

## Stochastic Oscillations at Stick-Slip Motion in the Boundary Friction Regime

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In this paper, the further development of the synergetic model describing the ultrathin lubricant film state clamped between two atomically smooth solid surfaces operating under boundary friction mode has been done based on the Lorentz model for the approximation of a viscoelastic medium. In all cases, the phase portraits have been built. It has been found that the friction surfaces' temperature increasing leads to the growth of stochasticity in the investigated system. In the phase plane the stochastic oscillation mode can be described as a strange attractor. Also, the behavior of two different types of tribosystems were described using current model. The first was the system with the unidirectional shear of the surfaces and, and the second was the system under an alternating external effect. Obtained results agree qualitatively with known experimental data.

Keywords: Synergetic model, Ultrathin lubricant film, Shear strains, Stochastic oscillation, Lorentz attractor.

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## 1. INTRODUCTION

The study of the boundary friction processes that develop in nanosized tribosystems has drawn active interest from many researchers. One of the evolving directions is the investigation of the friction of atomically smooth solid surfaces in the presence of an ultrathin film of a homogeneous lubricant between them. A synergetic representation of the boundary friction processes [1-4] makes it possible to describe the nontrivial behavior of an ultrathin lubricating film clamped between two solids that are in relative motion. The model is based on a system of three differential equations for the stresses, strains, and lubricating film temperature. In this work, the problem of the occurrence of oscillations in the system in the deterministic chaos mode that, according to the structure of the basic equations, can be observed in the synergetic representation has been studied.

## 2. BASIC EQUATIONS

The system of the basic equations for the stresses, strains, and lubricating film temperature is as follows [1-4]:

$$\dot{\sigma} = -\sigma + g\varepsilon, \tag{1}$$

$$\tau \dot{\varepsilon} = -\varepsilon + (T - 1)\sigma, \tag{2}$$

$$\delta \dot{T} = (T_e - T) - \sigma \varepsilon + \sigma^2, \tag{3}$$

where  $\sigma$  is the shear component of the stresses that arise in the lubricating film,  $\varepsilon$  is the shear component of the relative strains, T is the lubricating film temperature, and  $T_e$  is the friction surface temperature. All of these parameters are dimensionless, which makes it possible to carry out a qualitative analysis without considering the characteristics of a given tribosystem. The constant g < 1, which is numerically equal to the ratio of the lubricant shear modulus to its characteristic value, and the following parameters  $\tau = \tau_{\varepsilon} / \tau_{\sigma}$ ,  $\delta = \tau_T / \tau_{\sigma}$  where  $\tau_{\sigma}$ ,  $\tau_{\varepsilon}$  and  $\tau_T$  are the times of relaxation of the stresses, strains, and temperature, respectively.

In the general case, when all of the relaxation times in system (1)-(3) have nonzero values, it is reduced to the following third-order differential equation:

$$\ddot{\sigma} = \ddot{\sigma} \left( \frac{\dot{\sigma}}{\sigma} - \frac{1}{\tau} - \frac{\delta+1}{\delta} \right) + \frac{\dot{\sigma}}{\tau} \left( \frac{\dot{\sigma}(\tau+1)}{\sigma} - \frac{\sigma^2+1+\tau}{\delta} \right) +$$

$$+ \frac{\sigma \left( g \left( T_e + \sigma^2 - 1 \right) - \sigma^2 - 1 \right)}{\delta \tau}.$$

$$(4)$$

Since the behavior of the system described by Eq. (4) depends critically on the initial conditions, it is appropriate to present the solution as phase portraits. Figure 1 shows the corresponding phase portraits at different temperatures of the friction surfaces  $T_e$ . In all of the figures, the initial value of the second time derivative is  $\ddot{\sigma}_0 = 0$ . The initial values of  $\dot{\sigma}$  and  $\sigma$  can be seen directly on the phase planes and correspond to the origin of the phase trajectories.

