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# AUTOMATED SOFTWARE FOR ELASTIC ANISOTROPY RESEARCH

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The new automated software was developed to research the directional distribution of such elastic values as phase velocities of quasi-longitudinal wave, «quick» and «slow» quasi-transverse waves, angle between wave normal and elastic displacement vector of quasi-longitudinal wave, differential coefficients of elastic and transverse anisotropy, Young's modulus, Poisson's ratio and shear modulus. It was shown the importance of a standard acoustic coordinate system whose basis is the acoustic tensor's eigenvectors. It allows orienting the stiffness tensor of the geological environment in the direction of its main symmetry elements.

The software also allows to approximate the stiffness tensor by the nearest isotropic, transversely isotropic and rhombic symmetry which significantly simplifies the further calculations.

*Key words*: elastic anisotropy, stiffness tensor, phase velocity, anisotropy coefficient, Young's modulus, Poisson's ratio, stereo-projection

# АВТОМАТИЗОВАНА ПРОГРАМА ДЛЯ ДОСЛІДЖЕННЯ ПРУЖНОЇ АНІЗОТРОПІЇ

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Нове автоматизоване програмне забезпечення було розроблено для дослідження спрямованого розподілу таких пружних властивостей, як фазові швидкості квазіподовжньої хвилі, швидкості «швидкої» та «повільної» квазіпоперечних хвиль, кут між хвильовою нормаллю та вектором пружного зміщення квазіподовжньої хвилі, диференціальні коефіцієнти пружної та поперечної анізотропії, модуль Юнга, коефіцієнт Пуассона та модуль зсуву. Показано важливість стандартної акустичної системи координат, основою якої є власні вектори акустичного тензора. Це дозволяє орієнтувати пружний тензор геологічного середовища у напрямку його основних елементів симетрії.

Програмне забезпечення також дозволяє наблизити тензор жорсткості до найближчої ізотропної, поперечно ізотропної та ромбічної симетрії, що значно спрощує подальші розрахунки.

*Ключові слова:* пружна анізотропія, пружний тензор, фазова швидкість, коефіцієнт анізотропії, модуль Юнга, коефіцієнт Пуасона, стереопроекція

# АВТОМАТИЗИРОВАННАЯ ПРОГРАММА ДЛЯ ИССЛЕДОВАНИЯ УПРУГОЙ АНИЗОТРОПИИ

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Новое автоматизированное программное обеспечение было разработано для исследования направленного распределения таких упругих свойств, как фазовые скорости квазипродольной волны, скорости «быстрой» и «медленной» квазипоперечных волн, угол между волновой нормалью и вектором упругого смещения квазипродольной волны, дифференциальные коэффициенты упругой и поперечной анизотропии, модуль Юнга, коэффициент Пуассона и модуль сдвига. Показана важность стандартной акустической системы координат, основой которой есть собственные векторы акустического тензора.

Программное обеспечение также позволяет ориентировать упругий тензор геологической среды в направлении его основных элементов симметрии.

*Ключевые слова:* упругая анизотропия, упругий тензор, фазовая скорость, коэффициент анизотропии, модуль Юнга, коэффициент Пуассона, стереопроекция

## Introduction

Research into anisotropy of elastic properties and seismic waves of rocks is of great interest in terms of oil and gas deposits exploration and applying seismic methods and techniques in assessment of the abnormally high pressures at great depths, crack distribution, rocks bedding, as well as geophysical monitoring of such hazards as landslides, mudflows, soil subsidence, etc.

The new automated software was developed on C# WPF language for elastic anisotropy research of ordered geological medium with any symmetry using its stiffness tensor and density.

