DOI: http://dx.doi.org/10.15673/2072-9812.4/2014.42023
Proc. Intern. Geom. Center 2014 **7**(4) 60–71 $d\omega$

Geometry of a Relativistic Quantum Chaos: New approach to dynamics of quantum systems in electromagnetic field and uniformity and charm of a chaos

A.V. Glushkov, V.B. Ternovsky, V.V. Buyadzhi, G.P. Prepelitsa

Abstract Work is devoted to the development of the theoretical foundations of the universal new relativistic chaos-geometric and quantum-dynamic approach that consistently includes a number of new relativistic quantum models and a number of new or improved methods of analysis (correlation integral, fractal analysis, algorithms, average mutual information, false nearest neighbors, Lyapunov exponents, surrogate data, non-linear prediction, spectral methods, etc.) to solve problems of complete modelling relativistic chaotic dynamics in an electromagnetic field. For a number of atomic systems there are firstly discovered availability of a relativistic quantum chaos and obtained the corresponding quantitative data on the chaos characteristics.

Keywords relativistic quantum chaos, quantum systems in field, chaos-geometric and quantum-dynamics approach

Mathematics Subject Classification (2000)55R01-55B13

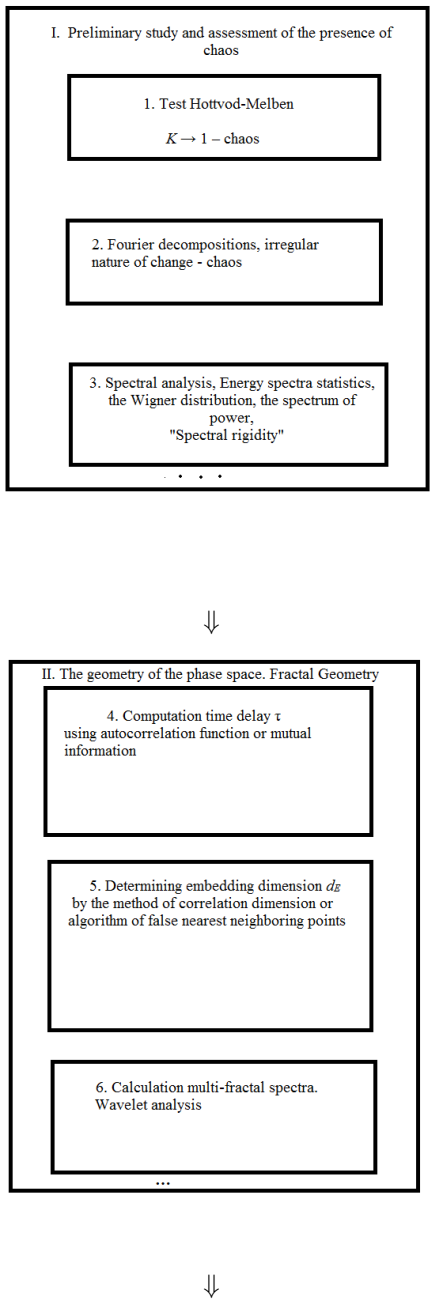
Introduction

At present time one of the extremely important and too complex areas of elements, systems theory is study of regular and chaotic dynamics of nonlinear processes in the different classes of quantum, quantum-generating systems (atomicr systems in an external electromagnetic field) [1-14].

It is worth to remind that dynamics of the cited systems in external electromagnetic field has features of the random, stochastic kind and its realization does not require the specific conditions. The importance of mathematical studying a phenomenon of stochasticity or quantum chaos in dynamical systems is provided by a whole number of technical applications, including a necessity of understanding chaotic features in a work of different electronic devices and systems. New field of investigations of the quantum and other systems has been provided by a great progress in a development of a chaos theory methods. In previous our papers [2-4] we have given a review of new methods and algorithms to analysis of different dynamical systems. In this paper we present the theoretical foundations of the new universal relativistic chaos-geometric and quantum-dynamic approach to modelling chaotic dynamics of heavy complex relativistic quantum systems in an external electromagnetic field, opening a new field of relativistic quantum chaos in geometry of a chaos. Chaos-geometric block includes a set of new or partially improved non-linear analysis methods (such as correlation (dimension D) integral, fractal analysis, average mutual information, false nearest neighbours, Lyapunov exponents (LE) and Kolmogorov entropy (KE) , power spectrum analysis, the surrogate data, nonlinear prediction, predicted trajectories, neural network methods etc), quantum-dynamical block – new relativistic approach to systems in a field. For a number of heavy systems there are firstly theoretically discovered availability of a relativistic quantum chaos and obtained the corresponding quantitative data on the chaos characteristics.

