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INCREASE OF GENERAL STABILITY AND CORRECTION OF DIRECT INDEXES OF QUALITY OF TRANSIENTS NONMINIMAL FINITE DURATION IN DIGITAL CONTROL SYSTEMS

The methods of supplementing the general method of synthesis of discrete (digital) control systems in solving problems associated with the transition from discrete transient minimum finite duration to transients nonminimal finite duration. This allows direct measures to improve the quality of the processes involved. The solution to this type of problem associated with certain difficulties due to discrepancies between the amounts of the coefficients of the characteristic equation of the synthesized system number of initial calculated equations. Using the proposed methods in many cases allows to circumvent this problem by using special relations estimated coefficients obtained from the analysis of discrete transient characteristics desired or by obtaining more robust synthesized system after increasing its general stability. This is accomplished by a formal chargeability to unstable of stable roots in direct proximity to the stability boundary. Also, the proposed methods in aggregate allow you to customize by a finite duration transients even those systems for which it was beginning not possible due to their small dimensions, or similar reason.

Key words: *planning, synthesis, digital system control, stability, robust of the system, factorization, transients.*

Introduction

A major task at planning of digital control systems (DCS) is development of algorithm of work of digital computing device (DCD) on making of the managing affecting of object control, able to provide on an output the systems transitional a process is the best on a fast-acting with optimum correlation of all direct indexes of quality [1-6]. It is considered that such algorithms can provide the best fast-acting the system managements, stopped up in DCD, a job of which performance on the output of the system are transients, having minimum and finite duration. Wide distribution at the decision of similar class of tasks was got by the so-called criterion of fast-acting, allowing forming the law of work of digital regulator so that discrete transitional description (DTD) of the synthesized system assumes an air of steady transient of minimum and finite duration.

At tuning of the systems in accordance with the criterion adopted higher it is necessary necessarily to take into account two conditions: condition of physical realizability of the system and condition of rudeness of synthesized DCS. A failure to observe of these terms can result in impossibility of hardware (physical) representation of system control as such, and also to the loss of the desired descriptions of transients by the designed system from correlations of internal parameters of component blocks of DCS, arising up in the process of functioning of the system after its hardware representation [5].

Other complication, arising up at the synthesis of the systems in obedience to the criterion of fast-acting, large the overshoot of the got transients of minimum and finite duration of DSC is, as a rule [6].

This problem can be removed by the waiver of minimum duration and passing to nonminimum and finite duration of transients, but there is new complication which consists in impossibility of receipt analytically of synonymous values of coefficients of tuning of DCD from the increase of their number because of afore-named transition.

Two methods are grounded in this work, which it is possible to use in the process of synthesis of the systems in obedience to the criterion of fast-acting consistently: method of increase of general stability of DCS and method of correction of direct indexes of quality of transients of synthesized DCS. Thus, the first method allows as early as beginning of synthesis of DCS to strengthen the condition of rudeness of the system, not violating the condition of its physical realizability, and the same to protect the system from negative influence of correlations of internal parameters of component blocks of DCS. The second method allows, in the case of unsatisfactory value of the overshoot of transient of minimum and finite duration in the synthesized system, to pass to the transient of unminimum eventual duration. Thus, the process of transition becomes simpler, with the less number of calculations of parameters of DCD, without the substantial loss of fast-acting the system and by possibility of prognostication of value of the overshoot of DTD of the system got as a result.

1. Analysis of general method of synthesis of algorithms of management of DCS on condition of minimum and finite duration of transients in the system

The synthesis of the digital systems of automatic control (SAC) consists of development of such program of treatment of information in DCD, at which the synthesized system satisfies the requirements put initially to the transient in it. As be said higher, at the synthesis of DCS it is necessary to take into account some special terms major from which is a condition of physical realizability and condition of rudeness of the system. In general formulation a condition of physical realizability of discrete (digital) SAC is the following: desired discrete transmission function (DTF) of the reserved system, taken in a kind $W_{res}(z) = P(z)/Q(z), P(z)$ - where is a polynomial of numerator, and $Q(z)$ is a polynomial of denominator DTF reserved SAC, must be the proper fraction, thus a degree $P(z)$ must be a less or equal degree $Q(z)$ $(m \le n)$. It should be noted that a condition of physical realizability is necessary, but in general case insufficient. During practical realization of discrete (digital) correctings chains of their description can a few to differ from initially necessary. If this difference will be caused by the small change of processes in reserved SAC, such SAC is the rough system. If the small difference of descriptions will change a process high-quality, the system will be not rough.

