A Simple Holdup Model with Two-sided Investments: the Case of Common-Purpose Investments

Makoto Hanazono

Abstract

This paper theoretically studies a simple, discrete holdup model with two-sided investments, in which each party's investment enhances the value of transaction by improving a common factor (the buyer's valuation, in text). We investigate whether contractual remedies to the holdup problem are available by adopting noncontingent or option contracts. The main result of this paper is that an option contract can completely remedy the holdup problem in several cases, whereas noncontingent contracts can only partly remedy it. Remedies by option contracts are more likely as the relative importance of the purely cooperative investment to the purely selfish investment becomes larger, and as the cost of investment becomes smaller.

Key words: Holdup, two-sided investments, common-purpose investments, option contracts.
JEL classification: D23, L14.

1. Introduction

Consider, for example, a production of highly-customized computer software created by a programmer. The programmer can make efforts in order to satisfy the user's wants and needs by gathering and analyzing information from the user. The user can also make efforts in order to enhance the value of the software by, say, learning skills specific to the programmer's software. If there is not much to do for reducing the cost of programming, this is a situation in which two parties can make ex ante, *relationship-specific*[1] investments for a *common purpose*; enhancement of the user's valuation[2].

In an ideal world of perfect information and perfect commitment, no problem arises to achieve efficient investments; the parties sign a contract that instructs them to make efficient investments. In reality, however, information and commitment are imperfect, and the parties cannot directly control the investment incentives. Accordingly, it has been recognized that many aspects of transaction, such as characteristics of the products, the level and quality of investment, and the state of nature, are too costly to verify or describe, and thus ex ante noncontractible. Namely, contracts are quite *incomplete*.

The central issue from contractual incompleteness and relationship-specificity of investment is the so-called *holdup problem*: firms' investment incentives can be undermined, thereby resulting in underinvestment, since the investing party fears that the fruit of investment is partly appropriated ex post by the opponent party (*holdup*), without a contractual protection. This observation leads to studies on how incomplete contracting can remedy this underinvestment issue[3]. However, many related problems still remain unsolved. In particular, contractual remedies to holdup with two-sided investments have not been extensively explored.

This paper studies a holdup model with two-sided investments for a common purpose. To fix ideas, we suppose that both the buyer's and the seller's investments enhance the buyer's valuation of the traded good. Trade and investment are assumed to be binary in the model. We follow

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[1] An investment is called to be *relationship-specific*, if the fruit of investment results only from transactions within the specific relationship. After relationship-specific investments are made, the parties find that leaving from the relationship is suboptimal. The ex post trading situation then becomes a *bilateral monopoly,* in which the terms of trade are typically determined by bargaining (without an ex ante agreement).

[2] Another good example is in the relationship of automakers and parts suppliers that mainly attempt cost-reduction by two-sided investments.

[3] See, for example, Hart and Moore (1988), Nöldeke and Schmidt (1995), Aghion et al. (1994), Chung (1991), Edlin and Reichelstein (1996), and Che and Hausch (1999).

the standard assumptions on commitment and incompleteness of contracts: i) the parties cannot commit not to renegotiate the initial contract, and they split the additional surplus according to the Nash bargaining solution. ii) contractible variables are trading decision and transfer payment. The main questions are twofold; under what condition, can incomplete contracting remedy the holdup problem in this situation? Which type of contracts is more effective for improving incentives?

We find the following results. First, noncontingent contracts cannot completely remedy the holdup problem. The reason for this result is that the seller's investment is purely cooperative[1]. As argued in Che and Hausch (1999), contracting is basically discouraging to purely cooperative investment, since such an investment improves the opponent's ex post bargaining position with a contract. This argument directly applies to our model with noncontingent contracts. Second, an option contract can completely remedy the holdup problem in some cases. If the relative importance of the seller's cooperative investment to the buyer's is large, a buyer-option contract might achieve the efficient investment profile. Because of the discreteness of choices, Che-Hausch's argument based on continuous choices might not apply with more complex contracts than noncontingent ones. For some parameter values about the relative importance and the cost of investment, the buyer can appropriately punish the seller only when the seller makes no investment, and the buyer can be a residual claimant. In this way, the holdup problem can be completely solved.

