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# METHODS OF SYSTEM POLICY IN CREATION OF INFOCOMMUNICATION SYSTEMS



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Abstract - Some results of scientific researches of the Department of Telecommunication Systems (KNURE) connected with employment of stochastic dynamic models in a state space are presented. It is argued that the representation of the model by equations of state is more general and more complete because it characterizes a stochastic process as a whole but not just its private probabilistic characteristic represented by the density or distribution function. The necessity to consider the interconnections for multidimensional systems that can improve the assessment accuracy of the state system is discussed. The level and quality of these connections contributes to the attainment of its highly-integrated emergence properties, a measure which can be the measure of Watanabe. The known threshold of the critical connectivity of multidimensional differential system at the level of 13% requires a more detailed study using the theory of stability. Methods of analysis and synthesis presented in the article are only a part of the department system researches and they aimed at creating a theory of infocommunication systems.

Анотація – Надаються деякі результати наукових досліджень кафедри телекомунікаційних систем ХНУРЕ, пов'язані з використанням стохастичних динамічних моделей у просторі станів. Стверджується, що представлення моделі рівняннями стану більш універсально і більш повно, оскільки характеризує стохастичний процес в цілому, а не тільки його окрему ймовірнісну характеристику, яка представляється через щільність або функцію розподілу. Обговорюється необхідність врахування взаємних зв'язків для багатовимірних систем, що дозволяє підвищити точність оцінки стану системи, а рівень і якість цих зв'язків сприяє набуттю нею зверхінтегральних властивостей емерджентності, мірою яких може служити міра Ватанабе. Відома межа критичної зв'язності багатовимірної диференціальної системи на рівні 13% вимагає більш детального дослідження з використанням теорії стійкості. Методи аналізу і синтезу, представлені в статті, є тільки частиною системних досліджень кафедри і спрямовані на створення теорії інфокомунікаційних систем.

Аннотация – Представляются некоторые результаты научных исследований кафедры телекоммуникационных систем ХНУРЭ, связанные с использованием стохастических динамических моделей в пространстве состояний. Утверждается, что представление модели уравнениями состояния более универсально и более полно, поскольку характеризует стохастический процесс в целом, а не только его частную вероятностную характеристику, представимую через плотность или функцию распределения. Обсуждается необходимость учета взаимных связей для многомерных систем, что позволяет повысить точность оценки состояния системы, а уровень и качество этих связей способствуют обретению ею сверхинтегральных свойств эмерджентности, мерой которых может служить мера Ватанабе. Известный предел критической связности многомерной дифференциальной системы на уровне 13% требует более детального исследования с использованием теории устойчивости. Методы анализа и синтеза, представленные в статье, являются только частью системных исследований кафедры и направлены на создание теории инфокоммуникационных систем.

We all notice how our way of life changes on the background of the rapid progress of infocommunications and telecommunications in particular, which is its material basis. Progressive development is mainly determined by the achievements of science and technology. A feature of our branch is that its development is primarily based on technological advances. However, new technological solutions emerge on the basis of reasonability and engineering intuition. At the same time, the sectorial science as an integral phenomenon has not yet been created. We can only talk about individual sections for analysis and synthesis, research and optimization of various models. Generalizing science based on the theory of systems does not exist yet.

We do not intend to infringe on any priority in the development of the theory of telecommunication systems. We can only say that we, like many thousands of scientific teams, are trying to make a contribution to its development as much as possible. "We" means our University and our Department of Telecommunication Systems in particular, which conducts considerable researches that are more or less related to the development of the theory of telecommunication systems. At our department there are 10 professors, doctors of science, 31 associate professors; we have Ph.D courses and external PhD courses. Let us briefly comment on some of the system scientific researches conducted at the Department.

A big team can afford conducting research in a rather wide field. We are engaged in field observations of model sets, subnets as well as in research of new technological solutions for analysis and synthesis of structural and functional specific features of telecommunication networks. For this we have a modern educational and scientific laboratory, the most comprehensive set of network elements, systems and technological software solutions, including equipment of Cisco, Awaji, Samsung, D-Link, Monis etc.