We will consider the method of synthesis of discrete SAC on the criterion of fast-acting, when basic is a requirement, that an output signal had finite and minimum duration. We will accept the followings denotations: – DTF of unalterable part; it is a transmission function of DCD. Then it is possible to write down for the transmission functions of the broken a secret and reserved system:

$$W_{sec}(z) = W_O(z)D(z), \quad (1)$$

$$W_{res}(z) = \frac{W_{sec}(z)}{1 + W_{sec}(z)} = \frac{W_O(z)D(z)}{1 + W_O(z)D(z)} = \frac{P(z)}{Q(z)}. \quad (2)$$

If DTF of unalterable part and reserved system is known, from (2) follows:

$$D(z) = \frac{1}{W_O(z)} \frac{W_{res}(z)}{1 - W_{res}(z)} = \frac{W_{sec}(z)}{W_O(z)}. \quad (3)$$

We will present the transmission function of unalterable part in the following kind:

$$W_O(z) = \frac{B(z)}{C(z)} = \frac{B^-(z)B^+(z)}{C^-(z)C^+(z)}. \quad (4)$$

Polynomials with the index of "+" have all roots into the circle of single radius, and polynomials with the index of "–" out of this circle. The operation of presentation of transmission function in a kind (4) is named factorization.

The condition of rudeness of the system requires, that the transmission function of the desired reserved system contained the zeros of polynomial $B^-(z)$ as the zeros, and a transmission function $1 - W_{res}(z)$ as the zeros contained the zeros of polynomial $C^-(z)$.

$$\begin{cases} W_{res}(z) = \dfrac{B^-(z)M(z)}{Q(z)}, \\ 1 - W_{res}(z) = \dfrac{C^-(z)N(z)}{Q(z)}. \end{cases} \quad (5)$$

Choice of polynomials $M(z)$, $N(z)$ and $Q(z)$ provide the receipt of the set high-quality indexes of process of adjusting in discrete moments of time. If necessary to get finite duration of adjusting process choose the characteristic polynomial of the reserved system in a kind:

$$Q(z) = z^l, \quad (6)$$

where l is a positive integer.

It is possible to get by virtue of expressions (4) and (5): $D(z) = \dfrac{C^+(z)M(z)}{B^+(z)N(z)}$;

$$W_{sec}(z) = \frac{B^-(z)M(z)}{C^-(z)N(z)}.$$

Then it is possible to write down for the characteristic polynomial of the reserved system:

$$Q(z) = C^-(z)N(z) + B^-(z)M(z) = z^l. \quad (7)$$

The observance of principle of physical realizability is provided, if

$$\left\|C^-(z)\right\| + \left\|N(z)\right\| \ge \left\|B^-(z)\right\| + \left\|M(z)\right\|. \quad (8)$$

A sign $\left\|\cdots\right\|$ is meant by the order of polynomial and at arbitrary polynomials $B^-(z)$ and $C^-(z)$ this condition (8) is executed, if

$$\begin{cases} \left\|N(z)\right\| \ge \left\|B^-(z)\right\|, \\ \left\|M(z)\right\| \ge \left\|C^-(z)\right\| - 1. \end{cases} \quad (9)$$

It ensues from expressions (8) and (9), that the minimum order of the desired characteristic polynomial of the reserved system is equal

$$l_{min} = \left\|C^-(z)\right\| + \left\|B^-(z)\right\|. \quad (10)$$

The select orders of polynomials $M(z)$ and $N(z)$ equalization of polynomials (9) decides development of it in the system of algebraic equalizations in relation to the coefficients of the indicated polynomials by equating of members with the identical degrees of

operator of z in the left and right part of initial equalization.

A choice $l = l_{min}$ is determined by the process of minimum and finite duration. In this case the number of equalizations of the got system is equal to the number of unknown coefficients and it has the unique decision. More frequent than all exactly at such choice of duration of process the synthesized system does not possess the sufficient supplies of stability and has high the overshoot.

For the exception of this phenomenon there are two ways. The first consists in the maintainance of finite duration of transient at the increase of adjusting time by a choice $l > l_{min}$. In this case the system of algebraic equalizations contains unknown more than equalizations and has numberless decision content. Difference between the number of equalizations and number unknown equal to the size of increase of order of the system as compared to minimum. The method of choice of "superfluous" unknown coefficients will be offered below, which is based on imposition of limits on the coefficients of numerator of transmission function of the closed system and allows considerably to narrow the ambiguousness of process of choice of unknown coefficients of DTF DCD.

