection and compensation, maintaining the stability of the economic system of the state. A key element in considering the insurance process is the definition of insurance risks in different approaches to the formation of insurance payments. Naturally, a more appropriate approach is to identify risks based on insurance models, with subsequent adjustments to the results. Despite the fact that the main activity of insurers is the attraction of insurance premiums and the implementation of insurance payments, a special role in the functioning of insurance companies plays an investment component.

Ensuring the stability of the functioning of the financial market in the conditions of the mutual integration of many of its parts is placed on the insurance and reinsurance sector as an instrument for ensuring financial stability of insurance companies. Probabilistic nature of insurance determines the critical role of a high level of solvency of the insurer, as the company should be able to meet the obligations under insurance at any time. After all, insurance companies are required to generate and post compulsory insurance reserves, including reserves and premium reserves losses.

Therefore, to determine the optimal volume of insurance payments is an important criterion for the assessment of the insurance company.

There are a number of classical economic and mathematical methods for forecasting the probability of occurrence of risk that are based on the apparatus of mathematical statistics, but the fuzzy-plural descriptions are, on the one hand, a set of adequate formalities for the simulation of financial systems in conditions of significant uncertainty, and, on the other hand, the field for a new interpretation of classical probabilistic and expert estimates.

The generalization of the classical theory of sets and the classical formal logic is a mathematical theory of fuzzy sets and fuzzy logic that allows for the creation of appropriate guidance for making managerial decisions to prevent a crisis.

A special feature is taking into account expert knowledge in the subject area, which is a quantitative and qualitative information on the operation environment of insurance companies as well as optimization of models by operating data from indicators of financially stable insurance companies and potential bankrupt.

Fuzzy logic model builds on the basis incorporated therein sets of logical rules and set parameters of membership functions. This means that they can operate even without real data.

The models are open and understandable, because they are presented in natural language. Also, unlike other methods, they are able to combine the possibility of adjusting their parameters to real data from the analysis of quantitative - qualitative indicators of insurance companies, because they have a set of advantages inherent in neural networks. And the use of decision-making rules in fuzzy logic models makes it possible to take into account in the analysis of expert knowledge in the subject area.

The example of mathematical modeling of diagnostics bankruptcy independent variables can hold the following sequence of calculations.

When building a model priority is to identify and list the formation of a set of input parameters (independent variables) based on fuzzy logic tools $X_i, i = \overline{1,N}$ that are selected to construct discriminant function (1) by checking multicollinearity.

For the evaluation of financial performance $X_i$ and forming a knowledge base, $i=\overline{1,N}$ that allow to characterize the financial position, forming a single list of three quality definitions: L - low $X_i$, I - intermediate $X_i$, *HL*- high level $X_i$.

In accordance with the current financial condition, the following definitions are used to evaluate the values of the resulting linguistic variable G, which includes a number of bankruptcy risk degrees (RD): L - low RD, A - average RD, H - high RD.

In case of uncertainty of the expert there are fuzzy descriptions of the methodological approach of conducting financial and economic analysis. He can not clearly distinguish the mean and high level of a particular parameter.

Therefore, it is necessary to build the functions of the membership of input and result variable definitions for the implementation of an adequate classification regarding the level of all indicators.

Possible range of change of input factors $X_i, i=\overline{1,N}$ and resultant parameter G, sets the appearance of membership functions {L, I, HL} of fuzzy definitions of the input variable $X_i, i=\overline{1,N}$ on the set X.

It is analytically possible to represent quasi-bell-like functions that belong to fuzzy definitions, both input $X_i, i=\overline{1,N}$ and output variable G, using the function [5]:

$$\mu^T(X)=\frac{1}{1+\left[\frac{X-b_T}{c_T}\right]^2} \quad (1)$$

C - concentration factor, extracting function; b - coordinate of the maximum of the function (μ (b) = 1); T - linguistic definition of the set { L, I, HL }. As in points of maximum, the values of the functions of the membership of the side definitions L and H of all variables outside of their maxima *b* are equal to 1.

If the configuration parameters of the model will not be in accordance with the normative values and by comparing these indicators for different companies in different periods of time necessary to set tentative limits of all the definitions of each indicator $X_i, i=\overline{1,N}$. We set the levels of indicators that are consistent with their linguistic definitions so that they are fairly consistent with the rules of bankruptcy assessment [6].

To factor mobility assets ($X_1$) distinction between the definitions of L and I was set at 0,3, and between I and H - level 1. Payable turnover ratio ($X_2$) defined boundary between the L and I is set at 0,5, and between I and H – 1,5. For coefficients of turnover of equity ($X_3$) and payback assets ($X_4$) defined at 0,4 and 1,2 respectively. For the coefficient of security of own working capital ($X_5$) - at -1 and 0. The coefficient of concentration of attracted capital ($X_6$)as the limits has a value of 0,6 and 1,2, and the ratio of debt to equity ($X_7$) is 0,4 and 1,4[7].

