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Ferrofluid Heating by a Rotating Field: Noise-Free Limit

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The deterministic spherical motion of the ferromagnetic fine particle with frozen magnetic moment is considered. The energy absorbed from circularly polarized field is transformed into thermal energy through the viscous friction between fine particle and carrier fluid. Using the classical equations of motion, the mode of uniform precession and the mode of torsional oscillation are described analytically in overdamped limit. The corresponding relationships for the power loss are obtained. Critical influence of the static field on the power loss value is discussed.

Keywords: Ferrofluid, Spherical motion, circularly polarized field, Energy absorption, Heating rate.

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1. INTRODUCTION

Ferrofluids are colloidal suspensions, which combine the properties of ferromagnetics as well as liquids. This causes a wide range of ferrofluid applications in engineering, such as damper devices [1, 2], sealing [3], hydrodynamic bearings [3], ironless loudspeakers [4], etc. Also it needs to note the high potential of applications such media in medicine and biotechnologies. Ferrofluids are successfully used for the separation of macromolecules [5], new therapy methods such as targeted drug delivery, and magnetic fluid hyperthermia [6, 7]. During this therapy the energy of external ac-field is absorbed, that leads to tissue heating.

The mechanisms of energy absorption by the fine particles, which are contained in a ferrofluid, were described by Rosenzweig [8]. In particular, it was derived, that for sufficiently of large particles or sufficiently low frequencies of the external field, the energy dissipation is a result of viscous friction during of mechanical rotation of the entire particle. Here implies that the magnetic moment rigidly bound with the crystal axis of the nanoparticle.

Despite a lot of studies are based on the results obtained in [8], the analysis presented there was carried out in the following assumptions: 1) ferrofluid is a continuous medium; 2) the thermal equilibrium takes place; 3) the Boltzmann distribution is valid. In this approach the structure of ferrofluid, which is arising from the dipole interactions, is neglected. Then, if the thermal energy is much smaller, than magnetic energy, the time of establishment of equilibrium can be larger, than the observation time. Therefore, the concept of thermal equilibrium becomes impractical. And, finally, application of the ac-field leads to inapplicability of Boltzmann statistics.

Approaches, based on analytical and numerical treatments of motions equations of motions of the particle were developed in [9], and [10], respectively. But detailed description of the dynamical modes of nanoparticle and consistent account of the thermal fluctuations in the presence of time-dependent external field, were not performed.

In present work we examine the other limit case, or case without thermal fluctuations. Using the classical equations, we consider the spherical motion of the fine ferromagnetic particle in a viscous carrier under the action of rotating field. The value of the power loss is calculated using the trajectories of motion.

2. MODEL AND MAIN EQUATIONS

Using the fundamental equation of rotatory motions [11] and the condition for spherical motion [12], one can write original system of equations in the form:

$$\begin{cases} \dot{\vec{\omega}} = \dfrac{1}{\tau_0^2}\vec{m}\times\vec{h} - \dfrac{1}{\tau_r}\vec{\omega}, \\ \dot{\vec{m}} = \vec{\omega}\times\vec{m}, \end{cases} \tag{2.1}$$

where $\vec{\omega}$ is the angular velocity of the particle. The first term in first equation of (2.1) corresponds to the torque origin from the external field $\vec{h}$, which is acting on the reduced magnetic moment $\vec{m}$ of the nanoparticle. Here $\tau_0 = \sqrt{I/\mu_0\mu^2 V}$ is the characteristic time of such action (I is the nanoparticle moment of inertia, $\mu_0 = 4\pi\cdot 10^{-7}$ H·m is the magnetic constant, μ is the nanoparticle magnetization, V is the nanoparticle volume). The second term in first equation of (2.1) is correspond to the friction torque in the most simple and conventional form. Here $\tau_r = I/8\pi\eta R^3$ is the time which characterizes free damped precession of the particle (η is the liquid viscosity, R is the particle radius).

We consider joined action of the circularly polarized and the static fields

$$\tilde{\vec{h}} = \vec{e}_x\tilde{h}\cos\tilde{\Omega}\tilde{t} + \vec{e}_y\tilde{h}\sin\tilde{\Omega}\tilde{t} + \vec{e}_z\tilde{h}_z, \tag{2.2}$$

where $\tilde{h} = h/\mu$, $\tilde{h}_z = h_z/\mu$ are the reduced field amplitudes, $\tilde{\Omega} = \Omega/\Omega_{\text{cr}}$ ($\Omega_{\text{cr}} = \tau_r/\tau_0^2$) is the reduced field frequency, $\tilde{t} = t\cdot\Omega_{\text{cr}}$ is the reduced time. Schematically representation of the particle, coordinate system, and applied fields are given in Fig. 1.