The second way consists in a refuse and from finite duration of transient. In this case the characteristic polynomial of the reserved system gets out in the following kind:

$$Q(z) = z^k (z-a)^{l-k}. \qquad (11)$$

In this case size of the overshoot and duration of transient, determined the set time of adjusting, it is frequently succeeded to get and at the minimum order of the system by the proper choice of sizes and to, the however limited exactness of DCS with such type of DTD allows to use this way of correction of direct indexes only in the very narrow class of digital SAC.

2. Increase of general stability of DCS in the process of their synthesis

As stated above, during practical realization of discrete (digital) correcting chains, a few their description can to differ from necessary, that at the insufficient rudeness of the realized system can result in the of principle difference of type of transients in this system from desired, up to violation of general stability of the system [5]. From the mathematical point of view such violation of general stability of the systems can be explained appearance in expression (4) in composition the factorized element $C^+(z)$ of coefficients which are subject to correlation during their practical realization in the proper elements of DCS. By the basic feature of element $C^+(z)$, that roots, being in a direct closeness to the border of stability of the system, appear in composition his becomes in this case, that to the circumference of single radius in the complex plane of roots of z. In order to avoid this negative phenomenon offered, in the process of synthesis, to extract the proper roots from composition of factorized element $C^+(z)$, and to attribute them to the element $C^-(z)$, that will allow to strengthen general stability of the designed system, but as investigation and its rudeness, and also, on occasion, to get in the separate contours of adjusting the transients of minimum and finite duration even then, when to it such were impossible, that easily to illustrate on a next simple example.

If to accept as a linear transmission function of unchanging part of one of contours of adjusting link with a transmission function $W_{LP}(p) = k/(T_O p + 1)$ (see fig.1), DTF of unchanging part of this contour of adjusting taking into account the proper discrete-continuous transformer will be equal

$$W_O(z) = \frac{z-1}{z} Z\{\frac{k}{p(T_O p+1)}\} = \frac{k(1-d)}{z-d},$$

where $d=\exp\{-(T_0/t)\}$; to is a transmission coefficient of element;

T_0 – permanent time of element;

T is time of discretization of the system;

Z is an operator of calculation of z-transformation of function.

Then after factorization will become obvious circumstance that the process of minimum and eventual duration is impossible in this contour of adjusting transitional, that optimum on a fast-acting ($l_{min} = 0$).

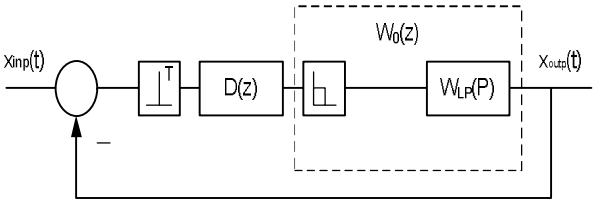

Fig. 1 The scheme of the simplest contour of adjusting in DCS

Is it enough to take into account for the removal of this failing, that at small time of discretization of the system of $T \to 0$ a value of size of d, being the root of the proper characteristic equalization, will be near to unit, but here will be in steady area of complex plane of roots of z. As a result, using the above-described method of increase of general stability of DCS, it is possible it will be in the process of synthesis of the system to attribute this root not to the factorized element $C^+(z)$ and to the element

$C^{-}(z)$, that as a result will allow to get in this contour adjusting transitional process of minimum and finite duration (see fig.2) and will promote general stability of the system, and also its rudeness, from subsequent structural indemnification of the proper root which can go out for the border of stability because of correlation of parameters of element during practical realization.

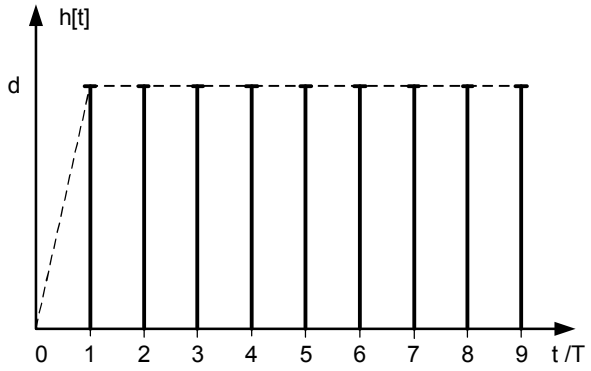

Fig. 2 DTD of contour of adjusting, resulted in an example, after a synthesis, on condition of the use of the offered method of increase of general stability of DCS

3. A correction of direct indexes of quality of transients is in digital system control

It was marked at the beginning of the article, that synthesized on the general method of DCS does not possess the sufficient supplies of stability and have a large volume of adjusting of transients. Promoting the supply of stability of such systems is possible, using a method, described in p.3 of this work. Unexposed is the second part of question, related to the sharp increase of values of the overshoot in the process of synthesis in accordance with the criterion of fast-acting [6]. We will consider on the example of digital automat of stabilizing, the flow diagram of which is resulted on the fig.3 calculation of algorithm of making of DCD of managing influence in the system, able to provide minimum and finite duration of transients.

