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The realization of higher-dimensional breaking mechanism

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Abstract We study D-branes on Calabi-Yau threefolds, which are realized through the blowing up the singularity of orbifold. This D-branes are represented as sheaves, which can be stable or unstable, what is connected with the transition in the Teichm*ü*ller space. Using the derived category of McKay quiver representations, which describe D-branes as quivers and open superstrings between them by Ext groups, we can represent Higgs multiplets by the moduli space of an open superstring, connecting two McKay quivers. Through the equivalence between the derived category of coherent sheaves and triangulated category of distinguished triangles over the abelian category of McKay quivers we can associate D-branes with quivers or with sheaves, defined on Calabi-Yau. After the dimensional reduction of the ten-dimensional space-time we can receive matter content of the four-dimensional space-time. Thus, a higher-dimensional breaking mechanism is associated with four-dimensional GUT Higgs multiplets and symmetry breaking higgs mechanism.

Keywords Triangulated category· McKay quiver representations· Higgs multiplets

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1 Introduction

The definition of D-branes as allowed endpoints for open strings [1], generalizes the notion of quarks on which the QCD string can terminate. In contrast to the quarks of QCD, D-branes are intrinsic excitations of the fundamental theory. D-particles can probe distances much smaller than the size of the fundamental string quanta. D-branes played a crucial role in the 'second string revolution'ex, the way to reconcile quantum mechanics and Einstein gravity. The D-brane concept [2], [3] is powerfull because of the relations between supersymmetric gauge theories and geometry.

The purpose of our article is connected with the searches of a higher-dimensional breaking mechanism in the context of D-branes, which is connected or associated with four-dimensional Grand Unification Theory Higgs multiplets and symmetry breaking higgs mechanism.

2 The Category of D-branes as Derived Category of Coherent Sheaves

Due to the important development in string theory through the discovery of D-branes we can use a compactification model: the string theory has a target space $R^{1,3} \times X$ for compact space X and focus on X. Let X be a topological space. On such space we can construct locally free sheaf $\mathcal{E}$. If we have embedding $i : S \to X$ and a sheaf $\mathcal{E}$ on S, than we can define a sheaf $i_* \mathcal{E}$ on X, through the following construction: a map $f : X \to Y$ between two algebraic varieties and a sheaf $\mathcal{F}$ on X define the sheaf $f_* \mathcal{F}$ on Y by $f_* \mathcal{F}(U) = \mathcal{F}(f^{-1}U)$. For embedding $i : X \to P^n$ the sheaf $i_* \mathcal{E}$ is given by $i_* \mathcal{E}(U) = \mathcal{E}(U \cap P^n)$ for all open subsets $U \subset P^n$. This embeds the sheaves on X into the sheaves on P^n. So we have locally free sheaf on P^n, which is associated with the D-brane.

From [4] it follows that more generally we can consider a complex of locally-free sheaves:

$$\dots \xrightarrow{d_{n-1}} \mathcal{E}^n \xrightarrow{d_n} \mathcal{E}^{n+1} \xrightarrow{d_{n+1}} \mathcal{E}^{n+2} \xrightarrow{d_{n+2}} \dots,$$

where morphisms $d_n : \mathcal{E}^n \to \mathcal{E}^{n+1}$, $d_n \in \mathrm{Ext}^0(\mathcal{E}^n, \mathcal{E}^{n+1}) = \mathrm{Hom}(\mathcal{E}^n, \mathcal{E}^{n+1})$, $d_{n+1} d_n = 0$ for all n are morphisms between locally-free sheaves $\mathcal{E}^n$ and $\mathcal{E}^{n+1}$. For a further aim we must use the notion of a category.

A category $\mathcal{L}$ consists of the following data:

1) A class Ob $\mathcal{L}$ of objects $A, B, C, \cdots$;

2) A family of disjoint sets of morphisms $\mathrm{Hom}(A, B)$ one for each ordered pair A, B of objects;

3) A family of maps

$$\mathrm{Hom}(A,B)\times\mathrm{Hom}(B,C)\to\mathrm{Hom}(A,C)\ ,$$

one for each ordered triplet A, B, C of objects.

These data obey the axioms:

a) If $f\ : A\to B,\ g\ : B\to C,\ h\ : C\to D$, then composition of morphisms is associative, that is, $h(gf)=(hg)f$;

b) To each object B there exists a morphism $1_B\ :\ B\to B$ such that $1_B f=f\ ,\ g1_B=g$ for $f\ : A\to B$ and $g\ : B\to C$.