Within the assumption of fine particle ($R \sim 10$ nm), the particle moment of inertia I is too small. It let us to neglect of the time derivative of $\vec{\omega}$ in the left of the first equation of (2.1) and rewrite the system in the form

$$\dot{\vec{m}} = \frac{\tau_r}{\tau_0^2}\vec{m}\times(\vec{m}\times\vec{h}), \tag{2.3}$$

that is widely used (see, for example [11]).

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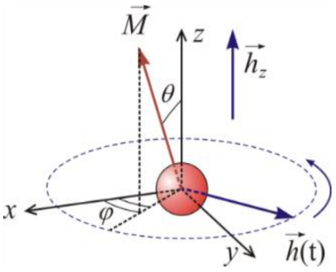


Fig. 1 – Sketch of the model

In the spherical coordinate system and the field given by (2.2), the vector equation (2.3) can be simplified to

$$\begin{cases} \dfrac{d\theta}{d\tilde{t}} = \tilde{h}\cos\theta\cos(\tilde{\Omega}\tilde{t}-\varphi) - \tilde{h}_z\sin\theta, \\ \dfrac{d\varphi}{d\tilde{t}} = \dfrac{\tilde{h}\sin(\tilde{\Omega}\tilde{t}-\varphi)}{\sin\theta}, \end{cases} \tag{2.4}$$

where θ and φ are the polar and azimuthal coordinates of $\vec{m}$, respectively.

In accordance with definition, power loss Q per unit volume can be written as:

$$Q = \mu_0\mu^2 \frac{\Omega}{2\pi}\int_0^{\frac{2\pi}{\Omega}} \tilde{h}\frac{\partial\vec{m}}{\partial t}dt ,$$

or in reduced form

$$\tilde{Q} = \frac{Q}{\mu_0\mu^2\Omega_{cr}} = \frac{1}{\Omega_{cr}}\cdot\frac{\Omega}{2\pi}\int_0^{\frac{2\pi}{\Omega}} \tilde{h}\frac{\partial\vec{m}}{\partial t}dt . \tag{2.5}$$

Using the known equation of heat balance, it is easy to write relationship between the power loss and the heating rate

$$\mathcal{H}_r = \tilde{Q}\frac{\mu_0\mu^2\Omega_{cr}}{C_1D_1\phi + C_2(1-\phi)D_2} , \tag{2.6}$$

where C_1 is the specific heat of the nanoparticles material; D_1 is the density of nanoparticles material; $\phi = dV_1/dV$ is the volume fraction of nanoparticles in ferrofluid; C_2 is the specific heat of the liquid carrier; D_2 is the density of the liquid carrier.

3. RESULTS AND ANALYSIS

3.1 Uniform precession mode

Precession is the obvious type of nanoparticle motion, which is follow from the symmetry of the problem. The precession velocity here is equal to the field frequency. The main characteristics of this type of motion are precession cone angle Θ and lag angle Φ. Therefore, the solution of (2.4) in this case we represent in the following form:

$$\varphi = \tilde{\Omega}\tilde{t} + \Phi,\ \theta = \Theta . \tag{3.1}$$

Substitution (3.1) into system (2.4) let us to obtain the relationships for Θ and Φ

$$\begin{cases} \tilde{h}\cos\Theta\cos\Phi = \tilde{h}_z\sin\Theta, \\ \tilde{\Omega}\sin\Theta = \tilde{h}\sin\Phi, \end{cases} \tag{3.2}$$

There are two solutions of (3.2) in the case of absence of static field ($\tilde{h}_z = 0$)

$$\begin{cases} \Theta = \pi/2 \\ \sin\Phi = \tilde{\Omega}/\tilde{h}, \end{cases} \tag{3.3}$$

$$\begin{cases} \sin\Theta = \tilde{h}/\tilde{\Omega} \\ \Phi = \pi/2. \end{cases} \tag{3.4}$$

The stability analysis these solutions using the Routh-Hurwitz criterion [13] shown, that (3.3) is stable while condition $\tilde{\Omega}/\tilde{h} \le 1$ holds. In other words, only restriction $\sin\Phi \le 1$ exists for solution (3.3). At the same time, (3.4) is always unstable.