Let us consider in more details Fig. 1a. The phase portrait is characterized by the only critical point at the origin of coordinates when  $\sigma = \dot{\sigma} = 0$ ; this point is a stable focus. In particular, at the initial value  $\sigma_0 \sim 0$ , the focus becomes more pronounced (trajectory 7). It can be seen that, for trajectories 2, 3, 5, and 6, an aperiodic mode is realized. Trajectories 1, 4, and 7 correspond to the damped oscillation mode. For the parameters of trajectory 7, damping is less pronounced, since several oscillations are observed in the vicinity of the stable value  $\sigma = 0$  during relaxation.

Fig. 1b illustrates a phase portrait at the temperature of the friction surfaces  $T_e > T_{c0}$  at which we have the nonzero value of the stresses and the rubbing block moves with the nonzero velocity. In the phase portrait, two criti-

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cal points occur with the coordinates  $(\sigma_0, 0)$  and  $(-\sigma_0, 0)$  that are symmetric with regard to the origin of coordinates. Both points are equivalent and represent stable focuses around which long oscillations develop until the block begins to move with a constant velocity.

As the temperature  $T_e$  elevates, the situation shown by the phase portrait in Fig. 1c is achieved. In this case, the set of the initial values has a critical effect on the behavior of the system. It can be seen that, at low absolute values of the stresses and the rate of their change, the same critical points occur in the phase portrait as in Fig. 2b (trajectories 2 and 3 in Fig. 1c). However, if in the beginning of motion, the initial values exceed critical values, a chaotic mode sets in. In this case, no stationary state is achieved with time, but constant phase transitions between the solid- and fluid-like lubricant states occur (trajectory 1). It can be seen in Fig. 1c that this mode is not periodic in time and represents a strange attractor, i.e., a realization of the deterministic chaotic mode.

Fig. 1d is plotted at a fairly high value of the temperature  $T_e$ . the arrow in the right part of the figure shows the point that corresponds to the origin of the trajectory (initial conditions during the solution of Eq. (4). In this figure, two symmetric critical points occur and are shown by the points on the abscissa axis. Unlike Fig. 1c in which the stable focuses are presented, these points are unstable focuses. We note that the phase trajectory is shaped as a Lorentz strange attractor [5, 6] and, in this case, the mode that occurs is characterized by a more pronounced stochastic behavior than that shown in Fig. 1c.

This work presents the subsequent development of the synergetic model, which describes the state of an ultrathin lubricating film clamped between two atomically smooth solid surfaces during boundary friction. It has been shown that the use of this model can make it possible to describe the behavior of various types of tribosystems. The stickslip mode frequently observed in experiments has been depicted. In this mode, consecutive transitions between the structural states of the lubricant occur. This work makes it possible to extend the results of the previous studies obtained in the synergetic model, since the described stick-slip mode has a deterministic nature. This has not been shown previously, but is observed in numerous experiments on studying the boundary friction processes. An equation of motion for the stresses has been derived in the form of a three-order differential equation and analyzed at various friction surface temperatures. It has been found that, depending on the temperature and the lubricant parameters, either the damped oscillation mode or the stochastic oscillation mode may occur. When the temperature exceeds a critical value, the system follows the mode described by the Lorentz attractor. In the wide range of parameters, the reverse motion of the rubbing surfaces occurs. Our results agree qualitatively with known experimental data. It has been shown that, in all of the considered modes, a similar transient mode occurs in a definite range of initial conditions. This mode involves damped oscillations and the subsequent stick of the surfaces together for a long time, after which a stationary mode of friction sets in.

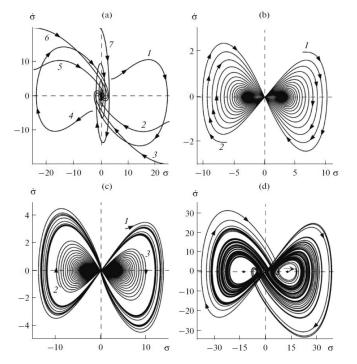


Fig. 1 – Phase portraits of system obtained during numerical solution of equations (1-3) at parameters g = 0.25,  $\tau = 3$ , and  $\delta = 95$ .

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