## Theory

As a priori elastic symmetry and spatial orientation of symmetry elements in ordered geological environment is unknown, acoustic symmetric tensor $\mu_{il}$ is used to define it. This tensor is a convolution of the stiffness tensor by a pair of external or internal indices (Fedorov, 1965):

$$\rho\mu_{il} = C_{ijjl} \tag{1}$$

In any direction of the wave normal it can be determined by the sum of squared phase velocities with orthogonal polarization:

$$\vec{\mu}(\vec{n}) = \mu_{il} n_i n_l = v_1^2 + v_2^2 + v_3^2 \quad (2)$$

As it follows from the characteristic equation of acoustic tensor:

$$(\mu_{il} - \lambda\delta_{il})X_l = 0, \quad (3)$$

where $\lambda$ – scalar; tensor has three eigenvalues $\mu_1, \mu_2, \mu_3$. Each eigenvalue corresponds to its own eigenvector $\vec{X}', \vec{X}'', \vec{X}'''$. Eigenvectors form three mutually orthogonal vectors. Directions of acoustic tensor's eigenvectors coincide with the orientation of rock anisotropic symmetry elements - they either collinear to symmetry axes or orthogonal to symmetry planes. So acoustic tensor can be used to make the clear natural choice of symmetry axes in monoclinic and triclinic crystals and rocks (Alexandrov, Prodayvoda, 2000; Fedorov, 1965).

Considering these peculiarities, right three eigenvectors of acoustic tensor are used as the basis of standard acoustic coordinate system (Prodayvoda, 1998).

For a detailed classification of the geological medium symmetry group the stiffness tensor symmetry in the standard acoustic coordinate system was used. The transformation of stiffness tensor's components from the initial coordinate system $C'_{mnpq}$ to the stiffness tensor's components in the standard acoustic coordinate system $C_{ijkl}$, which was found by the formula:

$$C_{ijkl} = a_{im} a_{jn} a_{kp} a_{lq} C'_{mnpq}, \quad (4)$$

where $a_{im}$ –cosines of angles between acoustic tensor's eigenvectors and initial coordinate system axes.

Table 1 shows the input stiffness tensor for Vosges Sandstone (a) taken from the article (Mensch, Rasolofosaon, 1997) and rotated stiffness tensor (b) and compliance tensor in the standard acoustic coordinate system. The density of Vosges Sandstone equals to 2080 kg/m$^3$.

**Table 1.** Elastic tensors for Vosges Sandstone: a) initial stiffness tensor from (Mensch, Rasolofosaon, 1997), in GPa; b) transformed stiffness tensor in standard acoustic coordinate system, GPa; c) compliance tensor in standard acoustic coordinate system, $TPa^{-1}$.

a)

| | | | | | |
|---|---|---|---|---|---|
| 10,3 | 0,9 | 1,3 | 1,4 | 1,10 | 0,8 |
| | 10,6 | 2,1 | 0,2 | -0,2 | -0,6 |
| | | 14,1 | 0 | -0,5 | -1 |
| | | | 5,10 | 0 | 0,2 |
| | | | | 6 | 0 |
| | | | | | 4,90 |

b)

| | | | | | |
|---|---|---|---|---|---|
| 10,27 | 0,32 | 1,42 | 1,2 | 0,76 | 0,32 |
| | 11,17 | 2,5 | 0,19 | -0,48 | -0,21 |
| | | 13,7 | -0,04 | -0,51 | -0,71 |
| | | | 5,12 | -0,1 | -0,25 |
| | | | | 6,57 | -0,15 |
| | | | | | 4,26 |

c)

| | | | | | |
|---|---|---|---|---|---|
| 103,4 | -0,73 | -11,81 | -25,24 | -13,65 | -11,67 |
| | 93,65 | -16,74 | -3,26 | 5,61 | 1,98 |
| | | 78,38 | 4,88 | 6,62 | 13,6 |
| | | | 202,42 | 6,56 | 14,79 |
| | | | | 154,92 | 8,14 |
| | | | | | 239,38 |

The advantages of standard acoustic coordinate system is shown on Figure 1. It represents the mean-quadratic difference (described in formula (11)) between triclinic stiffness tensor of Vosges Sandstone stiffness tensor and its nearest transversely-isotropic approximation. The maximum value of this coefficient decreased from 17,1% to 12,6%, and the mean value from 6,22% to 5,8%, indicating a more precise definition of transversely-isotropic approximation in standard acoustic coordinate system. For minerals this coefficient drops even higher. For example mean value of differential coefficient of relative mean-quadratic transverse anisotropy for kaolinite (Vyzhva, Prodayvoda, Vyzhva, 2011, 2012, 2013) decreased from 11,7% in initial coordinate system to 2,5% in standard acoustic coordinate system, and its maximum value from 33,1% to 16,8% respectively.