2. Chaos-geometric approach to treating a chaos dynamics

As our approach has been presented earlier [1-3], here we are limited only by the key moments. Let us formally consider scalar measurements $s(n) = s(t_0 + n\Delta t) = s(n)$, where t_0 is the start time, Δt is the time step, and is n the number of the measurements. Further it is necessary to reconstruct phase space using as well as possible information contained in the $s(n)$. Such a reconstruction results in a certain set of d-dimensional vectors $\mathbf{y}(n)$ replacing the scalar measurements. Packard et al. [9] introduced the method of using time-delay coordinates to reconstruct the phase space of an observed dynamical system. The direct use of the lagged variables $s(n+\tau)$, where τ is some integer to be determined, results in a coordinate system in which the structure of orbits in phase space can be captured.


I. Preliminary study and assessment of the presence of chaos
1. Test Hottvod-Melben
$K \to 1$ – chaos
2. Fourier decompositions, irregular nature of change - chaos
3. Spectral analysis, Energy spectra statistics, the Wigner distribution, the spectrum of power, "Spectral rigidity"
. . .
II. The geometry of the phase space. Fractal Geometry
4. Computation time delay τ using autocorrelation function or mutual information
5. Determining embedding dimension d_E by the method of correlation dimension or algorithm of false nearest neighboring points
6. Calculation multi-fractal spectra. Wavelet analysis
...

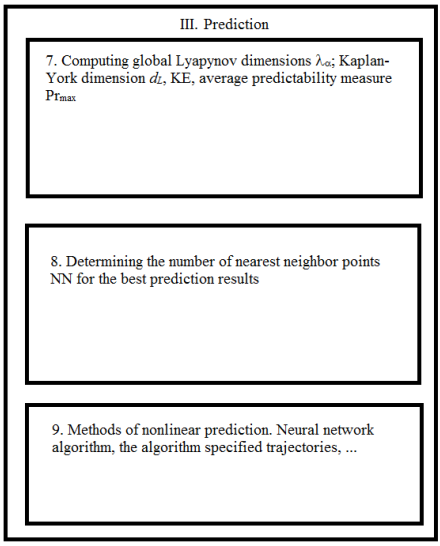


Figure 1. Chaos and neural network-geometric approach to nonlinear analysis and forecast chaotic dynamics processes in complex systems (devices). Then using a collection of time lags to create a vector in d dimensions,

$$\mathbf{y}(n) = [s(n), s(n+\tau), s(n+2\tau), \ldots, s(n+(d-1)\tau)], \tag{1}$$

the required coordinates are provided. In a nonlinear system, the $s(n+j\tau)$ are some unknown nonlinear combination of the actual physical variables that comprise the source of the measurements. The dimension d is called the embedding dimension, d_E.

According to Mane and Takens [12], any time lag will be acceptable is not terribly useful for extracting physics from data. If τ is chosen too small, then the coordinates $s(n+j\tau)$ and $s(n+(j+1)\tau)$ are so close to each other in numerical value that they cannot be distinguished from each other. Similarly, if τ is too large, then $s(n+j\tau)$ and $s(n+(j+1)\tau)$ are completely independent of each other in a statistical sense. Also, if τ is too small or too large, then the correlation dimension of attractor can be under- or overestimated respectively [3]. It is therefore necessary to choose some intermediate (and more appropriate) position between above cases. First approach is to compute the linear autocorrelation function

$$C_L(\delta) = \frac{\frac{1}{N}\sum_{m=1}^{N}[s(m+\delta)-\bar{s}][s(m)-\bar{s}]}{\frac{1}{N}\sum_{m=1}^{N}[s(m)-\bar{s}]^2}, \tag{2}$$

where $\bar{s} = \frac{1}{N}\sum_{m=1}^{N} s(m)$ and to look for that time lag where $C_L\delta$ first passes through zero. This gives a good hint of choice for τ at that $s(n+j\tau)$ and