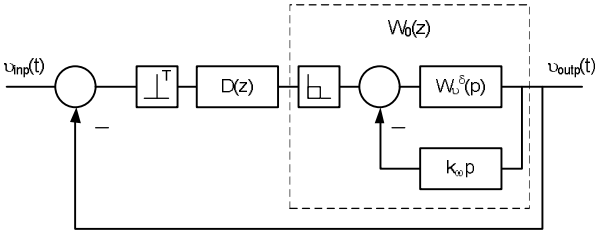

Fig. 3 Structural diagram of a digital automatic stabilization damped by an analog channel

It is possible to accept at the calculated coefficient of damping and beforehand preset parameter of object

$$W_O(z) = \frac{1.495 \cdot 10^{-2}(z^2 - 0.6397z - 0.354)}{(z - 3.5 \cdot 10^{-2})(z^2 - 2z + 1)}. \quad (12)$$

After a calculation in obedience to the general method of synthesis of DCS it is possible to draw conclusion in accordance with the criterion of fast-acting, that for providing of minimum and eventual duration of transients in this system it is necessary in DCD to provide making of managing influence in accordance with a next transmission function

$$D(z) = \frac{C^{+}(z)}{B^{+}(z)} \frac{2z-1}{1},$$

where $B^{+}(z) = 1.495 \cdot 10^{-2}(z^2 - 0.6397z - 0.354)$,

$C^{+}(z) = z - 3.5 \cdot 10^{-2}$.

DTF got reserved DCS will be equal

$$W_{res}(z) = \frac{2z-1}{z^2}.$$

DTD of the got system will look like thus, represented on fig. 4.

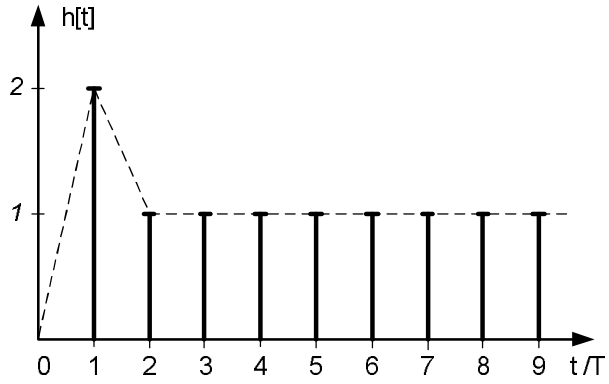

Fig. 4 Discrete transient response of digital automat of stabilizing after a synthesis in accordance with criterion of fast-acting

Transitional does a process, resulted on fig.4, have minimum and finite duration, however much a large value has the overshoot σ = 100%. By obvious possibility to be delivered this failing there is a waiver of minimum finite and passing to nonminimum finite duration of transient, that, as a result, entails the row of difficulties related to impossibility of synonymous calculation of whole set of internal parameters of algorithms of work of DCD in the synthesized system. Narrowing this problem is possible, if to analyze the discrete transients of nonminimum finite duration using properties of z-transformation.

In obedience to the system of equalizations (5) the discrete transmission function of got DCS must be evened

$$W_{res}(z) = \frac{B^{-}(z)M(z)}{Q(z)} = \frac{P(z)}{Q(z)}. \qquad (13)$$

If to present the polynomial of numerator of expression (12) in a kind

$$P(z) = \sum_{i=0}^{l-1} p_i z^i, \qquad (14)$$

where l is duration of selectable transient of nonminimum finite duration, then DTD of the system to the subject correction will be equal

$$H(z) = \frac{z}{z-1} \sum_{i=0}^{l-1} p_i z^{-(i+1)}. \qquad (15)$$

The original of function (15) taking into account properties of z-transformation will be equal:

$$h[t] = \sum_{i=0}^{l-1} p_i \cdot 1[t-(i+1)T]. \qquad (16)$$

Geometrical interpretation of expression (16) is presented on fig.5.