The phase velocities and polarization vectors of elastic waves were found by solving Green-Christoffel's equation (Prodayvoda, Kuzmenko, Vyzhva, 2015):

$$(\Gamma_{il} - \rho v^2 \delta_{il}) U_l = 0, \qquad (5)$$

where $\Gamma_{il} = C_{ijkl} n_j n_k$ – Christoffel's tensor; $v$ – phase velocity; $n_j$ – wave normal vector; $C_{ijkl}$ – stiffness tensor; $U_l$ – elastic displacement vector components; $\rho$ – density; $\delta_{il}$ – Kroneker's delta.

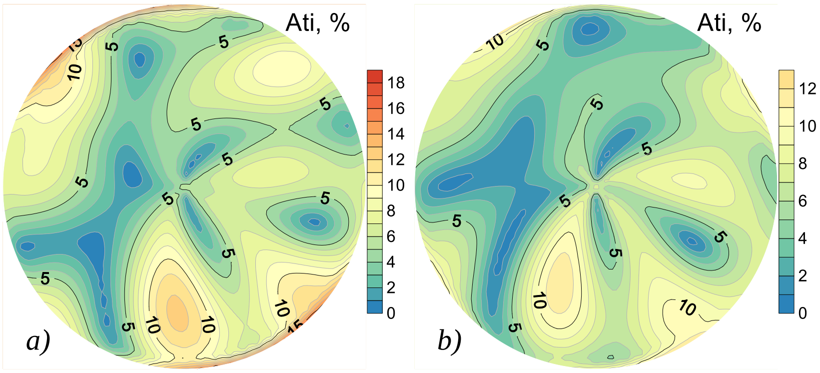


**Fig. 1.** Differential coefficient of relative mean-quadratic transverse anisotropy for Vosges Sandstone, in %: a) initial coordinate system; b) standard acoustic coordinate system.

If the wave normal vector is given then from the condition of equation (5) solution existence:

$$|\Gamma_{il} - \rho v^2 \delta_{il}| = 0, \quad (6)$$

eigenvalues of Christoffel's tensor $\Gamma_{il}$ can be found, that up to a constant ρ determine phase velocities of quasi-longitudinal ($v_1$) and two quasi-transverse waves - "quick" $v_2$ and "slow"$v_3$ ($v_1 > v_2 \geq v_3$).

Elastic displacement vectors can be found from equation (Fedorov, 1965):

$$U_j U_m = \frac{(\overline{\Gamma - \rho v^2})_{jm}}{(\overline{\Gamma - \rho v^2})_c}, \quad (7)$$

where $(\overline{\Gamma - \rho v^2})_{jm}$ – adjugate tensor to $(\Gamma - \rho v^2)_{jm}$; c – tensor trace.

Figure 2 shows the spherical projections and stereo-projections of phase velocities for Vosges Sandstone. This distribution coincides with those in article (Mensch, Rasolofosaon, 1997) but provides better and accurate visualization. Spherical projections Figure 2 a), b), c) represent three-dimensional distribution of elastic parameters and they show values with no distortion in 3D space. And stereo-projections (Figure 2 d), e), f)) that are made on modified Wulff's grid, give full 180° view compare to approximately 120° in sphere.

