Determination of critical load of elastic steel column based on experimental data

Sergii Bilyk

Kyiv National University of Construction and Architecture
Povitroflotskyy prosp., 31, Kyiv, Ukraine, 03037
vartist@ukr.net, orcid.org/0000-0001-8783-5892

Summary. We have developed a general approach for assessing critical-strength for the central-compressed steel elements, what have an initial imperfections. The metod is made on the basis of experimental data. The method is a development of the method Tymoshenko-Southwell. In work we show of the solutions several tasks. We demonstrate the possibility to determine of critical-strength for the central-compressed steel elements on the basis of experimental data initial geometric imperfections. The results can be used at checking the technical condition of the Central compressed rod in the survey metal structures trusses, columns, structural designs.

Key words: steel structures, stability, buckling, steel elements, slenderness, reduction factor for buckling, initial deflection, residual stresses initial imperfections, the critical force, method Tymoshenko-Southwell.

INTRODUCTION

The fundamental the methodology of buckling of columns is developed the hundreds years and in solving of the problem to determine of critical-strength for central-compressed steel elements is research [2 – 7].

Separate important task to determine the stability of the rods is to analyze the influence of initial imperfections [1, 8 – 11].

With the technical inspection of steel structures no data on initial deflection longitudinal bending of the compressed elements with initial imperfections [13, 14 – 17]. So it is important improve the method of finding of critical load the elements by measurement of deflection of the strut. [18 – 22]. This methodic makes it possible to determine the critical force based on experimental data.

Analysis of experimental data centrally compressed bars dedicated a number of outstanding works [3, 5, 7, 10, 13].

In the article the theoretical studies analyzing experimental data of elastic central-compressed steel elements with initial geometric imperfections.

PURPOSE AND METHODS

This methodology of buckling of columns with initial imperfections is based upon of research Tymoshenko S.P. and Southwell R.V. [2, 7].

But there is a need on synthesis methodology for the analysis of experimental data centrally compressed bars considering initial geometric imperfections.

The purpose of research, which set out in Article, is generalization theoretical approach to the analysis of the stability of the central-compressed steel element with the initial geometric imperfections. Research Methods based on analytical studies of centrally compressed rod with initial imperfections.

The initial imperfections is initial geometric imperfections, initial deflections, random eccentricities application of longitudinal force. So buckling centrally compressed rod with initial imperfections we see to as a deformation the noncentral-compressed element.

RESULTS AND DISCUSSION

The general equation buckling noncentral-compressed element has recording in the form of linear differential equation:

$$\eta^{//}+k^2\eta = -k^2\cdot(\delta_{f0}\sin(\pi z/l) + e_b), \quad (1, a)$$

де $k^2 = Nl^2/(EI_x)$.

Differential equation (1, *a*) is second order liear nonhomogeneous differential equation. The general solution of this differential equation (1, *a*) has unknown factors that depend on the boundary conditions of strut.

The general solution of the second order nonhomogeneous linear equation can be writing in the form.

$$\eta = C_1 f_m \sin(kz/l) + C_2 f_m \cos(kz/l) - C_3\delta_{f0}\sin(\pi z/l) - e_b. \quad (1, b)$$

In (1, *b*) $\eta_{gs} = C_1 f_m \sin(kz/l) + C_2 f_m\cos(kz/l)$ is a general solution of the corresponding homogeneous equation, $\eta_{ps} = -C_3\delta_{f0}\sin(\pi z/l) - e_b$, is any specific function (particular solution) that satisfies the nonhomogeneous equation (1, *a*).

