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THE APPLICATION OF A LINEAR PROGRAMMING METHOD WITH FUZZY CONDITIONS FOR DESIGNING OF COMPUTER SYSTEMS

The application of a linear programming method with fuzzy conditions for designing of computer systems is considered in this paper. This problem statement leads to the necessity of multi-criterion linear programming problems resolving.

linear programming, multicriterion problem, criterion function, automated design systems, optimal decision

Introduction

Now at modern computer systems (CS) design a lot of questions, related to acceptance of optimum decisions are examined. Most of the CS design tasks are multicriterion, and at their decision it is necessary to take into account a large number of factors. In addition, there is often a necessity to make project decisions in real-time operation mode. Decision-making at CS design consists in generation of all possible flow and functional schematics of a designed device, estimation of these charts and selection the best among them. An optimum decision is a variant of future device schematics, which optimizes some objective function (productivity, cost, etc.) in a field of examined variants. With appearance and development of CS computer aided design (CAD) systems the indicated processes of different variants of architecture generating and best decision selecting will be realized within a framework of complex CAD systems of electronic components. However, decisions generation is made directly by developers, but not by a CAD system. In this connection the task of formalizing and algorithmizing process of different variants of project decisions generation is actual.

Publication analysis. Efficiency of CAD systems rises at including into their composition optimization methods, allowing in this concrete situation to get the best of possible decisions [1]. A task of optimum object structure choice is always urgent for realization of given functions [2, 3]. It is known, that a stage of structural synthesis of design objects is very labour-intensive, especially at a search of optimum decisions. For obtaining optimum variant of a designed object structure it is necessary to have its mathematical model, representing a formal specification of a set of object structures at accepted level of detailing. In this case a task of structural synthesis comes to selection of compromise variant on a countable set, thus solving of nascent logic and combinatory tasks requires heavy time expenses. In connection to this a problem of combinatory supplying of CAD systems arises. It is natural to create combinatory supplying on a base of sufficient general mathematical models, allowing uniformly and compact description of large complexes of tasks and construction software packages for their solving, leaning onto a single arsenal of approaches and receptions. From a variety of formalizable tasks of technical objects structural synthesis their substantial part can be reduced to determining extreme value of an objective function [4]. However, in a task of mathematical programming there can be unclearness, both at description of a variety of alternatives, and at description of objective functions. Therefore a task of formalization of CS designs process as multicriterion task of linear programming is actual [4].

Types of tasks in fuzzy mathematical programming

With setting of concrete technical conditions in a process of CS design vagueness can take place, which leads to solving tasks of mathematical programming with fuzzy terms. Fuzziness in a task statement of mathematical programming can be both in describing of great number of alternatives and in describing of utility functions. Tasks, in which a great number of alternatives and (or) objective utility function are fuzzily described, are named tasks of fuzzy mathematical programming (FMP).

Primary objective of FMP is to help a person, who is making a decision, to understand accepted assumptions. Fuzzy approach is not substituted by simplest analysis in a search of reasonable exactness.

Analyzing tasks of FMP, it is possible to select two different approaches to their solving. At the approach, first offered by P. Bellman and A. Zade [6], the task of FMP is formulated as a task of performance of fuzzy purpose at fuzzy limitations, thus a decision is set by crossing fuzzy sets of purpose and limitations. At the second approach it is assumed that decisions must be selected just as it is done in tasks of multicriterion optimization.

Only decisions (alternatives), which not are strictly dominated by any other alternatives, are examined; so effective alternatives are chosen in understanding of Pareto. Such decisions use advantages of fuzzy relations vehicle.

Classic mathematical programming and its varieties are largely normative methodology of effective choice. Fuzzy programming selects natural multiplicity of roughly determined purposes, values, and limitations. Thus optimality is determined both in terms of behavior, and as a quality, inherent by a decision.

A standard task of FMP is usually formulated as a task of maximization (or minimizations) of a given function on a given set of possible alternatives, which is described by a system of equalities or inequalities.

For example:

$$f(x) \to \max, \varphi_i(x) \le 0, i = 1,...,m, x \in X,$$

where X – a set of alternatives, $f : X \to R^1$ and $\varphi : X \to R^1$ – given functions.

While designing in a fuzzy form of real tasks of decision-making, only fuzzy descriptions of f and φ functions, parameters, which these functions depend on, and a X set, can appear at researcher’s disposal. Thus a task of standard mathematical programming will grow into a task of FMP.

Forms of fuzzy description of initial information in a task of decision-making can be different; hence there are distinctions in mathematical formulations of corresponding tasks of fuzzy mathematical programming. Consider some of such statements [7].