Thus, the category of D-branes is the derived category of locally-free sheaves. Locally-free sheaves and morphisms don't form an abelian category. So we should replace the category of locally-free sheaves by the abelian category of coherent sheaves. Abelian category is characterized by the existence of an exact sequences. We can form the category of D-branes, which is the derived category of coherent sheaves $D(X)$. On a smooth space X, any coherent sheaf $\mathcal{A}$ has a locally-free resolution

$$0\to\mathcal{F}^{-3}\to\mathcal{F}^{-2}\to\mathcal{F}^{-1}\to\mathcal{F}^{0}\to\mathcal{A}\to 0\ ,$$

where $\mathcal{F}^k$ is locally free. This is a quasi-isomorphism $\mathcal{F}^\bullet\to\mathcal{A}$ between a complex of locally-free sheaves and a coherent sheaf. Thus, an abelian category of coherent sheaves $D(X)$ of X consists of objects $\mathcal{E}^\bullet$ - exact complexes of sheaves:

$$\ldots\xrightarrow{d_{n-2}}\mathcal{E}^{n-1}\xrightarrow{d_{n-1}}\mathcal{E}^{n}\xrightarrow{d_{n}}\mathcal{E}^{n+1}\xrightarrow{d_{n+1}}\ldots$$

and morphisms between them $\mathcal{E}^\bullet\to\mathcal{F}^\bullet$:

$$\begin{array}{ccccccccc}
\ldots & \xrightarrow{d_{n-2}} & \mathcal{E}^{n-1} & \xrightarrow{d_{n-1}} & \mathcal{E}^{n} & \xrightarrow{d_{n}} & \mathcal{E}^{n+1} & \xrightarrow{d_{n+1}} & \ldots \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
\ldots & \xrightarrow{d_{n-2}} & \mathcal{F}^{n-1} & \xrightarrow{d_{n-1}} & \mathcal{F}^{n} & \xrightarrow{d_{n}} & \mathcal{F}^{n+1} & \xrightarrow{d_{n+1}} & \ldots
\end{array}$$

3 Triangulated Category and Central Charge

For physical purposes we will work in future with Calabi-Yau threefolds X. According to [4], if X and Y are mirror Calabi-Yau threefolds then the derived category $D(X)$ is equivalent to the triangulated category $\mathrm{Tr}\mathcal{F}(Y)$. So, in such category $D(X)$ objects are distinguished triangles:

$$\begin{array}{ccc} & C & \\ {}_{[1]}\swarrow & & \nwarrow \\ A & \xrightarrow{f} & B \end{array} \qquad C = \mathrm{Cone}(f) \tag{1}$$

and morphisms of this category are morphisms of distinguished triangles [4]. Than the derived category is additive category, where exact sequences are exchanged by distinguished triangles. According to Douglas [5], [6] instead of physical D-branes, living in the boundary conformal field theory, we can work with topological D-branes, and the relationship between them is the notion of Π-stability. A topological D-brane is physical if it is Π-stable. For the precise definition of Π-stability we must compute the central charge of objects $\mathcal{E}^\bullet$ in $D(X)$:

$$Z(\mathcal{E}^\bullet) = \int_X e^{-(B+iJ)} ch(\mathcal{E}^\bullet)\sqrt{td(X)},$$

where $B + iJ$ is the complexified Kähler form. For a given $\mathcal{E}^\bullet$ we may choose a grading $\xi(\mathcal{E}^\bullet)$:

$$\xi(\mathcal{E}^\bullet) = \frac{1}{\pi}\arg Z(\mathcal{E}^\bullet) \pmod 2.$$

If we have a distinguished triangle in $D(X)$ of the form (1) with A and B stable, than C is stable with respect to the decay represented by this triangle if and only if $\xi(B) \prec \xi(A) + 1$. Also, if $\xi(B) = \xi(A) + 1$ then C is marginally stable and we may state that

$$\xi(C) = \xi(B) = \xi(A) + 1.$$

We may generalize this to the case of decays into any number of objects. For any object E we may define the set of distinguished triangles

$$\begin{array}{ccccccc} E_0 & \longrightarrow & E_1 & \longrightarrow & \ldots & \longrightarrow E_n & = E \\ {}_{[1]}\nwarrow & \swarrow & {}_{[1]}\nwarrow & \swarrow & {}_{[1]}\nwarrow & \swarrow & \\ & A_1 & & A_2 & & A_n & \end{array} \tag{2}$$