Crossing the functions of membership set above distinction between the values for each definition $T \in \{\mathrm{L, I, HL}\}$ input variables that can get the parameters $b_T$ and $c_T$ features (1). Thus, the boundaries of the intersection of the linguistic definitions of the output variable can be determined.

Definition H, which characterizes the high degree of RD, was assigned to the fact that they went bankrupt in less than 24 months from the statement of financial results. The average degree of RD (I) gave them that they became bankrupt more than in 24 months. For companies that have been working stably for more than five years after submitting for the analysis of financial statements, they set the definition - L. The starting point of the model is the time remaining before bankruptcy, rather than the division of companies on the basis of bankruptcy or stable functioning. So, not only is the model output variable set of three linguistic definitions set {L, I, HL}, but also predicts the time at which the company faces bankruptcy.

After converting the fuzzy value of the output variable to a clear number, that is, dephasing, the model calculates the time (monthly), which remains until the probable bankruptcy.

An expert system based on fuzzy knowledge should contain a mechanism allowing decision making, which should include knowledge relating to the financial side of bankruptcy and form a system of fuzzy logic rules. This will help predict bankruptcy based on analytics information.

The rules by which decisions show a low degree RD can be represented using membership functions and weighted coefficients [7]:

$$\begin{aligned}\mu^{H}(X_1,...,X_7) = \omega_1^{H}\left\lfloor \mu^{B}(X_2) * \mu^{B}(X_3) * \mu^{-B}(X_4) * \mu^{-H}(X_5) * \mu^{H}(X_6) \right\rfloor \vee \\ \vee\, \omega_2^{H}\left[\mu^{-H}(X_1) * \mu^{c}(X_3) * \mu^{H}(X_4) * \mu^{B}(X_5) * \mu^{-B}(X_6) * \mu^{B}(X_7)\right]\end{aligned} \quad (2)$$

$\mu^{d} j(X_1,...,X_N)$ - membership function of input variables vector $X_i, i = \overline{1,N}$ the value of output variable $d_j$ the set { L, I, HL };

$N$ - the number of input variables (in this problem N = 7);

$\mu^{a_i^{jp}}(X_i)$ - membership function of input variable $X_i$ linguistic definition $a_i^{jp}, j = \overline{1,m}, i = \overline{1,N}, p = \overline{1,k_j (\mu^{-a}(X_i)} = 1 - \mu^{a}(X_i))$;

$m$ - number of values of the output variable G (m = 3);

$k_j$ - the number of rules in the knowledge base that meet the definition of j-th output variable G ($k_1$=$k_3$= 2, $k_2$= 4);

$\omega_p^{d_j}$ - the weight of the $p$ rule among those that correspond to the definition $d_i$ of the output variable.

In terms of the theory of fuzzy logic, this means linguistic expression: "If the value $X_2$ (H) $X_3$ (H) $X_4$ not (H) $X_5$ no (L), $X_6$ (L) / IF $X_1$ no (L) $X_3$ (I) $X_4$ (L), $X_5$ (H) $X_6$ not (H) $X_7$ (H), then the degree of risk of bankruptcy G is low (L)".

Expert confidence in the chosen rule characterizes the weight, which is the number of interval [0, 1]. In the same way, all functional dependencies are formed, which embody decision-making rules in mathematical form.

Find optimal solutions using different number of rules should be performed with the maximum calculated value of membership function. In order for each rule to take into account the values of all input variables, when calculating the membership function of the output variable, the operation of minimizing the membership functions of all input variables is replaced by multiplication.

In the presence of statistical data, it is necessary to optimize the parameters, but this model can be used to assess the financial condition of the enterprise. Therefore, before evaluation of enterprise financial condition should adjust the model data to bankrupt companies and financially stable companies using optimization algorithms of a neuro-fuzzy models, such as back mistake propagation algorithm [12]. If to optimize the model on the available statistical material, the quality of its logical conclusion is significantly increased.

When configuring the model parameters are optimized all the rules and parameters of membership functions of input variables with certain restrictions. In order to provide a clear identification of enterprises that, according to these financial indicators, remained bankrupt for less than two years, for the initial variable, the parameters of membership functions of all definitions remain unchanged [13].

Then, based on indicators $X_i, i = \overline{1, N}$, Financial reports and expert opinions for different time periods after the construction and configuration parameters for the model estimated the current level of financial condition of the company G, to be able to monitor the dynamic changes in susceptibility to bankruptcy and enterprise decision. The final decision on G models that for which the function of target variable G is greatest for a given value $X_i^*, i = \overline{1, N}$:

$$G = \arg\max \left[\mu^{d_j}(X^*, ..., X_N^*)\right], j = \overline{1, m} \quad (3)$$

$d_j$ = {L, I, HL}.

As for determining the effective rate G maximum use out of all the rules and functions of target variable for each rule is calculated as the product of the functions of all input variables, the model output variable calculated by the following function:

$$G = \arg\max\left\{\omega_p^{d_j}\prod_{i=1}^{N}\mu^{a_i,jp}(X_i^*)\right\} \qquad (4)$$

$p = \overline{1,k_j}$, $j = 1,m$.