The basis for the future science of the theory of telecommunication (infocommunications) systems is various mathematical models [1]. In this direction, there is a place for research of mathematical models, including simple and complex, deterministic and stochastic, static and dynamic, structural and functional ones.

When conducting analysis and synthesis of models we use different mathematical tools: mathematical statistics, probability theory, stochastic processes, functional analysis, tensor and simplex analysis, theory of estimations and management, network theory, graph theory, optimization methods of structures and functions of systems. The methods of modeling, analysis and synthesis are based on the theory of systems and system policy rules (IETF REC 3198) (fig. 1).



Fig. 1. Presentation of system policy rules components

Here is one interesting fact. Usually models (physical or mathematical) represent reduced, simplified object as opposed to the system S(t, x) in general. At the same time an opposite tendency is used when a model turns out to be a more complicated variety in comparison to the system, which is being modeled. Models of neural networks can serve as an example [2].

Due to such varieties it is possible to set up these models or train them to give an adequate response. It this case it is worth mentioning Ashby's principle of requisite varieties [3, 4], according to which a variety with the power of  $V_D$  can be neutralized (compensated) only at the account of another variety  $V_N$  that has bigger power (fig. 2).



Fig. 2. Illustration to Ashby's principle

The basic mathematical models of telecommunication systems have stochastic nature. It is known that a telecommunications system is the largest artificially created system in the world. It is seems to be already stochastic because the traffic that it processes is random. Different mechanisms that influence the state of the system are random as well.

There are two most frequently used representations of stochastic models that are represented through the density or probability distribution function:

$$p(x) = dF(x)/dx, F(x) = \int_{-\infty}^{x} p(x)dx, \qquad (1)$$

where *x* is a random value,

$$p(x_1, x_2, ..., x_n; t_1, t_2, ..., t_n) = \partial^n F(x_n, t_n) / \partial^n x_n,$$
(2)

where  $x_i$  or x(t) is a random value.

An alternative view is the state of the stochastic system, formulated on the basis of diffusion-type processes:

- integrated model of the diffusion process [5]

$$x(t) = x(0) + \int_{T} \theta(t) dt + \int_{T} \Phi(t) dW(t), \qquad (3)$$

where *W*(*t*) is the Wiener process;

- differential state model [6]

$$dx(t)/dt = F(t)x(t) + G(t)\xi(t),$$
(4)

where  $\xi(t) = dW(t)/dt$  is the generating white Gaussian noise.

In addition, the given representations have their advantages and disadvantages. Thus the representation via the distribution density (1), (2) is more common and demonstrable. At the same time the state-space representation (3), (4) is more general and complete because it characterizes the stochastic process as a whole, not just its particular probabilistic characteristics. Both representations are applicable for static and dynamic systems. First of all we are interested in the dynamic states, the model of which is a random process, displayed by differential or difference equation (4).

To represent the process through the probability density function the Fokker-Planck-Kolmogorov equation is used [6, 7].

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$$\partial p(\lambda,t) / \partial t = -\partial [a(\lambda,t)p(\lambda,t)] / \partial \lambda + \frac{1}{2} \partial^2 [b(\lambda,t)p(\lambda,t)] / \partial \lambda^2.$$
(5)

For a stream of random events Erlang equation is used [7].

$$\begin{cases} dP_k(t)/dt = -(\lambda_k + \mu_k)P_k(t) + \lambda_{k-1}P_{k-1}(t) + \mu_{k+1}P_{k+1}(t), k \ge 1 \\ dP_0(t)/dt = -\lambda_0 P_0(t) + \mu_1 P_1(t), k = 0. \end{cases}$$
(6)

The complexity of the solutions of these equations is known. In particular, in queuing theory Erlang equations (6) are not solved at all, the derivative of the density equal to zero is immediately assumed and then we consider random values but not a process. Let us focus on the representation of models in state space.

The state-space representation of a random process (4) in addition to greater contentrichness allows to obtain universality of differential models, suitable for purely deterministic, mixed and purely random processes. Moreover, these representations adjust well with a number of tasks such as identification, adaptation, self-organization, management, and other tasks that play an increasingly important role in ensuring the reliability and quality of service in telecommunication systems [8].