$s(n+(j+1)\tau)$ are linearly independent. However, a linear independence of two variables does not mean that these variables are nonlinearly independent since a nonlinear relationship can differs from linear one. It is therefore preferably to utilize approach with a nonlinear concept of independence, e.g. the average mutual information. Briefly, the concept of mutual information can be described as follows. Let there are two systems, A and B, with measurements a_i and b_k. The amount one learns in bits about a measurement of a_i from measurement of b_k is given by arguments of information theory [3,7]

$$I_{AB}(a_i, b_k) = \log_2 \left(\frac{P_{AB}(a_i, b_k)}{P_A(a_i) P_B(b_k)} \right), \tag{3}$$

where the probability of observing a out of the set of all A is $P_A(a_i)$, and the probability of finding b in a measurement B is $P_B(b_i)$, and the joint probability of the measurement of a and b is $P_{AB}(a_i,b_k)$. The mutual information I of two measurements a_i and b_k is symmetric and non-negative, and equals to zero if only the systems are independent. The average mutual information between any value a_i from system A and b_k from B is the average over all possible measurements of $I_{AB}(a_i,\, b_k)$,

$$I_{AB}(\tau) = \sum_{a_i, b_k} P_{AB}(a_i, b_k) I_{AB}(a_i, b_k) \tag{4}$$

To place this definition to a context of observations from a certain physical system, let us think of the sets of measurements $s(n)$ as the A and of the measurements a time lag τ later, $s(n+\tau)$, as B set. The average mutual information between observations at n and $n+\tau$ is then

$$I_{AB}(\tau) = \sum_{a_i, b_k} P_{AB}(a_i, b_k) I_{AB}(a_i, b_k) \tag{5}$$

Now we have to decide what property of $I(\tau)$ we should select, in order to establish which among the various values of τ we should use in making the data vectors $\mathbf{y}(n)$. One could remind that the autocorrelation function and average mutual information can be considered as analogues of the linear redundancy and general redundancy, respectively, which was applied in the test for nonlinearity. The general redundancies detect all dependences in the time series, while the linear redundancies are sensitive only to linear structures. Further, a possible nonlinear nature of process resulting in the vibrations amplitude level variations can be concluded.

The goal of the embedding dimension determination is to reconstruct a Euclidean space R^d large enough so that the set of points d_A can be unfolded without ambiguity. In accordance with the embedding theorem, the embedding

dimension, d_E, must be greater, or at least equal, than a dimension of attractor, dA, i.e. $d_E > d_A$. However, two problems arise with working in dimensions larger than really required by the data and time-delay embedding [1,7,13,19]. First, many of computations for extracting interesting properties from the data require searches and other operations in R^d whose computational cost rises exponentially with d. Second, but more significant from the physical point of view, in the presence of noise or other high dimensional contamination of the observations, the extra dimensions are not populated by dynamics, already captured by a smaller dimension, but entirely by the contaminating signal. In too large an embedding space one is unnecessarily spending time working around aspects of a bad representation of the observations which are solely filled with noise. It is therefore necessary to determine the dimension d_A. There are several standard approaches to reconstruct the attractor dimension (see, e.g., [3,7-12]), but let us consider in this study two methods only. The correlation integral analysis is one of the widely used techniques to investigate the signatures of chaos in a time series. The analysis uses the correlation integral, $C(r)$, to distinguish between chaotic and stochastic systems. To compute the correlation integral, the algorithm of Grassberger and Procaccia [10] is the most commonly used approach. According to this algorithm, the correlation integral is

$$C(r) = \lim_{N\to\infty} \frac{2}{N(n-1)} \sum_{\substack{i,j \\ (1 \le i < j \le N)}} H\left(r - ||y_i - y_j||\right) \tag{6}$$

where H is the Heaviside step function with $H(u) = 1$ for $u > 0$ and $H(u) = 0$ for $u \le$, r is the radius of sphere centered on $\mathbf{y}_i$ or $\mathbf{y}_j$, and N is the number of data measurements. If the time series is characterized by an attractor, then the integral $C(r)$ is related to the radius r given by

$$d = \lim_{\substack{r \to 0 \\ N \to \infty}} \frac{\log C(r)}{\log r}, \tag{7}$$

where d is correlation exponent. The method of surrogate data [3,7-11] is an approach that makes use of the substitute data generated in accordance to the probabilistic structure underlying the original data. This means that the surrogate data possess some of the properties, such as the mean, the standard deviation, the cumulative distribution function, the power spectrum, etc., but