Special function η_{ps} can be represented as the sum of two functions:

$$\eta_{ps} = \eta_{ps\delta} + \eta_{psb}. \quad (1, c)$$

Partial solution $\eta_{ps\delta}$ to the differential equation (1, *a*) provides a formula for determining factor C_3.

$$\eta_{ps\delta} = C_3\delta_{f0}\sin(\pi z/l)$$

$$\eta^{//} + k^2\eta = -k^2\cdot\delta_{f0}\sin(\pi z/l),$$

$$-\pi^2 C_3\delta_{f0}\sin(\pi z/l) + k^2 C_3\delta_{f0}\sin(\pi z/l) =$$

$$= -k^2\cdot\delta_{f0}\sin(\pi z/l) - \pi^2 C_3 + k^2 C_3 = -k^2\cdot\delta_{f0}.$$

$$C_3 = -\frac{k^2}{-\pi^2+k^2} = \frac{1}{\frac{\pi^2}{k^2}-1}. \quad (1, d)$$

For element, which has boundary conditions of: hinged – hinged, and which load is noncentral compressing, we have decision without initial bend initial imperfections.

$$\eta = C_1 f_m\sin(kz/l) + C_2 f_m\cos(kz/l) - e_b. \quad (2, a)$$

Boundary conditions of: hinged – hinged is in form writing.

$$z = 0,\ \eta_0 = 0.\ z = l,\ \eta_0 = 0.$$

These boundary conditions give a system of linear inhomogeneous equations and can be expressed in the form.

$$C_2 f_m - e_b = 0,$$
$$C_1 f_m\sin(k) + C_2 f_m\cos(k) - e_b = 0. \quad (2, b)$$

Accordingly, it coefficients is.

$$C_1 f_m = e_b\left[\frac{1-\cos(k)}{\sin(k)}\right]. \quad (2, c)$$
$$C_2 f_m = e_b.$$

The general solution second order linear nonhomogeneous differential equations (1, *a*) will be.

$$\eta = e_b\left[\frac{1-\cos(k)}{\sin(k)}\right]\sin\left(\frac{kz}{l}\right) + e_b\left[\cos\left(\frac{kz}{l}\right)-1\right] + \frac{\delta_{f0}}{\frac{\pi^2}{k^2}-1}. \quad (2, d)$$

When coordinate $z/l = 1/2$ we have maximum displacement of the middle section of the column.

$$\eta_{z/l=1/2} = e_b\left[\frac{1-\cos(k)}{\sin(k)}\right]\sin\left(\frac{k}{2}\right) + e_b\left[\cos\left(\frac{k}{2}\right)-1\right] + \frac{\delta_{f0}}{\frac{\pi^2}{k^2}-1}.$$

$$\eta_{z/l=1/2} = e_b\left[\frac{1-\cos(k)+2\cos^2\left(\frac{k}{2}\right)-2\cos\left(\frac{k}{2}\right)}{2\cos\left(\frac{k}{2}\right)}\right] + \frac{\delta_{f0}}{\frac{\pi^2}{k^2}-1}.$$

$$\eta_{z/l=1/2} = e_b\left[\frac{1-\cos\left(\frac{k}{2}\right)}{\cos\left(\frac{k}{2}\right)}\right] + \frac{\delta_{f0}}{\frac{\pi^2}{k^2}-1}.$$

$$\eta_{z/l=1/2} = e_b\left[\frac{1}{\cos(k/2)}-1\right]+\frac{\delta_{f0}}{\frac{\pi^2}{k^2}-1}. \quad (3, a)$$

The last formula (3, *a*) is the formula secant and it is combined with the formula for the calculation of the growth-initial deflections at longitudinal bending. Taylor's theorem gives an approximation value secant by order Taylor polynomial:

$$\frac{1}{\cos(kl/2)}-1=\frac{1}{\frac{8}{k^2}-1}\approx\frac{1}{\frac{\pi^2}{k^2}-1}. \quad (3, b)$$

Finally, the formula of the maximum deflections of the middle section of the metal element has record.

$$\eta_{z=l/2}=\frac{e_b+\delta_{f0}}{\frac{\pi^2}{k^2}-1}. \quad (3, c)$$

$$\eta_{z=l/2}=\frac{e_b+\delta_{f0}}{\frac{\pi^2 EI_x}{l^2 N}-1}. \qquad N_{cr}=\frac{\pi^2 EI_x}{l^2}$$

$$\eta_{z=l/2}=\frac{e_b+\delta_{f0}}{\frac{N_{cr}}{N}-1}. \quad (4)$$

E – Modulus of elasticity in tension,

I_x – Quadratic moment of inertia,

l – Reduced (effective) strut length, equivalent length of column.

