Shchetinina E.K.,
Dr. of Physico-mathematical sciences, Prof.
Skripnik S.V.,
PhD, assoc. Prof.
Donetsk National University of Economy and Trade named after Michael Tugan-Baranovsky, Ukraine

Conference participant, National championship in scientific analytics, Open European and Asian research analytics championship

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ON THE THREE INVARIANT RELATIONS OF MOTION'S EQUATIONS OF THE SYMMETRIC GYROSTAT IN A MAGNETIC FIELD

The problem of the gyrostat motion in a magnetic field with the effect of Barnett-London is considered. It is assumed that gyrostatic moment depends on the time. The conditions for the existence of the motion equations of three invariant relations of a special kind are defined. These solutions of the motion equations are characterized by the elliptic functions of time.

Keywords: symmetric gyrostat, invariant relation, magnetic field.

The problem of a gyrostat motion in a magnetic field with the Barnett-London effect describes the motion of neutral ferromagnet (not initially magnetized) in a magnetic field, which is the rotation of the magnetization along the axis of rotation, which is magnetized along the rotation axis in the rotation [1]. This effect is called the Barnett effect and is characterized by the appearance of the magnetic moment, which depends on the angular velocity. A similar phenomenon appears when rotating superconducting solid at fast rotation in the magnetic field (the London effect). The mechanism of magnetization in both cases due to various reasons, but the motion equations can be presented in the same form [1–3].

The motion equations of a gyrostat with variable gyrostatic moment in a magnetic field with the effect of Barnett-London have the form [1]

$$\dot{x} = x \times ax + \lambda\alpha \times ax - \dot{\lambda}\alpha + ax \times B\nu + s \times \nu + \nu \times C\nu,$$
$$\dot{\nu} = \nu \times ax. \qquad (1)$$

Here we introduce the notation: $x = (x_1, x_2, x_3)$ – a moment-of-momentum body-transmitter; $\nu = (\nu_1, \nu_2, \nu_3)$ – an unit vector indicating the direction of the magnetic field; $\lambda = \lambda(t)$ – the value gyrostatic moment $\lambda(t)\alpha$; $\alpha = (\alpha_1, \alpha_2, \alpha_3)$ – constant unit vector; $a = (a_{ij})$ – gyration tensor; $s = (s_1, s_2, s_3)$ – a constant vector; $B = (B_{ij})$, $C = (C_{ij})$ – constant symmetric matrix of the third order; point above the variables denotes differentiation at time.

A case is examined when $s = (s_1, 0, 0)$, $\alpha = (1, 0, 0)$, $a = \mathrm{diag}\,(a_1, a_2, a_3)$, $B = \mathrm{diag}\,(B_1, B_2, B_3)$, $C = \mathrm{diag}\,(C_1, C_2, C_3)$. Then from equations (1) will get expressions

$$(x_1 + \lambda + B_2\nu_1)^{\bullet} = 0, \qquad (2)$$

$$\dot{x}_2 = (a_1 - a_2)x_1x_3 - a_2\lambda x_3 + B_2a_2x_3\nu_1 - B_1a_1x_1\nu_3 - s_1\nu_3 + (C_1 - C_2)\nu_1\nu_3, \qquad (3)$$

$$\dot{x}_3 = -(a_1 - a_2)x_1x_2 + a_2\lambda x_2 + B_1a_1x_1\nu_2 - B_2a_2x_2\nu_1 + s_1\nu_2 + (C_2 - C_1)\nu_1\nu_2, \qquad (4)$$

$$\dot{\nu}_1 = a_2(x_3\nu_2 - x_2\nu_3),$$
$$\dot{\nu}_2 = a_1x_1\nu_3 - a_2x_3\nu_1,$$
$$\dot{\nu}_3 = a_2x_2\nu_1 - a_1x_1\nu_2. \qquad (5)$$

Equation (2) allows to specify the first integral of (2)–(5)

$$x_1 + \lambda + B_2\nu_1 = \alpha_0, \qquad (6)$$

here α_0 – an arbitrary constant.