are otherwise postulated as random, generated according to a specific null hypothesis. Here, the null hypothesis consists of a candidate linear process, and the goal is to reject the hypothesis that the original data have come from a linear stochastic process. One reasonable statistics is obtained as follows. If we denote Q_{orig} as the statistic computed for the original time series and Q_{si} for ith surrogate series generated under the null hypothesis and let μ_s and σ_s denote, respectively, the mean and standard deviation of the distribution of Q_s, then the measure of significance S is given by $S = \frac{|Q_{orig}-\mu_s|}{\sigma_s}$. An S value of ~ 2 cannot be considered very significant, whereas an S value of $\sim$10 is highly significant. To detect nonlinearity in the amplitude level data, the one hundred realizations of surrogate data sets were generated according to a null hypothesis in accordance to the probabilistic structure underlying the original data. Often, a significant difference in the estimates of the correlation exponents, between the original and surrogate data sets, can be observed. In the case of the original data, a saturation of the correlation exponent is observed after a certain embedding dimension value (i.e., 6), whereas the correlation exponents computed for the surrogate data sets continue increasing with the increasing embedding dimension. The high significance values of the statistic indicate that the null hypothesis (the data arise from a linear stochastic process) can be rejected and hence the original data might have come from a nonlinear process. It is worth consider another method for determining d_E that comes from asking the basic question addressed in the embedding theorem: when has one eliminated false crossing of the orbit with itself which arose by virtue of having projected the attractor into a too low dimensional space? By examining this question in dimension one, then dimension two, etc. until there are no incorrect or false neighbours remaining, one should be able to establish, from geometrical consideration alone, a value for the necessary embedding dimension. Advanced version is presented in Ref. [3]

The LE are the dynamical invariants of the nonlinear system. In a general case, the orbits of chaotic attractors are unpredictable, but there is the limited predictability of chaotic physical system, which is defined by the global and local LE. A negative exponent indicates a local average rate of contraction while a positive value indicates a local average rate of expansion. In the chaos theory, the spectrum of LE is considered a measure of the effect of perturbing the initial conditions of a dynamical system. In fact, if one manages to derive the whole spectrum of the LE, other invariants of the system, i.e. KE and attractor's dimension can be found. The KE, K, measures the average rate at which infor-

mation about the state is lost with time. An estimate of this measure is the sum of the positive LE. The inverse of the KE is equal to an average predictability. Estimate of dimension of the attractor is provided by the Kaplan and Yorke conjecture:

$$d_L = j + \frac{\sum_{\alpha=1}^{j} \lambda_\alpha}{|\lambda_{j+1}|}, \tag{8}$$

where j is such that $\sum_{\alpha=1}^{j} \lambda_\alpha > 0$ and $\sum_{\alpha=1}^{j+1} \lambda_\alpha < 0$, and the LE λ_α are taken in descending order. There are a few approaches to computing the LE. One of them computes the whole spectrum and is based on the Jacobi matrix of system [3]. In the case where only observations are given and the system function is unknown, the matrix has to be estimated from the data. To calculate the spectrum of the LE from the amplitude level data, one could determine the time delay τ and embed the data in the four-dimensional space. In this point it is very important to determine the Kaplan-Yorke dimension and compare it with the correlation dimension, defined by the Grassberger-Procaccia algorithm. The estimations of the KE and average predictability can further show a limit, up to which the amplitude level data can be on average predicted.

3. Relativistic quantum chaos in atomic dynamics in a DC electric and electromagnetic fields

Further we present a new relativistic quantum approach to modeling the chaotic dynamics of atomic systems in a dc electric and ac electromagnetic fields, based on the theory of quasi-stationary quasienergy states, optimized operator perturbation theory, method of model-potential, a complex rotation coordinates algorithm method. The universal chaos-geometric block will be used further to treat the chaotic ionization characteristics for a number of heavy atomic systems.