As an example let us consider the estimation procedure used to solve many problems in the NGN technology, software-configurable networks (SDN) etc.

An optimal procedure for the assessment of random processes x(k) is the Kalman-Bucy filter (KBF) [9, 10].

$$d\hat{x}(t)/dt = F(t)\hat{x}(t) + K(k)[H\hat{x}(t) - y(t)],$$
(7)

where  $K(k) = HVN^{-1}$  - coefficient under a residual error,

y(t) = Hx(t) + v(t) - watch (measurement) equation.

However, in practice we often use a simpler procedure of stochastic approximation (SA), which is the transformation of the classical Robbins-Monro procedure [11]

$$x(k+1) = x(k)(1-\mu) + \mu y(k) = x(k) - \mu [x(k) - y(k)],$$
(8)

where  $\mu$  is a stepping constant that determines stability of the system and its convergence rate.

It is known that the SA procedure is optimal for assessment of random values. Its usage to estimate the states of random processes x(t) leads to the following: due to the presence of state changes x(t), the given procedure remains in the constant transient mode that causes errors in estimation. At the same time it is possible to considerably reduce these errors through reduction of the sampling interval  $\Delta t = (k+1) - k$  in comparison to the correlation interval  $\tau_{cor}$  of a random process. In practice it is shown that if the length of sampling rate is  $\Delta t = (0,01...0,001)\tau_{cor}$ , the influence of the transient process is slightly noticeable and the accuracy of the SA estimation procedure approaches to accuracy of KBF.

Due to these procedures RTT round-trip time in TCP technology is estimated and the procedure of congestion avoidance in router buffer (RED, WRED, etc.) works [8].

Note that the models presented in the state space are extremely popular in different scientific applications. It is sufficed to point out the fact that over 50% of publications in the "Avtomatika i Telemekhanika" magazine use exactly these models in the space state. The procedure of the Kalman-Bucy filter considered to be a powerful mathematical apparatus technique suitable for solving different tasks. For the first time it was used in the Apollo program during landing on the Moon.

Let us focus on another important system feature: connectivity. The property of connectivity for any system S(t, x) is determinative because "a system is a set of interconnected elements". If connectivity is lost, the system degrades and falls into elements. The increase of connectivity leads to enhancement of system properties, the so-called highly-integrated emergence properties appear.

Adequate illustration of highly-integrated system emergence properties can be Watanabe information measure [12], which is a generalization of Shannon measure:

$$W(x_1, x_2, ..., x_n) = \sum_{i=1}^n H(x_i) - H(x), \qquad (9)$$

where  $H(x_i)$  is an entropy of a random state of the *i*th element; H(x) is an entropy of a random state of the vector  $\vec{x}$ , which determines the state of the system in general. As long as the system S(t,x) has more strict and determined behavior towards its elements, Watanabe measure allows to find a quantitative measure of information increment in the group of interconnected elements at the account of connectivity itself. The given increment can serve as a measure of system emergence.

According to system properties models are divided into structural and functional. A connectivity matrix  $A = \{a_{ij}\}$  is a mathematical model for structural properties of the infocommunication system  $S(t, x_n)$ . It can be binary, showing presence or absence of connections between nodes. More inclusive model is represented by matrices, elements of which are quantitative characteristics of the properties of these connections, such as the probability of connectivity. Connectivity of networks is often associated with their reliability, because it seems logical to affirm that the increase in the number of connections (edges) leads to the increase in the reliability of connectivity with a large dimension network is extremely complex. It grows proportionally to the factorial n!. Thus for the simplest four-node bridge connection the formula to calculate the probability of connectivity has the 5th power. In order to obtain numerical results under large n different approximate estimations can be used. These are Esary-Proshan estimation [13], Polesskii estimation [14], etc. Calculated estimations  $P_{ij}$  characterize state probabilities of connectivity for the static state of the random system  $S(t, x_n)$ .