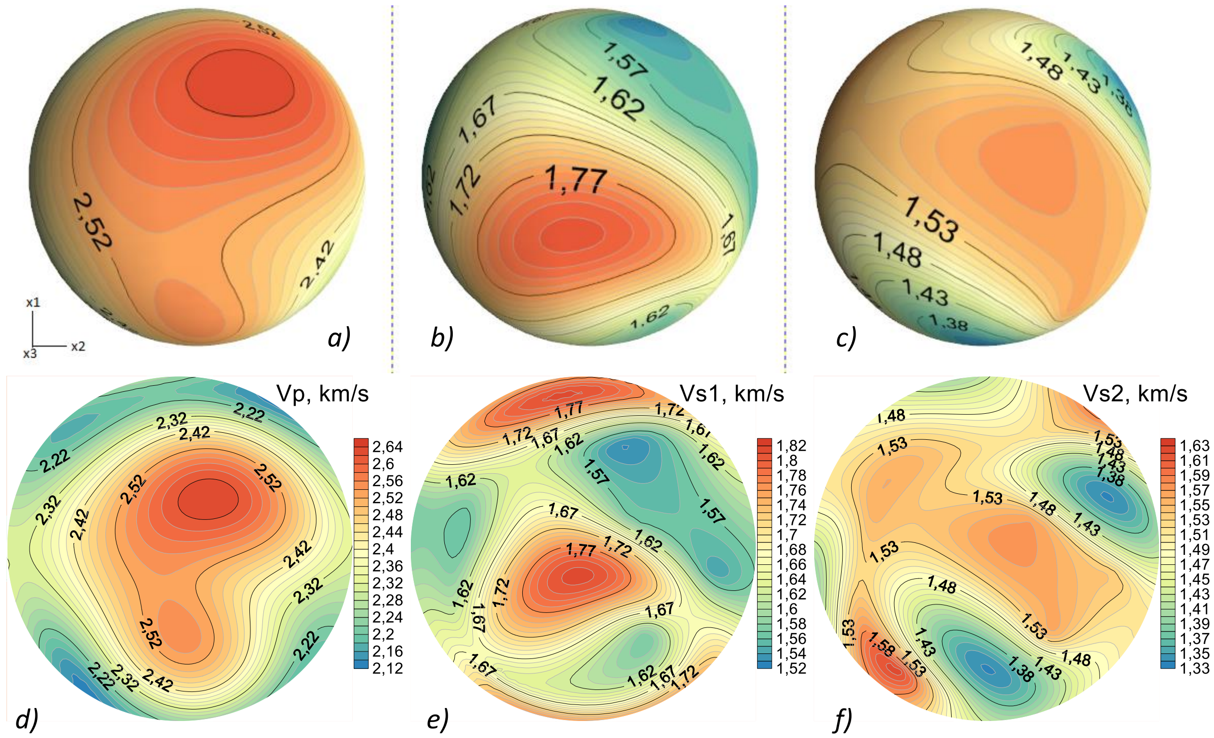


**Fig. 2.** 3D spherical projections (upper) and stereo-projections (lower) of phase velocities for Vosges Sandstone, in km/s: a),d) quasi-longitudinal wave; b),e) “quick” quasi-transverse wave; c),f) “slow” quasi-transverse wave.

To estimate the accuracy of the transversely isotropic approximation of the triclinic elastic constant matrix the transversely isotropic anisotropy coefficient (Alexandrov, Prodayvoda, 2000; Fedorov, 1965) was used, which characterizes the deviation of the triclinic elastic anisotropy from the most similar to it transversely isotropic medium. Analytically, the elastic constants of a transversely isotropic medium can be derived from the following condition (Prodayvoda G.T., Cholach P.Y., 1998):

$$\left[\left(\Lambda_{il}-\Lambda_{il}^{(t)}\right)^2\right]_C = \min, \tag{8}$$

where $\Lambda_{il} = C_{ijkl}\, n_j\, n_k / \rho$, $\Lambda_{il}^{(t)} = C_{ijkl}^{(t)}\, n_j\, n_k / \rho$; $C_{ijkl}^{(t)}$ – the stiffness tensor of an unknown transversely isotropic medium, which is most similar to the elastic medium of the triclinic matrix with stiffness tensor $c_{ijkl}$ and density ρ; c – tensor trace.

To solve this equation the covariant form of Kristoffel’s tensor was used when choosing the coordinate system:

$$\Lambda_{il}^{(t)} = a_0\delta_{il} + a_1 n_i n_l + a_2 e_i e_l + a_3 c_i c_l, \tag{9}$$

where $c_i = [\vec{e}\vec{n}]_i$; $n_i$ – components of the wave normal vector; $e_i$ – components of the vector, which determines the direction of the main symmetry axis of the transversely isotropic medium.