Let us remind that in the case of the electromagnetic field atomic Hamiltonian is usually as follows:

$$H = \frac{1}{2}p^2 + V_{at}(r) + zF_0 \cos(\omega t) \tag{9}$$

The field is periodic, of course one should use the Floquet theorem; then the eigen Floquet states $\Psi_{E_j}(r,t) >$ and quasienergies E_j are defined as the eigen functions and eigen values of the Floquet Hamiltonian $H_F = H - i\partial_t$. In the general form with using the method of complex coordinates the problem reduces to the solution of stationary SchrC†dinger equation, which is as follows in the model potential approximation:

$$(-1/2 \cdot \nabla^2 + V_{at}(r) + \omega L_z + F_o z)\Psi_E(r) = E\Psi_E(r) \tag{10}$$

i.e. to the stationary eigen value and eigen vectors task for some matrix A (with the consideration of several Floquet zones): $(A - -E_j B)E_j >= 0$. As a decomposition basis, system of the Sturm functions of the operator perturbation theory basis is used.

In our new theory we start from the Dirac Hamiltonian (in relativistic units):

$$H = \alpha p + \beta - \alpha Z/r_i + \sqrt{\alpha} F z, \tag{11}$$

Here a field strength intensity is expressed in the relativistic units ($F_{rel} = \alpha^{5/2} F_{at.un.}$; α is the fine structure constant). One could see that a relativistic wave function in the Hilbert space is a bi-spinor. Using the formal transformation of co-ordinates $r \to r\exp(i\theta)$ in the Hamiltonian

(**??**), one could get:

$$H(\theta) = (\alpha c p - Z/r)\exp(-i\theta) + \beta - \sqrt{\alpha} F z \exp(i\theta), \tag{12}$$

In comparison with an analogous non-relativistic theory, here there is arisen a technical problem. In formulae (11) there is term β, which can not be simply transformed. One of the solving receptions os a limitation of a sub-space of the Hamiltonian eigen-functions by states of the definite symmetry (momentum J and parity P). Thus states can be described by the following functions:

$$\Psi^M_{PJ} = 1/r \begin{pmatrix} f(r)\Upsilon^M_{lJ}(n,\sigma) \\ g(r)\Upsilon^M_{l'J}(n,\sigma) \end{pmatrix} \tag{13}$$

Here $l(l')$ and spin P... in the coupling scheme give a state with the total momentum J and its projection $M_J = M$. Action of the Hamiltonian (11) on the functions (13) with definite J results in:

$$\begin{array}{l} \widehat{H}\ (\theta)\Psi^M_{PJ} = \alpha_r(\widehat{p}_r - \frac{i\omega(J+1/2)}{r}\beta)\exp(-i\theta)\Psi^M_{PJ} + \\ +(\beta - \frac{\alpha Z}{r}\exp(-i\theta) - \sqrt{\alpha} F z \exp(-i\theta))\Psi^M_{PJ} \end{array} \tag{14}$$

where $\alpha_r = \begin{pmatrix} 0....\sigma n \\ \sigma n...0 \end{pmatrix}$, $\beta = \begin{pmatrix} 1....0 \\ 0..-1 \end{pmatrix}$, $p_r = -i(1/r)(d/dr)r$, $\mathbf{n} = \mathbf{r}/r$, σ - the Pauli matrices; parameter $\omega = -1$, if $l = J - 1/2$ and $\omega = 1$, if $l = J + 1/2$.

To further diahonalizuvaty Hamiltonian selecting the correct basis functions in subspace, including selecting the following functions (sleterovskoho vodnye-podibnoho or type):

$$\Psi^{a,M}_{PJ} = 1/r \begin{pmatrix} F(r)\Upsilon^M_{lJ}(n,\sigma) \\ 0 \end{pmatrix} \tag{15a}$$

$$\Psi_{PJ}^{b,M} = 1/r \begin{pmatrix} 0 \\ iG(r)\Upsilon_{l'J}^{M}(n,\sigma) \end{pmatrix} \qquad (15b)$$

It is easy to see that the matrix elements (14) will be no-zeroth only between the states with the same M_J. In fact this moment is a single limitation of the whole approach.

Transformation of co-ordinates in the Pauli Hamiltonian (in comparison with the Schrodinger equation Hamiltonian it contents additional potential term of a magnetic dipole in an external field) can be performed by the analogous way. However, procedure in this case is significantly simplified.

As illustration, below we present some results of our numerical modelling ionization dynamics for Rydberg atoms Rb, Cs, Fr (Rb: $n = 50-80; Cs, Fr : n = 60-80$) in a microwave field ($F = (1.2-3.2).10^{-9} a.u.; \omega/2\pi = 8.87, 36 HGz$). In particular, in Table 1 we present our new relativistic theory data on dependence of the Rb ionization probability P upon the F, interaction time "atom-field". For comparison there are listed the non-relativistic theoretical data by Krug-Buchleitner [15]; Th2 – Glushkov,Prepelitsa etal [3].