A few papers argue that ex ante contracts may remedy the holdup problem with two-sided investments. Earlier works assume additional contractibility; either contractible renegotiation process (Aghion et al., 1994; Chung, 1991), or contractibility on ex post delivery (Nöldeke and Schmidt, 1995). In contrast, other works are more parsimonious in contractibility, as in our model. Edlin and Reichelstein (1996) consider two-sided selfish investments, and show that under their separability condition, first-best is available. The separability condition is quite strong, however. Guriev (2003) explores a more general model with two-sided, *continuous* investments. He finds that first-best can be implemented if externality of investment is indeed negative or smaller than selfish effect, and the extent of each party's investment externality relative to selfish effect is symmetric. This condition certainly does not apply in our model, and our result differs from his because of the different nature of the investment space.

The structure of the rest is the following: the next section sets up the model. In Section 3, we analyze whether the holdup problem can be remedied by adopting contracts. Section 4 concludes.

2. The Model

We consider a seller (S) that produces one unit of good to a buyer (B). In advance of ex post trade, each party can make a noncontractible relationship-specific investment that affects the value of ex post trade.

Let i^s and i^b denote the seller's and the buyer's investments respectively. Assume that each party's investment is binary, i.e., $i^s \in \{0,1\}$, and $i^b \in \{0,1\}$. Let $\gamma > 0$ denote the monetary costs of investment, which is assumed to be common for both parties.

By the seller's investment $i^s = 1$, the buyer's valuation of the good increases by $a > 0$, whereas by the buyer's investment $i^b = 1$, it increases by $b > 0$. Suppose that investments do not affect the seller's cost of producing a good, which is normalized to 1. For simplicity, assume that $a + b = 2$, and $a \in [0,2]$.

The following table summarizes the consequences of investments:

[1]Following Che and Hausch (1999), an investment is called to be *cooperative* if it has a positive externality to the opponent's payoff from trade. For example, if the seller's investment improves the quality of the good for the buyer, it is cooperative. Meanwhile, an investment is called to be *selfish* if it has no externality. In our model, the buyer's investment is purely selfish whereas the seller's is purely cooperative.

Investment profile (i^s, i^b)	B's value of trade	S's costs of production	ex post efficient surplus	ex ante surplus
(1,1)	2	1	1	$1-2\gamma$
(1,0)	a	1	$[a-1]^+$	$[a-1]^+-\gamma$
(0,1)	2-a	1	$[1-a]^+$	$[1-a]^+-\gamma$
(0,0)	0	1	0	0

where $[\cdot]^+ = \max\{\cdot, 0\}$. Trade (no trade, respectively) is *ex post efficient* if it generates nonnegative (negative, respectively) surplus. The ex post efficient surplus obtains by an ex post efficient trade decision. The ex ante surplus is the ex post efficient surplus minus the costs of investments. All the variables are measured in the monetary term at the time of trade. Also, assume that both parties' payoffs are quasi-linear in monetary transfer.

We follow the standard assumption on contractibility; basically, trade decision and transfer payment are contractible and enforceable. In addition, messages from the parties can be verified by the court, by which option or more complex contracts are available.

We assume that $a \geq \gamma$, $b = 2 - a \geq \gamma$, and $\gamma \leq 1/2$. This implies that the first-best outcome is investment profile (1,1), and trade.

Using the terminology of Che and Hausch (1999), the buyer's investment is purely selfish and the seller's is purely cooperative. The parameter a measures the relative importance of the seller's investment to the buyer's.