Task 1. Maximization of a given ordinary function $f : X \to R^1$ on a given fuzzy set of possible alternatives $\mu\_ : X \to [0,1]$.

Task 2. A fuzzy variant of a standard task of mathematical programming. Let the following task of the mathematical programming be determined:

$$f(x) \to \max,$$
$$\varphi(x) \le 0, x \in X.$$

A fuzzy variant of this task is obtained, if to “ease” restrictions, i.e. to assume a possibility of their violation with one or another degree. Besides, instead of maximization of *f*(*x*) it is possible to aspire to achievement of a given value of this function. Thus, it is possible to add different degrees of admission to different deviations of the function value.

Task 3. The “maximized” function is fuzzily described, i.e. a reflection $\mu_\varphi : X \times R^1 \to [0,1]$ is set, where X – an universal set of alternatives, R^1 – numerical ax.

In this case a function $\mu_\varphi(x_0, r)$ at every fixed $x_0 \in X$ is a fuzzy description of alternative choice result estimation x_0 (fuzzy estimation of alternative x_0) or fuzzily known reaction of a controlled system on a control x_0. A fuzzy set of possible alternatives is set similarly $\mu_C : X \to [0,1]$.

Task 4. An ordinary maximized function $f : X \to R^1$ and a system of limitations $\varphi_i(x) \le b_i, i = 1,...,m$ are set, thus parameters in functions' descriptions $\varphi_i(x)$ are set in form of fuzzy sets.

Task 5. Both functions' parameters, determining limitations of a task, and a maximized function are fuzzily described.

Task of linear programming with fuzzy terms

Consider more detailed a task of linear programming with fuzzy coefficients. Fuzziness in a task of mathematical programming statement can be both in a description of a set of alternatives and in a description of an objective (goal) function.

$$f(x) \to \max, g(x) \le 0, x \in X . \qquad (1)$$

In practice there is often applying of the exact theory of optimization to the inexact models, where there are no bases to write exactly defined numbers and where calculable difficulties at description of large systems happen too often.

A fuzzy situation can be considered as a set X of alternatives together with its fuzzy subsets, which are fuzzily formulated criteria (goals and limitations), i.e. as a system $(X, f_0, f_1, ..., f_n)$. To take into account if possible all criteria in a such task means to build a function:

$$D = f_0 \cap f_1 \cap ... \cap f_n, \qquad (2)$$

in which goals and limitations enter in identical manner. A decision can be defined as a fuzzy subset of a universal set of alternatives. An optimum corresponds to that area X, elements of which maximize D. This is a case of fuzzy mathematical programming.

Obviously, it is unreasonable in real situations to conduct a sharp border for a set of possible alternatives, as it can happen that distributions, lying outside this border, will give an effect, exceeding less desirability for a face that accepts decision.

For example, it is clear, that at the incompatible distributions this area is empty. In this case a necessity of modification of limitations presents. It is desirable to find out how to change limitations of a task, in order to get feasible solutions and to obtain solvable task.

In such cases it is expedient to introduce a fuzzy set of possible elements and, consequently, to examine a problem as a task of UMP with usage of approach, which gives a man more freedom in the use of his subjective pictures of situation.

Forms of fuzzy description of initial information in a task of decision-making can be different; hence there are distinctions in mathematical formulations of corresponding tasks of fuzzy mathematical programming.

A fuzzy variant of a standard task of mathematical programming happens, if to "ease" a restriction, i.e. to assume possibility of their violation with one or another degree. Besides, instead of maximization of $f(x)$ it is possible to aspire to achievement of a given value of this function. Thus, it is possible to add different degrees of admission to different deviations of the function value (for example, the bigger rejection, the less degree of its admission).

Let a be a give size of a goal function $f(x)$, achievement of which is considered sufficient for performance of a decision-making goal, and let there be a threshold level of b of the kind, that inequality of $f(x) < a-b$ means strong violation of inequality $f(x) \ge a$. Then it is possible to define a function of belonging for a fuzzy goal function as follows:

$$\mu_G(x) = \begin{cases} 0, & if \quad f(x) \le a-b; \\ \mu_a(x), & if \quad a-b < f(x) < a; \\ 1, & if \quad f(x) \ge a, \end{cases} \qquad (3)$$

where $\mu_a(x)$ – a function of belonging, describing a degree of implementation of the corresponding inequality from a point of view of a person, who makes a decision.

A function of belonging $\mu_C(x)$ is determined similarly for fuzzy limitations. As a result, an initial task is formulated in a form of performance task of fuzzily defined goal, to which approach of Bellman – Zade can be applied (2).