Table 1. Dependence of ionization probability P for Rb ($l_0 = 0, m_0 = 0, n_0 = 60-66$) on n_0, F (at.units), (other parameters: $t = 327.2\pi/\omega$; frequency $\omega_c = \omega/2\pi = 36GHz, 8.87GHz$): Th1- numerical simulation of the SchrC†dinger equation by Krug-Buchleitner [15]; Th2 – Glushkov,Prepelitsa etal [3]; Th3 – this work

	Th.1	Th.2	Th.3	Th.1	Th.2	Th.2
n_0	$F = 3.1 \cdot 10^{-9}$ $\omega_c = 36$	$F = 3.1 \cdot 10^{-9}$ $\omega_c = 36$	$F = 3.1 \cdot 10^{-9}$ $\omega_c = 36$	$F = 2.8 \cdot 10^{-9}$ $\omega_c = 8.87$	$F = 3.1 \cdot 10^{-9}$ $\omega_c = 8.87$	$F = 3.1 \cdot 10^{-9}$ $\omega_c = 8.87$
60	0,25	0,252	0,27	0,20	0,21	0,24
63	0,36	0,358	0,38	0,30	0,31	0,33
64*	-	-	0,36	-	-	0,31
65	0,34	0,347	0,37	0,28	0,29	0,32
66	0,36	0,371	0,39	0,34	0,35	0,38

Comparison of theoretical results with experimental data by Munich group for Rb: ($n_0 > 60$, $F = (1.2 - 3.2).10^{-9}$ a.u., $\omega/2\pi = 8.87GHz$) [15,16] shows that all listed data are in a reasonable agreement with experiment, however, the best accuracy is provided by relativistic theory. In table 2 we firstly present new data on dependence of the Fr ionization probability P upon the F, interaction

time "atom-field" (the same parameters as in table 1). Unfortunately, here there are no any alternative theoretical or experimental results.

Table 2. Dependence of ionization probability P for Fr ($l_0 = 0, m_0 = 0, n_0 = 76 - 80$) on n_0, F (the same parameters as in table 1): this work

	this work	this work	this work	this work
n_0	$F = 2.8 \cdot 10^{-9}$ $\omega/2\pi = 36$	$F = 3.1 \cdot 10^{-9}$ $\omega/2\pi = 36$	$F = 2.8 \cdot 10^{-9}$ $8.87GHz$	$F = 3.1 \cdot 10^{-9}$ $8.87GHz$
77	0,47	0,50	0,43	0,46
80	0,58	0,61	0,54	0,56
83*	0,56	0,60	0,51	0,53
86	0,67	0,69	0,62	0,66

In whole, our modeling relativistic dynamics of ionization Rb, Cs, Fr Rydberg states in the microwave field for main quantum numbers $n_0 (n_0 \sim n^*)$ there are the local violations of probability smooth growth associated with the complex Floquet spectrum, link between the quasi-stationary states and a continuum, the growing influence of multiphoton resonances. The picture becomes by more complicated due to the single-photon near-resonance transitions with quasi-random detuning from resonance and quantum phase shift due to scattering Rydberg electron on the atomic core. It is in agreement with alternative comments in [3,15]. In conclusion we have used the chaos-geometric approach (ch. 2) to estimate parameters of relativistic chaotic dynamics for the Rydberg atoms Rb, Cs, Fr in microwave field: correlation dimension, LE, KE. In Table 3 there are listed the numerical LE values, Kolmogorov entropy K_{entr} , for three atomic systems Positivity first two LE showers certainly evidence of chaotic dynamics for studied systems in a microwave field.

Table 3. Numerical data for Hottvod-Melben parameter K, Lyapunov exponents λ_i, Kolmogorov entropy K_{entr} (our data)

Regim	Рль	λ_1	λ_2	K_{entr}
Chaos (Rb)	0.85	0.21	0.06	0.27
Chaos (Cs)	0.87	0.22	0.09	0.31
Chaos (Fr)	0.89	0.25	0.11	0.36

We have constructed the quantitative diagram of effects of the quantum fluctuations, stabilization, destabilization, delocalization and performance of the Kolmogorov-Arnold-Mozer theorem in relativistic atomic dynamics. We have found that the regime of the chaotic ionization for the Li, Rb in a microwave field at $\omega_o = \omega n_o^3 > 0.29(Rb), 0.25(Cs), 0.16(Fr)$ switches to dynamic stabilization one. In whole using the new relativistic chaos-geometric and quantum-dynamical

approaches there have been results, which confirm an universality and charm of relativistic chaotic phenomena.