3. Analysis

3.1. Investments without an Ex ante Contract

Suppose the parties do not adopt a contract ex ante. After making investments, they decide to trade if and only if trade is ex post efficient. Each party is assumed to receive a half of the ex post surplus by Nash bargaining.

The investment stage is represented by the following strategic game:

$i^s \setminus i^b$	0	1
0	0, 0	$\left[\frac{1-a}{2}\right]^+, \left[\frac{1-a}{2}\right]^+ - \gamma$
1	$\left[\frac{a-1}{2}\right]^+ - \gamma, \left[\frac{a-1}{2}\right]^+$	$\frac{1}{2}-\gamma, \frac{1}{2}-\gamma$

Nash equilibrium of this game depends on the parameter values. The following conclusions are straightforward; If

$$\gamma > \left|\frac{1-a}{2}\right| \Leftrightarrow \begin{cases} \gamma > \frac{1-a}{2} & \text{for } a \leq 1, \\ \gamma > \frac{a-1}{2} & \text{for } a \geq 1. \end{cases}$$

(0,0) is a Nash equilibrium. If

$$\left|\frac{1-a}{2}\right| \leq \frac{1}{2} - \gamma \Leftrightarrow \begin{cases} \gamma \leq \frac{a}{2} & \text{for } a \leq 1, \\ \gamma \leq 1 - \frac{a}{2} & \text{for } a \geq 1. \end{cases}$$

(1,1) is a Nash equilibrium. If

$$\gamma \le \left|\frac{1-a}{2}\right|, \text{ and } \left|\frac{1-a}{2}\right| \ge \frac{1}{2} - \gamma,$$

either (1,0) or (0,1) is a unique Nash equilibrium. For instance, if $a > 1$ and the conditions hold, (1,0) is a unique Nash equilibrium. Figure 1 summarizes equilibrium configurations for different parameter values.

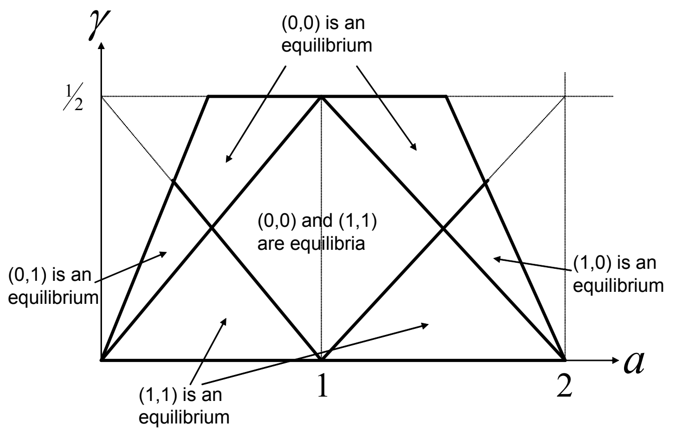
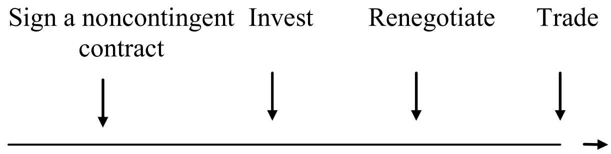


Fig. 1. Equilibria with no ex ante contract

Feasible parameter values are inside the largest trapezoid area. Each area surrounded by bold line segments has the same equilibrium property. The holdup problem is absent if (1,1) can be a Nash equilibrium of the game. Unlike in models with continuous investments, the holdup problem can be absent in some region due to discreteness of investment decision. Naturally we focus on the cases in which the holdup problem is present without an ex ante contract.

3.2. Noncontingent Contracts

We investigate how noncontingent contracts can alleviate the holdup problem. Suppose the parties sign a noncontingent contract before investing. Depending on the subsequent investment profile, the terms of trade in the contract may become ex post inefficient. As in the standard holdup model, we assume that the parties cannot commit not to renegotiate the initial contract. Renegotiation thus occurs if the terms of trade are ex post inefficient.