When $\tilde{h}_z \ne 0$, the values of Θ and Φ should be calculated numerically. It was shown numerically, that the only one solution of (3.2) exists, and this solution is stable.

In accordance with (2.5) and second expression in (3.2), the power losses in the mode of uniform precession, can be written as

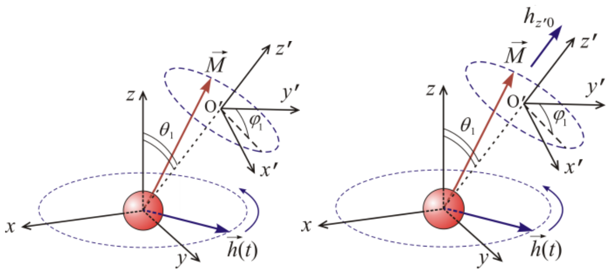
$$\tilde{Q} = \tilde{\Omega}^2\sin^2\Theta. \tag{3.5}$$

In particular, $\tilde{Q} = \tilde{\Omega}^2$, when $\tilde{h}_z = 0$. Since Θ grows with $\tilde{h}_z$, it is possible to control the power loss by the static field.

3.2 Torsional oscillation mode

When the condition $\tilde{\Omega}/\tilde{h} \le 1$ does not hold, the spherical motion of nanoparticle become more complicated. Polar and azimuthal angles are time-dependent here, and rotational velocity is smaller in average, than field frequency. Such motion is described numerically. But in the limit case, when the field frequency is much larger, than the amplitude ($\tilde{\Omega}/\tilde{h} \gg 1$), the particle performs oscillations around its initial position.

We assume the initial position is defined by the angles θ_1 and φ_1. The further solution of (2.3) is performed in the linear approximation. To this end we use new coordinate system x′y′z′, rotated with respect to original one by angles θ_1 and φ_1, as it is shown in



a) b)

Fig. 2 – Schematic representation of the rotated coordinate system: a) θ_1 and φ_1 are the angles, which define an initial state of the particle; b) θ_1 and φ_1 are the angles, which defines the orientation of static field with respect to circularly polarized one.

Fig. 2a. In other words, new coordinate system is defined by the initial position of $\vec{m}$, and oscillations occur around Oz′ axis.

When the condition $\tilde{\Omega}/\tilde{h} \gg 1$ holds, the amplitudes of components $m_{x'}, m_{y'}$ in coordinate system x′y′z′ are small enough and $\vec{m}$ can be represented as following

$$\vec{m} = \vec{e}_{x'} m_{x'} + \vec{e}_{y'} m_{y'} + \vec{e}_{z'}, \tag{3.6}$$

where $\vec{e}_{x'}, \vec{e}_{y'}, \vec{e}_{z'}$ are the unit vectors of coordinate system x′y′z′. The circularly polarized field in this system one can represent using the rotation matrix

$$\tilde{\vec{h}}' = \begin{pmatrix} \cos\theta_1\cos\varphi_1 & -\sin\varphi_1 & \sin\theta_1\cos\varphi_1 \\ \cos\theta_1\sin\varphi_1 & \cos\varphi_1 & \sin\theta_1\sin\varphi_1 \\ -\sin\theta_1 & 0 & \cos\theta_1 \end{pmatrix} \begin{pmatrix} \tilde{h}\cos\tilde{\Omega}\tilde{t} \\ \tilde{h}\sin\tilde{\Omega}\tilde{t} \\ 0 \end{pmatrix} \Rightarrow$$

$$\tilde{\vec{h}}' = \begin{pmatrix} \tilde{h}\cos\theta_1\cos\varphi_1\cos\tilde{\Omega}\tilde{t} - \tilde{h}\sin\varphi_1\sin\tilde{\Omega}\tilde{t} \\ \tilde{h}\cos\theta_1\sin\varphi_1\cos\tilde{\Omega}\tilde{t} + \tilde{h}\cos\varphi_1\sin\tilde{\Omega}\tilde{t} \\ -\tilde{h}\sin\theta_1\cos\tilde{\Omega}\tilde{t} \end{pmatrix}. \tag{3.7}$$

Using the expression (3.7), and assuming, that $m_{x'}, m_{y'} \sim \tilde{h}/\tilde{\Omega}$ and, finally, neglecting terms $\sim \tilde{h}^2$, we obtain from (2.3) the linearized system of equations for $\vec{m}$

$$\begin{cases} \dfrac{dm_{x'}}{d\tilde{t}} = \tilde{h}_{x'}, \\ \dfrac{dm_{y'}}{d\tilde{t}} = \tilde{h}_{y'}. \end{cases} \tag{3.8}$$