References

1. Gutzwiller M.,Chaos in Classical and Quantum Mechanics.-N.-Y.:Springer-Verlag, 1990.-720p.
2. Glushkov A.V., Svinarenko A.A., Buyadzhi V.V., Zaichko P.A., Ternovsky V.B., Chaos-geometric attractor and quantum neural networks approach to simulation chaotic evolutionary dynamics during perception process// Advances in Neural Networks, Fuzzy Systems and Artificial Intelligence, Series: Recent Advances in Computer Engineering (Gdansk,EU).-2014.-Vol.21.-P.143-150.
3. Glushkov A.V., Khetselius O.Yu., Svinarenko A.A., Prepelitsa G.P.Energy approach to atoms in a laser field and quantum dynamics with laser pulses of different shape///In: Coherence and Ultrashort Pulsed Emission, Ed. Duarte F. J. (Intech, Vienna).-2011.-P.159-186.
4. Glushkov A.V., Prepelitsa G.P., Lepikh Ya.I., Buyadzhi V.V., Ternovsky V.B., Zaichko P.A., Chaotic dynamics of non-linear processes in atomic and molecular systems in electromagnetic field and semiconductor and fiber laser devices: new approaches, uniformity and charm of chaos// Sensor Electronics and Microsystems Techn.-2014.-Vol.11,N4.-P.43-57.
5. Ternovsky V.B., Geometry and Dynamics of a Chaos: Modelling non-linear processes in relativistic backward-wave tubes chain// International Scientific Journal "Proceedings of International Geometry Center".- 2014.-Vol.7,N3.-P.77-82.
6. Gallager R.G., Information theory and reliable communication, Wiley, New York.-1986.
7. Kennel M., Brown R., Abarbanel H., Determining embedding dimension for phase-space reconstruction using geometrical construction//Phys.Rev.A.-1992.-Vol.45.-P.3403–3411.
8. Packard N., Crutchfield J., Farmer J., Shaw R., Geometry from time series// Phys.Rev.Lett-1988.-Vol.45.-P.712–716.
9. Grassberger P., Procaccia I., Measuring the strangeness of strange attractors// Physica D.-1983.-Vol.9.-P.189–208.
10. Fraser A., Swinney H., Independent coordinates for strange attractors from mutual information// Phys Rev A.-1986.-Vol.33.-P.1134–1140.
11. Takens F., Detecting strange attractors in turbulence. In: Dynamical systems and turbulence, ed. Rand D., Young L.- (Springer, Berlin-New York).-1981.-P.366–381.
12. Zimmerman M., Littman M., Kash M., Kleppner D., Stark and Zeemane structure of Rydberg states of alkali-metal atoms// Phys.Rev.A.-1979.-V.20,N6.-P.2251-2275.
13. Gasati G., Guarneri I., Mantica G., Classical Stabilization of periodically kicked hydrogen atoms// Phys.Rev.A.-1994.-V.50(**??**).-P.5018-5024.
14. Glushkov A.V., Ivanov L.N., DC Strong-Field Stark-Effect: consistent quantum-mechanical approach // J.Phys.B:At.Mol.Opt.Phys.-1993.-V.26.-P.L379-388.
15. Krug A., Buchleitner A., Microwave ionization alkali-metal Rydberg states in a realistic numerical experiment//Phys. Rev. A.-2002.-Vol.66.-P.053416 (10p.);
16. Gallagher T., Mahon C., Dexter J., Pillet P., Ionization of sodium and lithium Rydberg atoms by 10-MHz to 15-GHz electric fields//Phys. Rev. A.-1991.- Vol.44.-P. 1859-1872.

A.V. Glushkov, V.B. Ternovsky, V.V. Buyadzhi, G.P. Prepelitsa
Odessa State Environmental University, 15, L' vovskaya str., Odessa, 65016, Ukraine
E-mail: dirac13@mail.ru