If renegotiation occurs, the parties split the surplus generated by renegotiation, which is the difference between the ex post efficient surplus and the surplus generated from the initial contract. Assuming the Nash bargaining solution, each party's ex post payoff (EPP) equals the status quo payoff (SQ) that this party obtains by enforcing the initial contract, plus a half of the renegotiation surplus (RS). To calculate ex ante payoffs (EAP), the costs of investments need to count.

Now, we consider a noncontingent contract "trade at price 1". The level of price is indeed immaterial. Since trade is ex post efficient if the investment profile is (1,1), (1,0) for $a \ge 1,$ (0,1) for $a \le 1,$ renegotiation to no trade occurs otherwise.

For example, suppose $a > 1$ and the investment profile is $(0,1)$. Trade is ex post inefficient because the valuation $2-a$ is less than the cost of production, 1. Renegotiation of the initial contract generates the renegotiation surplus of $a-1$. The buyer's status quo payoff from the initial contract is $(2-a)-p = 1-a$, whereas the seller's status quo is $p-1=0$. The buyer's ex post payoff is thus

$$(\text{SQ}) + (\text{RS})/2 = 1 - a + \frac{a-1}{2} = -\frac{a-1}{2},$$

while the seller's is just $(a-1)/2$.

The following tables summarize the consequences of the investing stage: if $a \geq 1$,

(i^s, i^b)	B's SQ	S's SQ	RS	B's EPP	S's EPP	B's EAP	S's EAP
(1,1)	1	0	0	1	0	$1-\gamma$	$-\gamma$
(1,0)	$a-1$	0	0	$a-1$	0	$a-1-\gamma$	$-\gamma$
(0,1)	$1-a$	0	$a-1$	$-\frac{a-1}{2}$	$\frac{a-1}{2}$	$-\frac{a-1}{2}-\gamma$	$\frac{a-1}{2}$
(0,0)	-1	0	1	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$

If $a < 1$,

(i^s, i^b)	B's SQ	S's SQ	RS	B's EPP	S's EPP	B's EAP	S's EAP
(1,1)	1	0	0	1	0	$1-\gamma$	$-\gamma$
(1,0)	$a-1$	0	$1-a$	$-\frac{1-a}{2}$	$\frac{1-a}{2}$	$-\frac{1-a}{2}$	$\frac{1-a}{2}-\gamma$
(0,1)	$1-a$	0	0	$1-a$	0	$1-a-\gamma$	0
(0,0)	-1	0	1	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$

First of all, the seller has a dominant strategy, $i^s = 0$. This immediately implies that the holdup problem cannot completely be remedied.

For the buyer's incentive to invest, consider $a \geq 1$ first. The buyer invests if

$$-\frac{a-1}{2} - \gamma + \frac{1}{2} \geq 0 \Leftrightarrow \gamma \leq 1 - \frac{a}{2}.$$

However, with $a \geq 1$, this region already has (1,1) for an equilibrium. This noncontingent contract thus does not improve the buyer's incentive in this region.

Next, consider $a \leq 1$. The buyer invests if

$$1 - a - \gamma + \frac{1}{2} \geq 0 \Leftrightarrow \gamma \leq \frac{3}{2} - a.$$

In fact, the area satisfying the above inequality covers the entire feasible region with $a \leq 1$.

One might say that the noncontingent contract could induce a better investment profile for the parameter values such that (0,0) is a unique Nash equilibrium without an ex ante contract. This is not entirely true; if $(2-a)-1-\gamma<0 \Leftrightarrow \gamma>1-a,$ the buyer's investment generates less ex post surplus than the cost of investing, and (0,0) is thus more desirable. Therefore, the noncontingent contract partially alleviates the holdup problem if $\gamma \leq 1-a$ (Figure 2).