N_{cr} – Euler's buckling load.

The same conclusion can be justified using the approach in [2, 3, 7] for any boundary conditions of the column.

Record the total-solutions of the corresponding homogeneous second order differential equation ($EI_x\eta^{//} + N\eta = 0$) has this form.

$$\eta_{sn}=c_{s1n}\sum_{n=1}^{\infty}\sin\frac{n\pi z}{l}+\sum_{n=1}^{\infty}c_{s2n}\cos\frac{n\pi z}{l}. \quad (5)$$

If the element has initial geometric imperfections, it is in part the decision of general differential equation of longitudinal bending column $EI_x\eta^{//} + k^2N\eta = -k^2(\delta_{f0} + e_b)$ can also be represented as a series of trigonometric functions.

Since the initial geometric imperfections are common to the rod, its record is different from the coefficients of trigonometric functions and will vary by operator a_{s0n}.

$$\eta_{e0}=\sum_{n=1}^{\infty}c_{s0n}\sin\frac{n\pi z}{l}+\sum_{n=1}^{\infty}a_{s0n}c_{s0n}\cos\frac{n\pi z}{l}. \quad (6)$$

Substituting these solutions to the differential equation (1, *a*), that describes the stability of the element.

If you equate the coefficients of the same trigonometric functions in equation (6), then you get a recurrence formula to determine the coefficients of the trigonometric functions, which describe the different possible harmonics deflections of the rod at the loss of stability.

$$c_{s1n}=\frac{Nc_{s0n}}{\left(\frac{\pi^2 n^2 EI_x}{l^2}-1\right)};\quad N_n=\frac{\pi^2 n^2 EI_x}{l^2};$$

$$c_{s1n}=\frac{c_{s0n}}{\left(\frac{N_n}{N}-1\right)};\ c_{s2n}=\frac{a_{s0n}c_{s0n}}{\left(\frac{N_n}{N}-1\right)}. \quad (7, a)$$

Full deflection of the column will be equal to the amount of deflection longitudinal bending and of additional longitudinal deflection from the initial bend initial imperfections.

$$\eta_s=\eta_{gs}+\eta_{e0}$$

$$\eta_s=\sum_{n=1}^{\infty}\left[\frac{c_{s0n}}{\left(\frac{N_n}{N}-1\right)}+c_{s0}\right]\sin\frac{n\pi z}{l}+$$
$$+\sum_{n=1}^{\infty}a_{s0n}\left[\frac{c_{s0n}}{\left(\frac{N_n}{N}-1\right)}+c_{s0n}\right]\cos\frac{n\pi z}{l}. \quad (7, b)$$

Then have

$$\eta_s = \sum_{n=1}^{\infty} \frac{c_{s0n}}{1-\frac{N}{N_n}} \sin\frac{n\pi z}{l} + \sum_{n=1}^{\infty} \frac{a_{s0n}c_{s0n}}{1-\frac{N}{N_n}} \cos\frac{n\pi z}{l}. \quad (8)$$

Problem 1. Maximum displacement determine for the central-compressed steel element considering initial geometric imperfections. The rod has boundary conditions: hinged – hinged (column pivoted in both ends). These conditions give the formula.

$$\eta_{sz=0} = 0;\ \eta_{sz=l} = 0;$$

$$\eta_s = \eta_{sn} + \eta_{e0} = \sum_{n=1}^{\infty} \frac{a_{s0n}c_{s0n}}{1-\frac{N}{N_n}} = 0 \rightarrow a_{s0n} = 0. \quad (9, a)$$

For the cross section, which has a maximum displacement, $z=l/2$, we have.

$$\eta_{smax} = \sum_{n=1}^{\infty} \frac{c_{s01}}{1-\frac{N}{N_1}} \sin\frac{\pi}{2} + \frac{c_{s02}}{1-\frac{N}{N_2}} \sin\pi + \frac{c_{s03}}{1-\frac{N}{N_3}} \sin\frac{3\pi}{2} + \frac{c_{s03}}{1-\frac{N}{N_3}} \sin\frac{3\pi}{2} + ... \quad (9, b)$$