The connectivity matrix A(t) of a dynamic system  $S(t, x_n(t))$  possesses greater generality and its elements  $a_{ij}(t)$  are already random processes. Under such representation it is possible to show not only structural but also functional properties of the modeled system  $S(t, x_n(t))$ , the dynamics of which is reflected by the system of differential or difference equations (4). In scientific and technical literature much attention is paid to recursive filtering procedures, obtaining assessments, statistical output, etc. Univariate procedures (1) and (8) are well investigated and their stability is ensured by an appropriate choice of the stepping constant  $\mu_{ii}$  under rather general limitations of statistics properties  $y \in Y$ .

In addition, we often have to solve problems of finding estimations of two- or more dimensional processes  $x^{T}(k) = (x_1, x_2, ..., x_n), n = 2, 3, ..., n$ , that are functionality or statistically interconnected. The real examples of availability of processes connectivity can be network control processes, signals on neighboring antennas in MIMO technology, the processes in the interacting routers, etc.

When analyzing the structure connectivity in the multidimensional dynamical system not only nominal values  $\mu_{ij}$  are important but also the amount of connections themselves determined by the appropriate off-diagonal elements. From the works of Gardner, Ashby [15] and other authors it is known that a certain number of off-diagonal elements (for example, a complex 10-dimensional differential system) can cause an unstable mode. It is argued that such a coupled model falls apart with an increase in connectivity [16] (Fig. 3). Here we also can see data on the critical connectivity, when the level of connectivity due to the offdiagonal elements  $\mu_{ij}$  reaches 13% from their possible maximum number  $n \times n$ . Besides, such model instability is typical by for both linear and non-linear dynamic systems.



*Fig.* 3. Diagram of sustainability probability of *n* - dimensional model in accordance to the level of connectivity

To study the effect of interconnections  $a_{ij}$  it is sufficed to choose a 2-dimentional procedure of state estimation:

$$\begin{cases} \hat{x}_{1}(k+1) = \hat{x}_{1}(k) + a_{11}(\hat{x}_{1}(k) - y_{1}(k)) + a_{12}(\hat{x}_{2}(k) - y_{1}(k)); \\ \hat{x}_{2}(k+1) = \hat{x}_{2}(k) + a_{12}(\hat{x}_{2}(k) - y_{2}(k)) + a_{22}(\hat{x}_{1}(k) - y_{2}(k)), \end{cases}$$
(10)

where values  $a_{ii} = const$ ,  $a_{jj}$  change from 0 to  $a_{ij}$  and the impact of these dependences should be investigated. The block diagram (10) is represented on Fig. 4.



*Fig.* 4. Block diagram of the two-dimensional algorithm of process estimation  $x_i(k)$ 

Using the algorithm (10) the quality of the two-dimensional assessment was analyzed. Fig. 5 shows three different diagrams of analysis of the values accuracy assessment  $\hat{x}(k)$  according to the values of the sampling variance.



$$\sigma_{xi}^2 = \frac{1}{K-1} \sum_{k=1}^{K} (x_i(k) - \hat{x}_i(k))^2 .$$
(11)

*Fig.* 5. Diagrams of posteriori variance changes, estimation errors  $\tilde{x}_k = (\hat{x}_k - x_k)$  under signal-to-noise ratio  $P_C^{(x)}/N_v = 5 \,\text{dB}$ 

The diagram 1 is given for comparison. It characterizes the potential abilities of SA procedure in the evaluation of a random variable x = const. It follows from the diagram that with the increase in the amount of steps k > (50...100) a posteriori variance (11) asymptotically decreases to zero. The diagrams 2 and 3 state estimate of the random process x(t).

There is no posteriori asymptotic zero variance at  $k \to \infty$ . There is a residual value  $\sigma_{xi}^2$  due to the influence of unsteady transient states. The main result of this experiment is that the posteriori variance  $\sigma_{xi}^2$  for connectivity processes is somewhat lower (5...10)% than for independent processes under other equal terms. Accordingly, the accuracy of the estimates for the connected processes is higher.

Fig. 6 shows diagrams of the posteriori variance depending on the magnitude of the coupling coefficient  $a_{ij}$ . The diagram confirms the previous conclusion and shows that with the increase in levels of connectivity estimation accuracy increases as well. These studies have used relatively weak connections. With the increase in the level of connectivity and the number of these connections it was impossible to make more definite conclusions.