Define invariant relations of equations (2)–(5) as

$$x_1 = b_0 + b_1\nu_1,$$
$$x_2 = c_0 + c_2\nu_2,$$
$$x_3 = d_0 + d_3\nu_3. \qquad (7)$$

Equality on the parameters of equations (1) and invariant relations (7)

$$c_0(a_1b_0 - a_2\alpha_0) = 0,$$
$$d_0(a_1b_0 - a_2\alpha_0) = 0, \qquad (8)$$

$$b_0a_1(d_3 - c_2 - B_1) - a_2d_3\alpha_0 - s_1 = 0, \qquad (9)$$

$$b_0a_1(d_3 - c_2 + B_1) + a_2c_2\alpha_0 + s_1 = 0, \qquad (10)$$

$$d_3(a_1b_1 + a_2c_2 + 2a_2B_2) - a_1b_1(c_2 + B_1) + C_1 - C_2 = 0, \qquad (11)$$

$$c_2(a_1b_1 + a_2d_3 + 2a_2B_2) - a_1b_1(d_3 + B_1) + C_1 - C_2 = 0, \qquad (12)$$

are the conditions for the existence of invariant relations (7).

Consideration of the case $a_1b_0 - a_2\alpha_0 \neq 0$ leads to the relations

$$\nu_2^2 = \frac{1}{c_2 - d_3} \times (B_2\nu_1^2 - \alpha_0\nu_1 + k_* - d_3),$$
$$\nu_3^2 = \frac{1}{c_2 - d_3} \times [(d_3 - c_2 - B_2)\nu_1^2 + \alpha_0\nu_1 + c_2 - k_*], \qquad (13)$$

$$\dot{\nu}_1 = a_2\sqrt{(B_2\nu_1^2 - \alpha_0\nu_1 + k_* - d_3) \ldots \to \ldots \to [(d_3 - c_2 - B_2)\nu_1^2 + \alpha_0\nu_1 + c_2 - k_*]}. \qquad (14)$$

Function $\nu_1(t)$ is found from integral

$$\int_{\nu_1^{(0)}}^{\nu_1} \frac{d\nu_1}{\sqrt{(B_2\nu_1^2 - \alpha_0\nu_1 + k_* - d_3) \ldots \to \ldots [(d_3 - c_2 - B_2)\nu_1^2 + \alpha_0\nu_1 + c_2 - k_*]}} = t - t_0, \qquad (15)$$

which is reduced to elliptic integral in the Legandre form. Thus, $\nu_1 = \nu_1(t)$ is an elliptic function of time. According to the formula

$$\lambda = \frac{b_0(2a_1 - a_2)}{a_2} - \frac{B_2}{a_1}(a_1 - a_2)\nu_1$$

$\lambda(t)$ is also an elliptic function of time. The other variables of the problem $\nu_2 = \nu_2(t)$, $\nu_3 = \nu_3(t)$, $x_i = x_i(t)$ $(i = \overline{1,3})$ can be determined respectively from (13), (7). The above functions describe a new solution of system (1).

To reduce the problem of integrating the motion equations in case $c_0^2 + d_0^2 \neq 0$ to quadrature we introduce new variables θ and φ instead ν_i

$$\nu_1 = \cos\theta,$$
$$\nu_2 = \sin\theta\cos\varphi,$$
$$\nu_3 = \cos\theta\sin\varphi. \qquad (16)$$

The equation

$$\int_{\theta_0}^{\theta} \frac{\sin\theta\, d\theta}{\sqrt{4a_2^2(c_0^2+d_0^2)\sin^2\theta - F^2(\theta)}} = \frac{1}{2}(t - t_0), \qquad (17)$$

is obtained for the finding of function $\theta = \theta(t)$. On the basis (16) one can determine a function $\varphi(t)$

$$\varphi(t) = \varphi_0 + \arcsin\frac{F(\theta(t))}{2a_2\sqrt{c_0^2+d_0^2}\,\sin\theta(t)}. \qquad (18)$$

The functions $\theta(t)$, $\varphi(t)$ in (17), (18) can use in (7), (16) to obtain the dependence main variables x_1, x_2, x_3, ν_1, ν_2, ν_3 from time. Because of the structure of formula (17), all of the functions are elliptic functions of time.

Thus, in the paper, we obtained conditions for the existence of the equations (1) of three linear invariant relations of a special form (7). Two classes partial solutions of motion equations, which are expressed in the form of elliptic functions of time, are specified.

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