Members in (9, *b*) with even indices disappear, a true formula is.

$$\eta_{smax} = \frac{c_{s01}}{1-\frac{N}{N_1}} - \frac{c_{s03}}{1-\frac{N}{N_3}} + \frac{c_{s05}}{1-\frac{N}{N_5}}. \quad (10)$$

Coefficients c_{s0i} accept the results of measurements of samples rods. For rod, that has hinged support in both ends, a first harmonic is a dominant, and it's the sinusoidal shape. Now the maximum displacement will depend on the ratio of the current force and of the critical load.

$$\eta_{smax} = \frac{c_{s01}}{1-\frac{N}{N_{cr}}};\ N_{cr} = \frac{\pi^2 EI_x}{l^2}. \quad (11)$$

This decision is important for determine the maximum deflection rod for any value of the longitudinal force provided $N<N_{cr}$.

Problem 2. Determine the initial geometrical imperfections at a certain meaning full load and moving of the middle cross section of the rod. This is the inverse problem to problem 1.

We know of longitudinal force N_i, of critical force N_{cr} and we know displacement of rod, it is possible to determine the initial geometric imperfections.

$$c_{s01} = \eta_s \left(1 - \frac{N}{N_{cr}}\right). \quad (12)$$

Problem 3. Method is of determining additional deflection η_{sf}. Since the total deflection consists of initial deflection and deflection of buckling, we have a right to record this at $\eta_{e0} = c_{s01.}$

$$\eta_{sf} = \eta_{smax} - \eta_{e0} = \frac{c_{s01}}{1-\frac{N}{N_{cr}}} - c_{s01.}$$

$$\eta_{sf} = \frac{N_{cr}c_{s01} - N_{cr}c_{s01} + Nc_{s01}}{N_{cr} - N} = \frac{c_{s01}}{\frac{N_{cr}}{N} - 1}. \quad (13)$$

$$\eta_{sf} = \frac{c_{s01}}{\frac{N_{cr}}{N} - 1}. \quad (14)$$

Equation (14) coincides with the formula (4). Thus, the approach shows the relationship between the method of Timoshenko S.P. and the method of Southwell R.V.

Problem 4. Determination of initial imperfections elastic buckling element and its of critical load. A few experimental data of additional deflections (bending) the central-compressed element is: η_{s1}, η_{s2}, η_{s3}. Corresponding values of compressive strength we know too. We proposed the following sequence calculation and determination of initial geometric imperfections of column and definition of critical force.

According to the formula (14) we can compose system of two a linear equations with two unknown variable members. The first unknown member is variable parameter of bending*:* η_{s1}, η_{s2},, the second unknown members is the critical load buckling shapes of the column.

$$\begin{cases} \eta_{s1} = \dfrac{c_{s01}}{\dfrac{N_{cr}}{N_1} - 1} \\ \eta_{s3} = \dfrac{c_{s01}}{\dfrac{N_{cr}}{N_3} - 1} \end{cases} . \rightarrow \begin{matrix} c_{s01} = \eta_{s1}\left(\dfrac{N_{cr}}{N_1} - 1\right) \\ c_{s01} = \eta_{s3}\left(\dfrac{N_{cr}}{N_3} - 1\right) \end{matrix} \quad (15,\ a)$$

Divide the first equation of the system (15, *a*) on the second equation and thereby we exclude the unknown parameter.

$$\frac{\eta_{s1}}{\eta_{s3}} = \frac{\dfrac{N_{cr}}{N_3} - 1}{\dfrac{N_{cr}}{N_1} - 1} \rightarrow \frac{\dfrac{N_{cr} - N_3}{N_3}}{\dfrac{N_{cr} - N_1}{N_1}} = \frac{N_1\left(N_{cr} - N_3\right)}{N_3\left(N_{cr} - N_1\right)} .$$

Next, solution of system linear algebraic equations (15, *a*) leads to linear equation with one unknown parameter: N_{cr}.