Then E decays into $A_1, A_2, \ldots A_n$ so long as

$$\xi(A_1) \succ \xi(A_2) \succ \ldots \xi(A_n).$$

As we have pointed out, X and Y are mirror Calabi-Yau threefolds. According to [7] mirror map is defined between the complex moduli space of a Calabi-Yau manifold X and the Kähler moduli space of its mirror manifold Y. So we can work with Kähler moduli space, that is characterized by the complexified Kähler form $B + iJ$. Then we can use the Teichmüller space $\mathcal{T}$ as the universal cover of the moduli space of $B + iJ$ and we expect the set of stable D-branes to be well-defined at any point in $\mathcal{T}$.

The following rules are applied:

• We begin with a stable set of D-branes with value of grading ξ for each D-brane.

• During the moving along a path in moduli space the gradings will change continuously.

• Two stable D-branes may bind to form a new stable state.

• A stable D-brane may decay into other stable states.

So, we can write **Conjecture** from [4].

Conjecture. At every point in the Teichmüller space of $B+iJ$ there is a set of stable objects in $D(X)$ such that every object E can be written in the form (2) for some n and for stable objects A_k.

Every object in $D(X)$ is stable or unstable for a given point in the Teichmüller space of $B+iJ$. Thus the open string corresponding to f in (1) go from tachyonic to massive as we pass in the Teichmüller space.

Now as we work in ten-dimensional space-time $R^{3+1} \times \mathbb{C}/G$ and knowing that after blowing up the singularity of additional six-dimensional space-time - orbifold $\mathbb{C}/Z_3 \rightarrow \mathcal{O}_{P_2}(-3)$ we have the sheaf $\mathcal{O}_{P_2}(-3)$ on two-dimensional projective space P_2. Here we must consider **Theorem** from [4]:

Theorem. Suppose X is a smooth resolution of the orbifold $\mathbb{C}/G$ with G a finite subgroup of $SU(d)$ and $d \leq 3$. Then the derived category $D(X)$ is equivalent to the derived category of G-equivariant sheaves on $\mathbb{C}^d$.

Now we are dealing with the derived category of sheaves on P_2 and we can use the statement [4]: D-branes on the orbifold $\mathbb{C}/G$ and open strings between them are described by the derived category of McKay quiver representations.

In future we will work with the derived category of distinguished triangles over the abelian category of McKay quivers. Objects of this category are distinguished triangles (Figure 1)

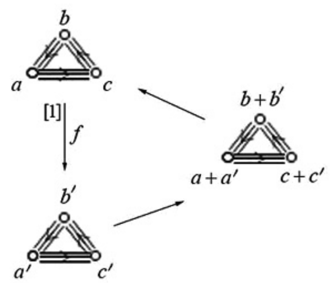

Fig.1. *Construction of distinguished triangle.*

(numbers a, b, c and a', b', c' denote orbifold charges characterizing McKay quivers); morphisms of this category are morphisms of distinguished triangles.

4 Grand Unification Theory Breaking and Dimensional Reduction

Our further work will be connected with the efforts to take many attractive features of the basic Grand Unification Theory and implement this ideas in four-dimensional models, for example, in the minimal four-dimensional supersymmetric SU(5) GUT with standard Higgs content. Moreover, because no appropriate four-dimensional GUT Higgs field is typically available to break the GUT group to the Standard Model gauge group, it is necessary to employ a higher-dimensional breaking mechanism. For type IIB theories, the corresponding vacua are realized as compactifications of F-theory on Calabi-Yau fourfolds. We will consider the left part of Figure 2 of the general overview of how GUT breaking constrains the type of GUT model [7].