At a situation simulation in a form of linear programming task

$$\min\{cx \mid Ax \le b, x \ge 0\} \quad (4)$$

about the coefficients of a_{ij}, b_i and c_i it is only known, that they are in some set, reflecting all real possibilities.

On some cases an exactly described set of limitations (possible alternatives) can be just approaching of reality in a sense, that in a real problem alternatives out of a limitations' set can not allowed, but only in one or another degree less desirable for a person, who makes decision, that alternatives inside this set.

Consider a problem of minimum search on a given area. Let an area be set:

$$P = \{x \in R_+^n \mid a_{i1}x_1 + a_{i2}x_2 + ... + a_{in}x_n \subseteq b_i, i = \overline{1,m}\}, \quad (5)$$

where a_{ij}, b_i – are fuzzy subsets of a set R, and binary operation "+" identifies addition of fuzzy sets. It is required to find $\min\limits_{x \in P}\langle c, x\rangle$ on a given area.

It is possible to consider a coefficient at every variable in limitations as a utility function, determined on a numerical ax. It is possible to consider that these coefficients give subjective estimation of different possibilities, including in that manner others, not determined limitations.

We will reduce solving of the initial task to solving of a set of tasks of linear programming. For this purpose we will enter discrete Δ-levels. As a result fuzzy limitations assume the following interval view:

$$P = \begin{cases} \sigma_\alpha(a_{i1})x_1 + \sigma_\alpha(a_{i2})x_2 + ... + \sigma_\alpha(a_{in})x_n \subseteq \sigma_\alpha(b_i); \\ x_j \ge 0, \end{cases} \quad (6)$$

$$i = \overline{1,m}, \quad \alpha = \overline{1,p}, \quad j = \overline{1,n}.$$

Thereby, we've passed from fuzzy sets to expressly determined ones, and now, knowing that Δ -ordinary interval, we can write down the task as the following (case 2□ 2):

$$(a_{11}, a_{12})\, x_1 + (c_{11}, c_{12})\, x_2 \subseteq (b_{11}, b_{12})$$

$$(a_{21}, a_{22})x_1 + (c_{21}, c_{22})\, x_2 \subseteq (b_{21}, b_{22}). \quad (7)$$

Now, to lead a problem to an ordinary task of linear programming, it is enough to write down inequalities separately on the left and on the right edges of intervals, taking into account the signs of inequality. I.e., we will reduce the system to the following:

$$\begin{aligned} a_{11}x_1 + c_{11}x_2 &\ge b_{11} \\ a_{12}x_1 + c_{12}x_2 &\le b_{12} \\ a_{21}x_1 + c_{21}x_2 &\ge b_{21} \\ a_{22}x_1 + c_{22}x_2 &\le b_{22}. \end{aligned} \quad (8)$$

By simple transformations we've passed from a problem with fuzzy coefficients to a problem of linear programming with clear coefficients, here the amount of limitations increased in two times and the obtained task we can decide by simplex method.

Thus, from the considered example the algorithm of problem solving with fuzzy coefficients is obviously seen. Following a chain of arguments in this example, let's make the algorithm of problem solving. It looks like:

initial task
⇓
enter discrete and Δ - levels
⇓
limitations get an interval view
⇓
write down inequalities separately on the left and on the right edges taking into account the signs of inequality(a dimension is increased thus)
⇓
get the task of LP with clear coefficients
⇓
decide the obtained task by simplex method

As we see, the initial task of FMP appears as a mix of ordinary jobs of linear programming on various sets of a level of a possible alternatives set. If an alternative x_0 is a decision of a task $\min\langle c,x\rangle$ on a of level Δ then it is possible to consider that number Δ is a degree of alternative x_0 belonging to a fuzzy set of decisions of the initial task. Sorting out thus various values Δ, we obtain a function of fuzzy decision belonging.

If components of an objective function of c_i are fuzzy, it is necessary to choose proper scopes of sets $\sigma_\alpha(c_J)$, $J=\overline{1,n}$ for every level in accordance with the rules of interval arithmetic, minimizing preliminary thus $\langle c,x\rangle$.

From this example can be evidently seen, that for flexibility it is necessary to pay the cost of task dimension increasing.

Actually an initial task with limitations on including will be transformed in a task with limitations as inequalities with which it is easily to treat; thus such price is not too high, since a possibility of well-developed classic methods usage is kept.

Conclusion

The obtained results can be practically used in computer equipment design on a base of programmable logic arrays, because sharp increase of integrated circuits nowadays requires formal methods of design decision evaluation.

Usage of the developed methods allows increasing a degree of reliability and fault tolerance of devices and systems, which are developed on a base of PLD.

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