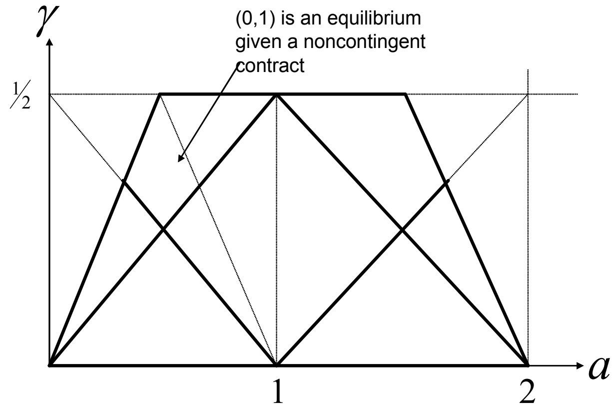


Fig. 2. Partial remedy by noncontingent contracts

What about other noncontingent contracts? Indeed, they cannot achieve first-best either. By quasi-linearity, changing price from $p=1$ does not have any effect on the parties' investment incentives. A noncontingent contract specifying no trade is essentially the same as in no ex ante contract. Thus we have already covered all the noncontingent contracts.

Proposition 1. Noncontingent contracts cannot completely remedy the holdup problem. A noncontingent contract can only improve the buyer's incentives to invest if the cost of investment is high and the buyer's investment has relatively larger effects on its own valuation to the seller's investment.

Two remarks are worth mentioning. First, noncontingent contracts do not improve the seller's investment incentives at all. This is inconsistent with Che and Hausch (1999), who show that, for *cooperative* investments, the null contract (equivalent to no ex ante contract) best enhances the incentives. Second, the above result is asymmetric between the buyer and the seller, because the buyer's investment enhances its own, whereas the seller's investment does not, but the opponent's.

3.3. Buyer-Option Contracts

Consider that the parties adopt the following buyer-option contract: "the buyer can choose either trade at price $1+p$, $p \geq 0$, or no trade". This option is exercised after each party's investment decision is completed. Again, renegotiation occurs if the terms of trade the buyer chooses are not ex post efficient. The seller receives $1+p-1=p$ as its status quo payoff if trade is selected, and 0 otherwise. If renegotiation occurs, it generates renegotiation surplus of either 1, $a-1\,(a \geq 1),\ 1-a\ (a \leq 1)$, which will be equally split to each party.

Case 1: $a \geq 1$.

For each of the buyer's option choices (trade for the upper row, no trade for the lower row), the consequences of investment profiles are as follows:

(i^s, i^b)	B's SQ	S's SQ	RS	B's EPP	S's EPP	B's EAP	S's EAP
(1,1)	$1-p$ 0	p 0	0 1	$1-p$ $\frac{1}{2}$	p $\frac{1}{2}$	$1-p-\gamma$ $\frac{1}{2}-\gamma$	$p-\gamma$ $\frac{1}{2}-\gamma$
(1,0)	$a-1-p$ 0	p 0	0 $a-1$	$a-1-p$ $\frac{a-1}{2}$	p $\frac{a-1}{2}$	$a-1-p$ $\frac{a-1}{2}$	$p-\gamma$ $\frac{a-1}{2}-\gamma$
(0,1)	$1-a-p$ 0	p 0	$a-1$ 0	$-\frac{a-1}{2}-p$ 0	$p+\frac{a-1}{2}$ 0	$-\frac{a-1}{2}-p-\gamma$ $-\gamma$	$p+\frac{a-1}{2}$ 0
(0,0)	$-p-1$ -1	p 0	1 0	$-\frac{1}{2}-p$ 0	$\frac{1}{2}+p$ 0	$-\frac{1}{2}-p$ 0	$\frac{1}{2}+p$ 0

The buyer chooses an alternative that gives a higher *ex post* payoff to itself (recall that the investment costs are already sunk ex post). Since $a \geq 1$ and $p \geq 0,$ it is straightforward that no trade is chosen if the investment profile is either (0,1) or (0,0). The buyer chooses trade for (1,1), if $p \leq 1/2,$ and for (1,0) if $p \leq (a-1)/2 \leq 1/2.$ If $p > 1/2,$ no trade is chosen all the time, and this option contract is thus the same as a noncontingent contract. To provide different incentives by an option contract, $p \leq 1/2,$ and the buyer needs to choose trade at least for (1,1).