The steady-state solution of (3.8) we search in the form

$$\begin{cases} m_{x'} = a_1\cos\tilde{\Omega}\tilde{t} + b_1\sin\tilde{\Omega}\tilde{t}, \\ m_{y'} = c_1\cos\tilde{\Omega}\tilde{t} + d_1\sin\tilde{\Omega}\tilde{t}. \end{cases} \tag{3.9}$$

Substituting expressions (3.9) into the system (3.8), we find the unknown constants:

$$\begin{aligned} a_1 &= \frac{\tilde{h}}{\tilde{\Omega}}\sin\varphi_1, \\ b_1 &= \frac{\tilde{h}}{\tilde{\Omega}}\cos\theta_1\cos\varphi_1, \\ c_1 &= -\frac{\tilde{h}}{\tilde{\Omega}}\cos\varphi_1, \\ d_1 &= \frac{\tilde{h}}{\tilde{\Omega}}\cos\theta_1\sin\varphi_1. \end{aligned} \tag{3.10}$$

The relationships for the power loss (2.5) one can represent in the coordinate system x′y′z. Then, using the expressions (3.7)-(3.10) we calculate the integrals and, finally obtain

$$\tilde{Q} = \frac{1}{2}\tilde{h}^2 (1 + \cos^2\theta_1). \tag{3.11}$$

3.3 Arbitrary orientation of the static field and polarization plane of the circularly polarized field

It is easy to expand the analysis in the previous subsection on the case, when the external static field does not perpendicular to the polarization plane of the circularly polarized field. To this end we use the coordinate system x′y′z′, which are introduced earlier. But here angles θ_1 and φ_1 define the orientation of the static field $\vec{h}_{z'0}$, as it is depicted in Fig. 2b.

If $\vec{h}_{z'0}$ is directed along O′z′ axis and one of the conditions $\tilde{h}_{z'0} \gg \tilde{h}$ or $\tilde{\Omega}/\tilde{h} \gg 1$ holds, the linear expansion (3.6) can be applied. Using the approach, that was described above, when the external field is $\tilde{\vec{h}}' + \vec{e}_{z'}\tilde{h}_{z'0}$, the linearized system of equations that is corresponding to (2.3) can be written as

$$\begin{cases} \dfrac{dm_{x'}}{d\tilde{t}} = \tilde{h}_{x'} - m_{x'}\tilde{h}_{z'0}, \\ \dfrac{dm_{y'}}{d\tilde{t}} = \tilde{h}_{y'} - m_{y'}\tilde{h}_{z'0}. \end{cases} \tag{3.12}$$

The steady-state solution of (3.12) can be found in the form

$$\begin{cases} m_{x'} = a_2\cos\tilde{\Omega}\tilde{t} + b_2\sin\tilde{\Omega}\tilde{t}, \\ m_{y'} = c_2\cos\tilde{\Omega}\tilde{t} + d_2\sin\tilde{\Omega}\tilde{t}, \end{cases} \tag{3.13}$$

where the unknown constants straightforwardly obtained as

$$\begin{aligned} a_2 &= \tilde{h}\frac{\tilde{h}_{z'0}\cos\theta_1\cos\varphi_1 + \tilde{\Omega}\sin\varphi_1}{\tilde{\Omega}^2 + \tilde{h}_{z'0}^2}, \\ b_2 &= -\tilde{h}\frac{\tilde{h}_{z'0}\sin\varphi_1 - \tilde{\Omega}\cos\theta_1\cos\varphi_1}{\tilde{\Omega}^2 + \tilde{h}_{z'0}^2}, \\ c_2 &= \tilde{h}\frac{\tilde{h}_{z'0}\cos\theta_1\sin\varphi_1 - \tilde{\Omega}\cos\varphi_1}{\tilde{\Omega}^2 + \tilde{h}_{z'0}^2}, \\ d_2 &= -\tilde{h}\frac{\tilde{h}_{z'0}\cos\varphi_1 - \tilde{\Omega}\cos\theta_1\sin\varphi_1}{\tilde{\Omega}^2 + \tilde{h}_{z'0}^2}. \end{aligned} \tag{3.14}$$