*Fig.* 6. Diagram of a posteriori variance of error estimates under different values of the off-diagonal matrix connectivity value  $\alpha_{ii}$ 

## Conclusion

1. The presence of connectivity between elements of systems provides that that it will acquire new highly-integrated properties: consistency, integrity and emergence. The presence of connectivity in the network structures such as infocommunication systems provides the acquisition of the properties of network reliability and survivability.

2. For fixed networks (time invariant parameters) connectivity nature is displayed by the connectivity matrix which may have a binary structure or consist of quantitative data determining the level of connectivity (probability, number of channels, distance, etc.). The coefficient of connectivity or connectivity probability for such networks is found via the approximate methods of Esary-Proshan, Polesskii, etc.

3. For dynamic (time-varying elements of connectivity) networks connectivity values can be obtained through the current assessment of the connectivity status for each node from all adjacent nodes. For real-posteriori assessment of connectivity it is advisable to use the model in the state space that allows us to characterize the dynamic deterministic and stochastic systems. Practice shows that the state of the process is allowed to use the methods of stochastic approximation provided selection of the sample rate  $\Delta t \ll \tau_{cor}$ .

4. The analysis of multidimensional differential systems shows that the existence of connections between components of the system cannot have arbitrary values. A large number of connections leads to unstable modes. At the other extreme case: in the absence of interconnections a system loses its system properties (integrity, emergence).

5. The connection between random processes improves accuracy of estimates (decrease the value of a posteriori variance) of the processes components in comparison to the case when the process data are independent.

## References

1. *Mesarovic M.D., Takahara Y.* General Systems Theory: Mathematical Foundations. - NY : Academic Press, 1975. – 268 p.

2. *Simon Haykin*. Neural Networks: A Comprehensive Foundation. Second Edition. – NJ : Prentice Hall, Inc., 1998. – 842 p.

3. Ashby W.R. An Introduction to Cybernetics. – London : John Wiley & Sons, 1963. – 295 p.

4. *Ashby W.R.* Systems and their informational measures. // Treds in General Systems Theory. – NY : Wiley-Intarscience, 1972. – P. 78-97.

5. *Ito K., McKean Jr. H. P.* Diffusion processes and their sample paths. – Springer, 1965.

6. *Elliott R.J.* Stochastic Calculus and Applications. Applications of Mathematics. - Springer-Verlag, 1982. – 302 p.

7. *Kleinarck L.* Queueing Systems VI. – NY : J. Wiley and Sons, 1975. – 409 p.

8. *Popovskij V., Barkalov A., Titarenko L.* Control and Adaptation in Telecommunication Systems. – Berlin Heidelberg : Springer-Verlag, 2011. – 172 p.

9. *Kalman R.E., Bucy R.S.* New results in linear filtering and prediction theory // Trans. ASME. Ser. D.J. Basic. Eng. – 1961. – Vol. 83. – P. 95-107.

10. *Kalman R.E.* System identification from noisy data // Dynamical System II. – NY: Academic Press, 1982. – P. 135-164.

11. *Nevelson M.B., Khasminskiy R.Z.* Stochastic approximation and recurrent evaluation. – M.: Nauka, 1972. – 304 p.

12. *Watanabe S.* Pattern recognition as a quest for minimum entropy // Pattern Recognition 13. – 1981. – N 5. – P. 381-387.

13. *Esary J., Proshan F.* Cohernet Structures of Non-Identical Components. // Technometrics – 1963. – Vol.5, No. 2. – P. 191-309.

14. *Polesskij V*. Employment of graph characteristics for lagre-scale data network topological optimization. // Proceedings of International Workshop. Teory and Application, 1998. – P. 1-19.

15. *Gardner M.R., Ashby W.R.* Connectance of large dynamics (cybernetic) systems: Critical values of stability // Nature. – 1970. - N 228. – P. 784.

16. *Klir G.J.* Architecture of systems problem solving. – NY: Plenum Press, 1985. – 520 p.