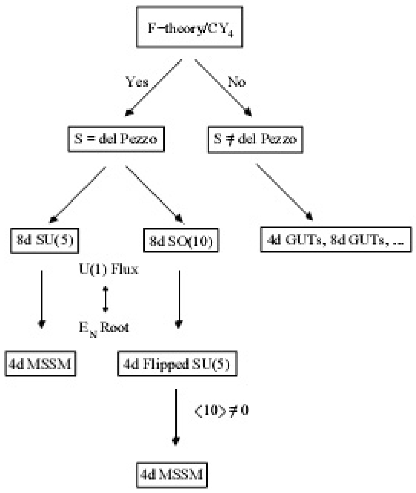

Fig.2. *General overview of how GUT breaking constrains the type of GUT model.*

We will consider elliptically fibered Calabi-Yau fourfold X with the base B and elliptic fiber ε:

$$\begin{array}{ccc} \varepsilon & \rightarrow & X \\ & & \downarrow \\ & & B \end{array}$$

This Calabi-Yau fourfold can be represented by the Figure 3.

Dimension	Space	Ingredient
10	$R^{3+1} \times B$	Gravity
8	$R^{3+1} \times S$	Gauge Theory
6	$R^{3+1} \times \Sigma$	Chiral Matter
4	R^{3+1}	Cubic Interaction Terms

Table 1 Dimensional reduction and matter content of the corresponding space-time.

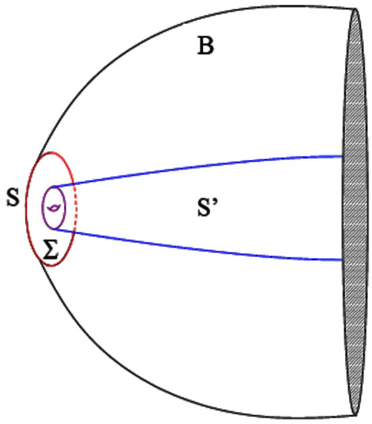

Fig.3. *Depiction of F-theory compactified on a local model of a Calabi-Yau fourfold.*

We shall assume that there exists a Calabi-Yau fourfold which contains the corresponding local enhancement in singularity type. When a del Pezzo surface S intersect $S^{'}$ on a Riemann surface Σ, the singularity type enhances further. In this case, additional six-dimensional hypermultiplets localize along Σ. In terms of four-dimensional superfields, the matter content, localized on a curve Σ, consists of chiral superfields. Schematically this can be represented by the Table 1.

As we can see, we have two points of view, connected with Calabi-Yau fourfolds. One is that Calabi-Yau is the sheaf on P_2 and as the object in $D(X)$ is stable or unstable in the Teichm$\ddot{u}$ller space of $B + iJ$. The other is that it is the fibered bundle, that can be reduced to four-dimensional theory with the corresponding matter content. After implementation of a higher-dimensional breaking mechanism to obtain four-dimensional models, we can receive the minimal four-dimensional supersymmetric $SU(5)$ Grand Unification Theory with standard Higgs content.

The moduli space of an open superstring [8] which is described by $\mathrm{Ext}^i(Q, Q^{'})$ groups and determined by the diagram [4] in Figure 4

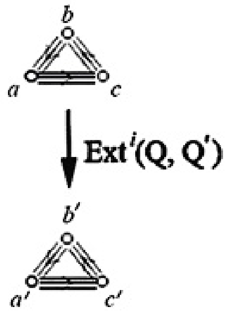

Fig.4. *Open superstring that is described by Ext$^i(Q, Q^{'})$ group.*

has the form

$$\begin{aligned} \mathrm{Ext}^0(Q, Q^{'}) &= \mathbb{C}^{\,aa^{'}+bb^{'}+cc^{'}}, \\ \mathrm{Ext}^1(Q, Q^{'}) &= \mathbb{C}^{\,3ab^{'}+3bc^{'}+3ca^{'}}. \end{aligned} \tag{3}$$

Substituting in (3) orbifold charges

$$a = b = c = a' = b' = c' = 4$$

and using the Langlands hypothesis [9], we obtain the realization of (3) in terms of $SU(5)$ multiplets

$$3 \times (24 + 5_H + \overline{5}_H + 5_M + \overline{5}_M + 10_M + \overline{10}_M) \ ,$$

where 5_H and $\overline{5}_H$ are Higgs multiplets, $\overline{5}_M^{(i)}$ and $10_M^{(j)}$ are multiplets of quark and lepton superpartners.

As the transition in the Teichmüller space is connected with stability of the object and this stability is characterized by the moduli space of an open superstring connected with Higgs multiplets, we can see how a higher-dimensional breaking mechanism is connected or associated with four-dimensional GUT Higgs multiplets and symmetry breaking higgs mechanism.

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