The solution of equation (8) yields the unknown factors:

$$a_0 = \frac{(1+n_3^2)[\vec{e}\vec{c}]\Lambda[\vec{e}\vec{c}] - \vec{n}\Lambda\vec{n} + n_3^2\vec{e}\Lambda\vec{e}}{2n_3^2}, \qquad a_1 = \frac{\vec{n}\Lambda\vec{n} - n_3^2\vec{e}\Lambda\vec{e} - [\vec{n}\vec{e}]^2[\vec{e}\vec{c}]\Lambda[\vec{e}\vec{c}]}{2n_3^2[\vec{n}\vec{e}]^2},$$

$$a_2 = \frac{(1-2n_3^2)\vec{n}\Lambda\vec{n} + n_3^2\vec{e}\Lambda\vec{e} - [\vec{n}\vec{e}]^2[\vec{e}\vec{c}]\Lambda[\vec{e}\vec{c}]}{2n_3^2[\vec{n}\vec{e}]^2}, \qquad a_3 = \frac{3n_3^2\vec{c}\Lambda\vec{c} - n_3^2\Lambda c + \vec{n}\Lambda\vec{n} - [\vec{e}\vec{c}]\Lambda[\vec{e}\vec{c}]}{2n_3^2}. \tag{10}$$

In order to compare the elastic properties of the triclinic and transversely isotropic symmetry, the value of differential coefficient of the relative mean-squared transversely isotropic elastic anisotropy ($A_t$) was used. This coefficient can be derived from the following formula (Prodayvoda, Cholach, 1998):

$$A_t = \left[\frac{(\Lambda^2)_c - (\Lambda^{(t)2})_c}{(\Lambda^2)_c}\right]^{1/2} \cdot 100\%, \tag{11}$$

where $(\Lambda^2)_c = \Lambda_{11}^2 + \Lambda_{22}^2 + \Lambda_{33}^2 + 2(\Lambda_{12}^2 + \Lambda_{13}^2 + \Lambda_{23}^2)$; $(\Lambda^{(t)2})_c = \Lambda_{11}^{(t)2} + \Lambda_{22}^{(t)2} + \Lambda_{33}^{(t)2} + 2(\Lambda_{12}^{(t)2} + \Lambda_{13}^{(t)2} + \Lambda_{23}^{(t)2})$.

In a similar way, the integral and differential coefficients of elastic anisotropy can be calculated, which indicate how the specified anisotropic medium with an arbitrary symmetry differs, in an average or certain direction, from the most similar to it isotropic medium. To do this the isotropic medium tensor should be entered into the formula (8):

$$\Lambda_m = a + b\vec{n}\cdot\vec{n}, \quad a = \frac{1}{2}(\langle\Lambda\rangle_c - \langle\vec{n}\Lambda\vec{n}\rangle), \quad b = \frac{1}{2}(3\langle\vec{n}\Lambda\vec{n}\rangle - \langle\Lambda\rangle_c). \tag{12}$$

To research the azimuthal anisotropy of elastic moduli their characteristic surfaces can be used. The characteristic surface of Young's modulus $E(\vec{l})$, Poisson's ratio $\nu(\vec{l})$ and shear modulus $G(\vec{p},\vec{q})$ can be calculated from works (Prodayvoda, Kuzmenko, Vyzhva, 2015; Alexandrov, Prodayvoda, 2000).

Figure 3 displays stereo-projections of elastic values for Vosges Sandstone and shows its triclinic symmetry. The maximum coefficient value of relative mean-quadratic elastic anisotropy (Fig. 3,b) is 19,5%, that almost coincides with work (Mensch, Rasolofosaon, 1997), where anisotropy is 21,1%. Transverse anisotropy coefficient (Fig. 3,c) maximum is 12,6%, and the mean value is 5,8%. The Young's modulus values change from 8,62 GPa to 13,93 GPa, Poisson's ratio from 0 to 0,23 and shear modulus 3,79 GPa to 6,5 GPa that shows a relatively high anisotropy of Vosges Sandstone.