We find conditions under which (1,1) can be an equilibrium by adopting this contract.

Case 1a: B selects trade for both (1,1) and (1,0)

This case arises if

$$\frac{a-1}{2} \geq p. \tag{1}$$

In this case, the buyer has indeed an incentive to invest, since

$$(1-p-\gamma)-(a-1-p) = 2-a-\gamma \geq 0$$

(the last inequality holds by assumption).

To check the seller's incentive to invest, first recall that the buyer chooses no trade for (0,1). Since the seller receives $p-\gamma$ for (1,1), we need

$$p-\gamma \geq 0 \tag{2}$$

to induce the seller's incentive to invest.

The conditions (1) and (2) hold if

$$\frac{a-1}{2} \geq \gamma.$$

In the previous analysis, we have seen that there exist parameter values satisfying the above inequality and $a \geq 1,$ such that (1,0) is a unique Nash equilibrium without an ex ante contract. A buyer-option contract can completely remedy the holdup problem for such values.

Case 1b: B selects trade for only (1,1)

This case arises if

$$\frac{a-1}{2} \le p \le \frac{1}{2}.$$

The buyer has an incentive to invest if

$$(1-p-\gamma)-\frac{a-1}{2} \ge 0 \Leftrightarrow \frac{3}{2} \ge p+\gamma+\frac{a}{2}.$$

This condition and (2) hold if (by setting $p=\gamma$)

$$\frac{3}{2} \ge 2\gamma+\frac{a}{2} \Leftrightarrow \gamma \le \frac{3}{4}-\frac{a}{4}.$$

Interestingly, complete remedy can be achieved by adopting the option contract, even in the region where (0,0) is a unique Nash equilibrium without an ex ante contract. This option contract indeed enhances not only the buyer's but also the seller's incentives to invest.

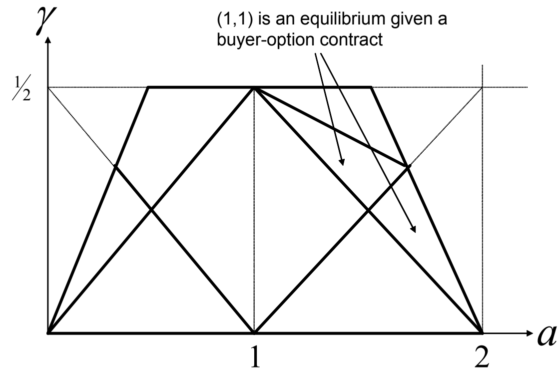


Fig. 3. Complete remedy by buyer-option contracts

Figure 3 shows the two triangular regions in which the buyer-option contract can remedy the holdup problem. In the lower triangle (satisfying $\gamma \le (a-1)/2$), the buyer has no incentive to invest without an ex ante contract. This is due to holdup; the buyer only receives a half of its contribution, $(2-a)/2$, which is smaller than the cost of investment, γ. Indeed, this holdup effect is absent by adopting the buyer-option contract. By setting $p=\gamma$, the seller is just indifferent between investing and not. Since $\gamma \le (a-1)/2$, $p=\gamma$ implies that the buyer selects trade for (1,0). The buyer thus becomes the residual claimant for (1,1) and (1,0), and has the right incentive to invest.

In the upper triangle (satisfying $\gamma \ge (a-1)/2$ and $\gamma \le 3/4-a/4$), the buyer must choose no trade for (1,0) by setting $p=\gamma$. The buyer can hold up the seller by not investing, receiving $(a-1)/2$. The buyer has an incentive to invest if being the residual claimant for (1,1) by paying the investment cost is more profitable than holding up the seller without investing. This holds if $\gamma \le 3/4-a/4$. This inequality is more likely to hold if γ is smaller, and if a is smaller.