$$\left(N_{cr} - N_1\right)N_3\eta_{s1} = N_1\eta_{s3}\left(N_{cr} - N_3\right).$$

$$\frac{N_{cr}}{N_1}\left(\frac{N_1}{N_3}\frac{\eta_{s3}}{\eta_{s1}} - 1\right) = \left(\frac{\eta_{s3}}{\eta_{s1}} - 1\right). \quad (15,\ b)$$

Solution of linear algebraic equation (15, *b*) can be represented by.

$$N_{cr} = \frac{N_1\left(\dfrac{\eta_{s3}}{\eta_{s1}} - 1\right)}{\left(\dfrac{N_1}{N_3}\dfrac{\eta_{s3}}{\eta_{s1}} - 1\right)}; N_{cr} = \frac{N_3\left(1 - \dfrac{\eta_{s1}}{\eta_{s3}}\right)}{\left(1 - \dfrac{N_3\eta_{s1}}{N_1\eta_{s3}}\right)}. \quad (15,\ c)$$

According to the formula (15, *c*) can calculate the upper critical load, but values lower the critical load is calculated by the formula (15, *d*), what is obtained using other experimental data.

$$\frac{N_{cr}}{N_2}\left(\frac{N_2}{N_3}\frac{\eta_{s3}}{\eta_{s2}} - 1\right) = \left(\frac{\eta_{s3}}{\eta_{s2}} - 1\right)$$

$$N_{cr} = \frac{N_2\left(\dfrac{\eta_{s3}}{\eta_{s2}} - 1\right)}{\left(\dfrac{N_2}{N_3}\dfrac{\eta_{s3}}{\eta_{s2}} - 1\right)}; N_{cr} = \frac{N_3\left(1 - \dfrac{\eta_{s2}}{\eta_{s3}}\right)}{\left(1 - \dfrac{N_3}{N_2}\dfrac{\eta_{s2}}{\eta_{s3}}\right)}. \quad (15,\ d)$$

The initial geometric imperfection will be writing.

$$\eta_{s01} = c_{s01} = \eta_{s1}\left(\frac{\left(\dfrac{\eta_{s3}}{\eta_{s1}} - 1\right)}{\left(\dfrac{N_1}{N_3}\dfrac{\eta_{s3}}{\eta_{s1}} - 1\right)} - 1\right) \quad (16)$$

According to the formula (12) maximum additional deflection $\eta_{sf\max}$ of the column excluding initial imperfections look like so.

$$\eta_{sf\max} = \frac{\eta_{s01}}{\dfrac{N_{cr}}{N_2} - 1}. \quad (17)$$

Total deflection of critical load of the loss of stability element we can to calculate at formula.

$$\eta_{smax} = \eta_{sf\max} + \eta_{s01}.$$

$$\eta_{smax} = \eta_{s1}\left(\frac{\left(\dfrac{\eta_{s3}}{\eta_{s1}} - 1\right)}{\left(\dfrac{N_1}{N_3}\dfrac{\eta_{s3}}{\eta_{s1}} - 1\right)} - 1\right)\left(\frac{1}{\dfrac{N_{cr}}{N_2} - 1} + 1\right). \quad (18,\ a)$$

$$\eta_{smax} = \eta_{s1}\left(\frac{\left(\dfrac{\eta_{s3}}{\eta_{s1}} - 1\right)}{\left(\dfrac{N_1}{N_3}\dfrac{\eta_{s3}}{\eta_{s1}} - 1\right)} - 1\right)\left(\frac{1}{1 - \dfrac{N_2}{N_{cr}}}\right). \quad (18,\ b)$$

Thus generalized approach makes it possible to identify critical load and initial geometric deflections on the results of a small number of experimental data.