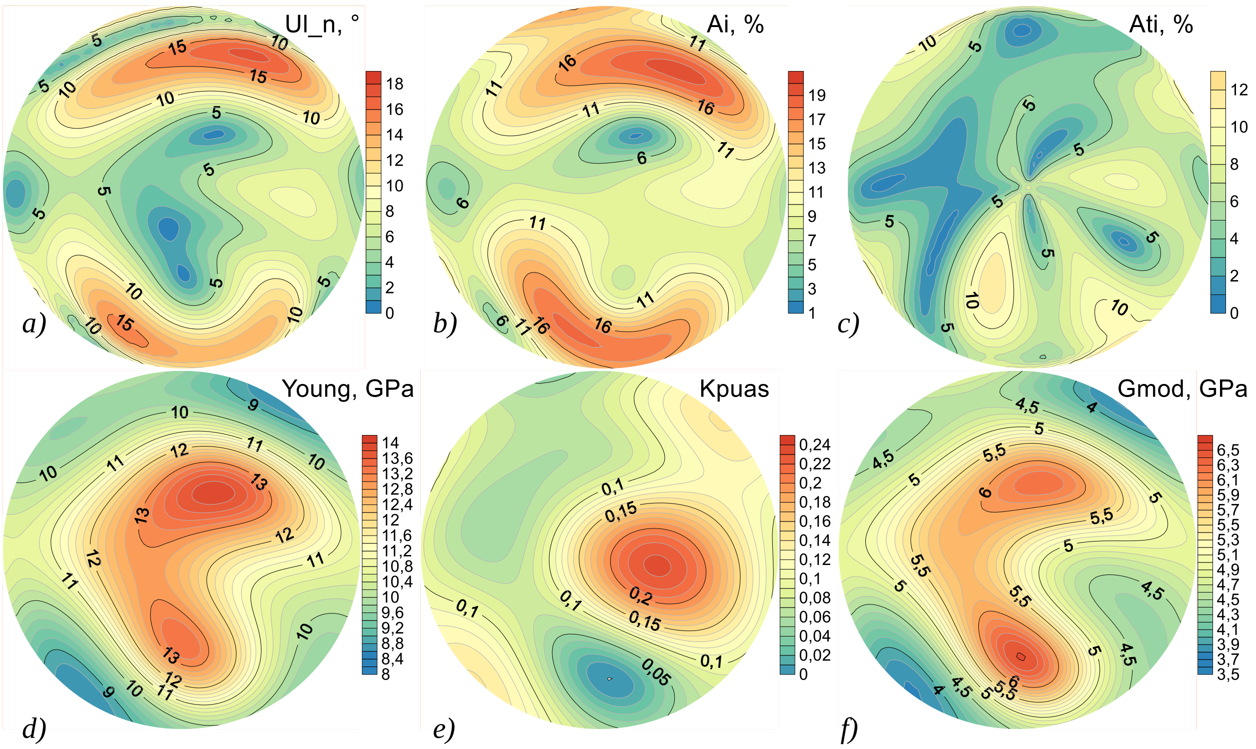


**Fig. 3.** Stereo-projections of elastic values for Vosges Sandstone: a) angle between elastic displacement vector of quasi-longitudinal wave and wave normal vector, in degrees; b) differential coefficient of relative mean-quadratic elastic anisotropy, in %; c) differential coefficient of relative mean-quadratic transverse anisotropy, in %; d) Young's modulus, in GPa; e) Poisson's ratio; f) Shear modulus, in GPa.

## Conclusions

The new automated software was developed to research the directional distribution of such elastic values as phase velocities of quasi-longitudinal wave, "quick" and "slow" quasi-transverse waves, angle between wave normal and elastic displacement vector of quasi-longitudinal wave, differential coefficients of elastic and transverse anisotropy, Young modulus, Poisson's ratio and shear modulus. It was shown the importance of a standard acoustic coordinate system whose basis is the acoustic tensor's eigenvectors. It allows orienting the stiffness tensor of the geological environment in the direction of its main symmetry elements. The software also allows to approximate the stiffness tensor by the nearest isotropic, transverse and rhombic symmetry which significantly simplifies the further calculations.

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