Is it possible to induce (1,0) by adopting a buyer-option contract in the region where $\gamma > 3/4-a/4$? The answer is no. If such investment profile is attainable, the buyer must choose trade for (1,0) (remember that a noncontingent contract cannot provide an incentive to invest to the seller). Then, the same conditions as in Case 1a must hold, i.e., $\gamma \le (a-1)/2$, which is inconsistent with $\gamma > 3/4-a/4$.

Case 2: $a<1$.

Recall that for $a<1$, (0,1) can be an equilibrium. We only need to check if the seller's investment incentive is restored by an option contract and thus (1,1) can be an equilibrium. Indeed, such remedy is not available as the following argument shows.

Consider the following partial table for $a < 1$:

(i^s, i^b)	B's SQ	S's SQ	RS	B's EPP	S's EPP	B's EAP	S's EAP
(1,1)	$1-p$ 0	p 0	0 1	$1-p$ $\frac{1}{2}$	p $\frac{1}{2}$	$1-p-\gamma$ $\frac{1}{2}-\gamma$	$p-\gamma$ $\frac{1}{2}-\gamma$
(0,1	$1-a-p$ 0	p 0	0 $1-a$	$1-a-p$ $\frac{1-a}{2}$	p $\frac{1-a}{2}$	$1-a-p-\gamma$ $\frac{1-a}{2}-\gamma$	$p-\gamma$ $\frac{1-a}{2}$

The buyer selects trade for (1,1) if $p \le \frac{1}{2}$, and for (0,1) if $p \le \frac{1-a}{2} \le \frac{1}{2}$. This implies that the only possibility that the buyer selects different alternatives is to select trade for (1,1) and no trade for (0,1) by setting

$$\frac{1-a}{2} \le p \le \frac{1}{2}.$$

The seller has an incentive to invest if

$$p - \gamma \ge \frac{1-a}{2} \Leftrightarrow p \ge \frac{1-a}{2} + \gamma.$$

These conditions hold only if

$$\frac{1}{2} \ge \frac{1-a}{2} + \gamma \Leftrightarrow \gamma \le \frac{a}{2}.$$

However, for these parameter values, (1,1) can be a Nash equilibrium without an ex ante contract. Consequently, for $a \le 1$, buyer-option contracts do not improve investment incentives.

In this case, the seller's opportunity to hold up the buyer by not investing is substantial ($(1-a)/2$). This requires the price p to be high in order for the seller to invest and to trade at p. To be consistent with these, γ must be low enough, and a buyer-option turns out to be ineffective to enhance incentives.

Proposition 2. Buyer-option contracts can completely remedy the holdup problem for some parameter values. Remedy is available only if the seller's investment is more important than the buyer's $(a \ge 1)$.

3.4. Seller-Option Contracts

We investigate the performance of seller-option contracts, but indeed show that they do not help improve investment incentives. Note that we can use the same tables as above to study seller-option contracts. The only difference is the choice of options; now the seller chooses an alternative that gives a higher ex post payoff to itself.

Case 1: $a \ge 1$.

Consider the region where the buyer has no incentive to invest without an ex ante contract or with a noncontingent contract ($\gamma > 1 - a/2$). If a seller-option induces the buyer to invest, then the seller must choose different alternatives, say, for (1,1) and (1,0). This is only possible if

$$\frac{a-1}{2} \le p \le \frac{1}{2},$$

and the seller selects trade only for (1,0). The buyer has an incentive to invest if

$$\frac{1}{2} - \gamma \geq a - 1 - p.$$

These conditions imply

$$\frac{1}{2} - \gamma \geq \frac{a-1}{2} \Leftrightarrow \gamma \leq 1 - \frac{a}{2}.$$

However, this condition is inconsistent with $\gamma > 1 - a/2$.