The result of the equation - the linguistic description of the risk of bankruptcy and quantification of time until a possible bankruptcy - quantitative interpretation derived from (4) quality assessment.

If modeling task is only to determine the possibility of bankruptcy, not predictions since its onset, the dephazyfication cannot be done.

By moving from quantitative values to linguistic definitions in fuzzy models, the meaning of multicollinearity can be lost and there is the possibility of constructing a model on those informative indicators that a financial analyst considers appropriate to use, without conducting a preliminary study on the existence of correlation links between them.

**The conclusions and recommendations for further research.** An analysis of existing approaches to ensure the financial stability of insurance companies can conclude that since the author's model is based on the same sets of explanatory variables and configured on the overall statistical results of comparative experiments with prediction of bankruptcies allow to draw conclusions about the ability of various mathematical tools to classify objects research. Fuzzy logic provides a convenient tool for representing expert knowledge about the development of the studied systems and processes in mathematical form. Involvement in fuzzy models ensures the automatic adjustment of their parameters, taking into account quantitative and qualitative factors, and gives a number of other undeniable advantages for the simulation of financial and economic systems, which is reduced to the development of a system of response that would be most in line with the requirements of a particular situation, which is the reasoned choice of tools fuzzy logic as the basis for constructing the corresponding mathematical models.

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**Застосування нейро-нечіткої економіко-математичної моделі для визначення оптимальних об'ємів страхових виплат**

У статті розглянуто теоретичне узагальнення методологічного забезпечення фінансової стійкості страхової компанії в умовах невизначеного зовнішнього середовища, для визначення оптимальних об'ємів страхових виплат з застосуванням інструментарію нечіткої логіки. Результати отримані за рахунок застосування: класичних економіко-математичних методів прогнозування ймовірності настання ризику, які базуються на апараті математичної статистики; нечітко-множинні описи, що являють собою, набір адекватних формалізмів для моделювання фінансових систем в умовах значної невизначеності, а також поле для нової інтерпретації класичних ймовірнісних та експертних оцінок; математична теорія нечітких множин і нечітка логіка, яка дозволяє формувати відповідні положення стосовно прийняття управлінських рішень у напрямку попередження кризових явищ. Особливістю є урахування експертних знань в предметній області, що складає кількісну та якісну інформацію щодо середовища функціонування страхових компаній, а також здійснення оптимізації моделей оперуючи даними показників діяльності фінансово стабільних страхових компаній та потенційних банкрутів. Запропонована побудова моделі, де першочерговим є визначення та формування набору переліку вхідних показників - незалежних змінних, на основі інструментарію нечіткої логіки, що відібрані для побудови дискримінантної функції шляхом перевірки на мультиколінеарність. Показано побудову функції належності вхідних і результуючих змінних визначень для здійснення адекватної класифікації щодо рівня усіх показників. Доведено, що виходом моделі є не поділ компаній за ознакою належності до банкрутів чи стабільно функціонуючих, а час, що залишався до банкрутства, адже, модель не лише знаходить вихідну змінну із множини трьох заданих лінгвістичних визначень, а й прогнозує час, протягом якого компанії загрожує банкрутство.

Наведено концептуальний підхід щодо забезпечення можливості автоматичного налагодження параметрів моделі з урахуванням кількісних та якісних факторів з наданням низки інших незаперечних переваг для моделювання фінансово-економічних систем, що зводиться до розробки системи реагування, яка найбільшою мірою відповідала б вимогам конкретної ситуації. Запропоновано рекомендаційні інструкції щодо ідентифікації лінгвістичного опису ризику банкрутства кількісної інтерпретації отриманої з якісної оцінки.

***Ключові слова:*** страховий ринок, фінансова стійкість, фінансовий ризик,страхові виплати, економіко-математичні моделі, аналіз, нейро-нечіткі технології.

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**Применение нейро-нечеткой экономико-математической модели для определения оптимальных объемов страховых выплат**

В статье рассмотрены теоретические обобщения методологического обеспечения финансовой устойчивости страховой компании в условиях неопределенной внешней среды, для определения оптимальных объемов страховых выплат с применением инструментария нечеткой логики. Проведен анализ по разработке оптимального методического обеспечения для формирования и оценки финансовой устойчивости страховых компаний. Выделен комплексный подход финансового анализа страховой компании в условиях обострения кризисных явлений, с использованием методов и инструментов нейро-нечеткого экономико-математического моделирования. Обоснована возможность повышения эффективности функционирования современных страховщиков с использованием особенностей теории нечетких множеств и нечеткой логики, которые позволяют формировать принятия управленческих решений в направлении предупреждения кризисных явлений.

***Ключевые* слова:** страховой рынок, финансовая устойчивость, финансовый риск, страховые выплаты, экономико-математические модели, анализ, нейро-нечеткие технологии.

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