In the way, we calculated (3.11), finally, we derive the power loss for the case of arbitrary orientation of the circularly polarized and the static fields

$$\tilde{Q} = \frac{1}{2}\tilde{h}^2\tilde{\Omega}^2\frac{1 + \cos^2\theta_1}{\tilde{\Omega}^2 + \tilde{h}_{z'0}^2}. \tag{3.15}$$

The form of denominator of (3.15) let us to state, that two limiting cases are present: 1) $\tilde{\Omega}^2 \gg \tilde{h}_{z'0}^2$; 2) $\tilde{\Omega}^2 \ll \tilde{h}_{z'0}^2$ (see Fig. 3a, b). The first of them is characterized by almost constant, close to the maximal values of the power loss. And in the second case $\tilde{Q}$ is smaller, than in the previous one, and tends to zero as $\tilde{\Omega}^2$. The transition between these limiting cases can be realized by change of $\tilde{h}_{z'0}$ in reasonable interval of values (see Fig. 3b). This fact gives the possibility of control of ferrofluid heating rate using the static field.

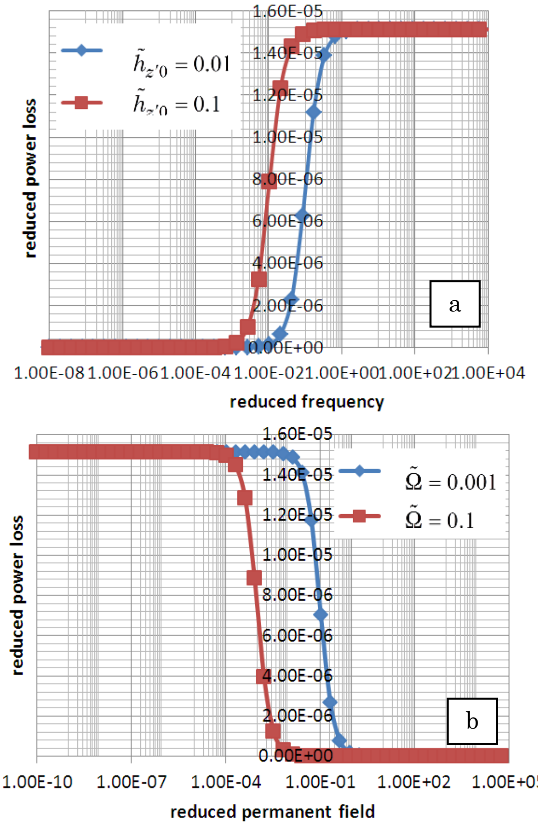


Fig. 3 – Influence of the static field on the power loss given by Eq. (3.15): a) $\tilde{Q} = \tilde{Q}(\tilde{\Omega})\big|_{\tilde{h}_{z'0}=\text{const}}$, b) $\tilde{Q} = \tilde{Q}(\tilde{h}_{z'0})\big|_{\tilde{\Omega}=\text{const}}$. $\tilde{h} = 5.5\cdot 10^{-3}$

The approach developed here is also can be useful for account of the influence of the dipole fields, which are present in real ferrofluid, on the heating rate.

4. CONCLUSIONS

We have studied the rotation dynamics of ferromagnetic spherical nanoparticle, whose magnetic moment is locked to the crystal axis, in a viscous liquid currier under action of the circularly polarized field. The angular momentum equation and on the equations of spherical motion are used. Since the particle size is rather small, the inertia term was neglected.

Within this framework, the two types of motion are investigated. The first is the precession, during which the magnetic moment $\vec{m}$ of the particle follow the rotating magnetic field. It was shown, that in absence of the static field $\vec{m}$ lies in the plane of the field polarization. The angle Φ between vectors of field and magnetic moment, as function of the reduced field frequency ($\tilde{\Omega}$) and amplitude ($\tilde{h}$), was found. It was established, this mode is realized while the condition $\tilde{\Omega}/\tilde{h} < 1$ holds. When the static field is applied perpendicular to the polarization plane of the circularly polarized field, the relationship for lag angle Φ and precession cone angle Θ was obtained.

The second type of nanoparticle motions consist is the torsional oscillations around the initial conditions when the static field is absent. The trajectory expression, which account the initial position of $\vec{m}$, was derived in the linear approximation. The approach presented was expanded on the case of action of static field, which is directed arbitrary to the polarization plane of the circularly polarized field.

The exact expressions for the power loss, for all described modes were found. The obtained power loss can be easy convert into heating rate. It was shown, that the static field can be utilize for control of the ferrofluid heating, despite this field is not absorbed by nanoparticle.

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