Alternatively, suppose the seller must choose different alternatives, say, for (0,1) and (0,0). This is only possible if

$$-\frac{1}{2} \leq p \leq -\frac{a-1}{2}.$$

The buyer has an incentive to invest if

$$-\gamma \geq -\frac{1}{2} - p.$$

These conditions imply

$$-\frac{1}{2} + \gamma \leq -\frac{a-1}{2} \Leftrightarrow \gamma \leq 1 - \frac{a}{2}.$$

However, this condition is also inconsistent with $\gamma > 1 - a/2$.

Consider the region where (0,0) is an equilibrium without an ex ante contract or with any other noncontingent contract ($\gamma > (a-1)/2$ and $\gamma > 1 - a/2$). We show that inducing (1,0) is not available by a seller-option contract. If these were available, the seller selects trade for (0,0) and no trade for (1,0), by setting

$$-\frac{1}{2} \leq p \leq \frac{a-1}{2}.$$

The seller has an incentive to invest if

$$\frac{1}{2} + p \leq \frac{a-1}{2} - \gamma \Leftrightarrow p \leq \frac{a-2}{2} - \gamma.$$

These conditions hold if

$$-\frac{1}{2} \leq \frac{a-2}{2} - \gamma \Leftrightarrow \gamma \leq \frac{a-1}{2},$$

incompatible with $\gamma > (a-1)/2$.

Case 2: $a < 1$.

Recall that (0,1) can be already an equilibrium in the region where (1,1) is not an equilibrium. For a seller-option contract to induce seller's investment incentive, the seller must select an alternative for (1,1) different from one for (0,1). This only arises if

$$\frac{1}{2} \geq p \geq \frac{1-a}{2},$$

and the seller selects no trade for (1,1) and trade for (0,1). The seller has an incentive to invest if

$$\frac{1}{2} - \gamma \geq p.$$

These conditions hold if

$$\frac{1}{2} - \gamma \geq \frac{1-a}{2} \Leftrightarrow \gamma \leq \frac{a}{2},$$

which is the region where the null contract achieves efficiency. Therefore seller-option contracts do not improve investment incentives.

4. Conclusion

This paper theoretically studies a simple, discrete holdup model with two-sided investments, in which each party's investment enhances the value of transaction by improving a common factor (the buyer's valuation, in text). In other words, we analyze a holdup model with a purely cooperative investment on the one hand, and a purely selfish investment on the other hand. We investigate whether contractual remedies to the holdup problem are available by adopting noncontingent or option contracts.

The main result of this paper is that an option contract can completely remedy the holdup problem in several cases. Remedies by option contracts are more likely as the relative importance of the purely cooperative investment to the purely selfish investment becomes larger, and as the cost of investment becomes smaller. By granting an option to trade to the party owning a selfish investment, this party has the right to punish the other party if no investment is made. This can limit the holdup power of the other party, as well as may give the right incentive to invest. This mechanism does not always work, however, because the party granted an option might want to abuse it for its own sake. In particular, an option granted to the party owning a cooperative investment does not work. Observe that exercising trade option with cooperative investment improves the opponent's bargaining position. This hinges to limit the opponent's holdup power, and it turns out that option contracts are at most as welfare improving as noncontingent contracts.

This result is in contrast to Che and Hausch (1999), who show that incentives to make purely cooperative investment are best enhanced without an ex ante contract, and that it is not possible to achieve the first-best outcome with purely cooperative investments (except that one party has all the ex post bargaining power). The combination of two-sided, common-purpose investments, and the discreteness of the model are keys to our conclusion that an option contract, not a noncontingent contract, is optimal, and the first-best outcome can be achieved in the presence of purely cooperative investment.

One limitation of our analysis is that we have not considered a more complex contract involving message games. An optimal contract is thus unknown except for the cases in which first-best can be achieved in our analysis. This is left for the future research.

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