Problem 5. Equation (15, *b*) has a detailed record.

$$\frac{N_{cr}}{N_1}\left(\frac{N_1}{N_3}\frac{\eta_{s3}}{\eta_{s1}}-1\right)=\left(\frac{\eta_{s3}}{\eta_{s1}}-1\right). \qquad (19)$$

We introduce notation for relationship experimental data of deflection of column $-\zeta_\eta$, and attitude critical force of the column to the value longitudinal force of the column at testing – p_N, that corresponds derived displacement.

$$\frac{\eta_{s3}}{\eta_{s1}}=\zeta_\eta^2; \qquad \frac{N_{cr}}{N_1}=p_N^2 . \qquad (20)$$

We obtain the equation, which connects two parametric functions: deflections and critical load.

$$p_N^2-\frac{N_3}{N_1}p_N^2\zeta_\eta^2+\zeta_\eta^2=1. \qquad (21)$$

This is the equation (21) is the hyperbolic function, and is the universal equation of physical and mathematical model describing the loss of the stability column.

We determine the asymptotes of the hyperbolic function and use this entry.

$$p_N^2=\frac{1-\frac{1}{\zeta_\eta^2}}{\left(\frac{N_1}{N_3}-\frac{1}{\zeta_\eta^2}\right)}. \qquad (22)$$

Thus, when $\zeta_\eta^2\to\infty$, we has the coordinate of the horizontal asymptote.

$$p_N^2=\frac{1-\frac{1}{\zeta_\eta^2}}{\left(\frac{N_1}{N_3}-\frac{1}{\zeta_\eta^2}\right)}\to\zeta_\eta^2\to\infty\to p_N^2\to\frac{N_{\zeta_\eta^2\to\infty}}{N_1}.$$

If we have in sufficient quantity of experimental results, the experimental value of the critical force column at loss buckling will approach to the theoretical value of the maximum critical load at deflections growth to infinite.

$$N_{cr}\to N_{cr\max}$$

Due to our research is calculated the upper limit of critical load and the lower limit of critical load and we can identify difference between their values to determine the nature of the buckling columns. If the difference between the upper and lower values of critical load is small, it means that the lost stability of the column is elastic. If the difference between the upper and lower the critical load is big, it we have to the conclusion that the buckling took place with the development of limited plastic deformation, it is inelastic buckling.

The research results are important when analyzing the results of the inspection of metal structures for civil use and engineering facilities operating in difficult conditions, surface and underwater structures [23].

CONCLUSIONS

In experimental studies of stability the elements always buckling occurs suddenly and the maximum critical force will not to determined indeed.

Fixing critical load and corresponding strain measurement is always the difficulty, since the loss of stability takes place fast at increasing deflections. When examining and monitoring structures we have the ability to fix the deflections of rods under load, it the deflections and force is not close to critical.

Therefore, it the research allows to define the critical load and initial imperfection by the analysis of data relationships deflections and of the load, which is less than the critical load.

It is shown that the maximum value and the actual deflections of elastic elements with initial imperfections at longitudinal bending, which will be defined by the methodology Tymoshenko S.P. and by the approach Sausvella R.V. are identical.

We have shown that the method Timoshenk-Sausvella can be used in the analysis of experimental results of research elastic buckling of columns and inelastic buckling of columns

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Определение критической нагрузки упругих колонн по экспериментальным данным

Сергей Билык

Киевский национальный университет строительства и архитектуры
Воздухофлотский просп, 31, Киев
Украина, 03037, vartist@ukr.net
orcid.org/0000-0001-8783-5892

Аннотация. Разработан обобщенный подход для оценки критической и силы стальных центрально-сжатых стержней с начальными геометрическими несовершенствами и с учетом использования экспериментальных данных. Предоставлено обоснование теоретического подхода. В статье обобщается и развивается методологический подход для определения критической нагрузки предложенный Тимошенко С.П. и Southwell R.V. При определении максимальных прогибов при потере устойчивости упругих стержней с учетом начальных несовершенств и случайных эксцентриситетов показано тождество обоих подходов. Приведены решения нескольких задач. Показана возможность определять на основании экспериментальных данных начальные геометрические несовершенства. Результаты работы могут быть использованы при проверке технического состояния центрально-сжатых стержней при обследовании металлических конструкций ферм, колонн, структурных конструкций.

Ключевые слова: стальные конструкции, устойчивость, начальные несовершенства, критическая сила, метод Тимошенко-Саусвелла.