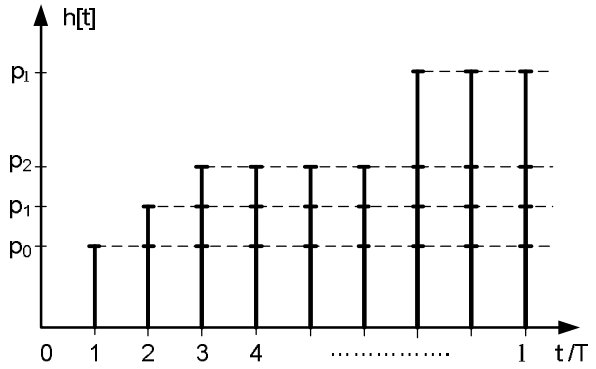

Fig. 5 Geometrical interpretation of original DTD the corrected system taking into account properties of z-transformation

Resulted the higher given (fig.5) mean that a resulting transitional process of nonminimum finite duration is a sum of discrete single step signals rationed on the sizes of p_i, thus that stabilizing of the system was carried out it is necessary at the level of unit, that the following equality was executed:

$$\sum_{i=0}^{l-1} p_i = 1. \qquad (17)$$

It is also necessary to take into account that every value of transitional description is determined as a sum

$$h[t_k] = h[kT] = \sum_{i=0}^{k-1} p_i, \qquad (18)$$

that allows in advance to define the desired value of the overshoot of transient of the system.

We will return for example with the synthesis of digital automat of stabilizing and will give up minimum finite duration transitional and will accept duration of transient equal l = 3, a characteristic polynomial must be evened as a result

$$m_0 z^2 + m_1 z + m_2 + (n_0 z + n_1)(z^2 - 2z + 1) = z^3. \qquad (19)$$

Due to limitations (17) and (18) and the analysis of transients offered higher is not difficult to set analytical connections between the values of coefficients of the proper polynomials of M(z) and N(z) (see table 1) and expose their optimum correlation with the purpose of lowering of correction.

Table 1

Comparative information for the choice of optimum correlation of coefficients in the polynomials of M(z) and N(z)

№ set	n_0	n_1	$m_0=2-n_1$	$m1=2\cdot n_1-1$	$m_2=-n_1$	the overshoot of transient (%)
1	1	0.1	1.9	-0.8	-0.1	90
2		0.2	1.8	-0.6	-0.2	80
3		0.3	1.7	-0.4	-0.3	70
4		0.4	1.6	-0.2	-0.4	60
5		0.5	1.5	0	-0.5	50
6		0.6	1.4	0.2	-0.6	60
7		0.7	1.3	0.4	-0.7	70
8		0.8	1.2	0.6	-0.8	80

Analyzing the data presented in the table it can be concluded that considerably is possible to reduce overshoot in the transition to nonminimum finite duration of the transition process following the minimum order (see Fig.6), when used in the system settings polynomials M (z) and N (z) are in pos. 5 Table 1, at the same time that is important, do not need the big mathematical calculations.

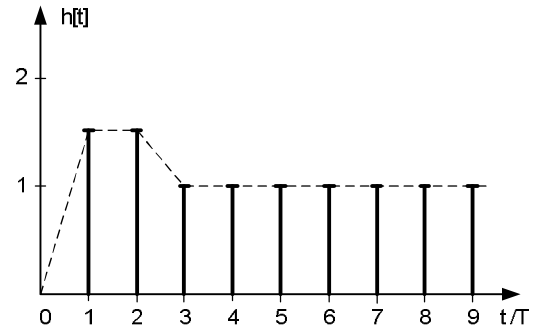

Fig. 6 Discrete transitional description of digital automat of stabilizing after the waiver of minimum and finite duration of transient

Conclusions

Analyzed the general method of synthesis of digital systems control based on the criterion of performance as a result of which he was made a number

of conclusions that can be useful in the design of the DCS, using them can be improved algorithms corresponding DCD. The most important may be considered a factorization with discrete transfer function of the constant part (DTFCP) system, which is synthesized unstable parts of factorization DTFCP except unstable roots of the characteristic polynomial can be classified as persistent roots, but which are close to the circle of radius, which is the limit of sustainability complex z-plane roots. This can lead to a significant increase in the overall stability of the system with its internal parameters of correlation due to a slight increase in the duration of transients in the DSC.

There was obtained and reasonable method that lets you enter in the system of equations for calculation settings DCD provided nonminimum finite duration transients there, additional equations that are based on the restrictions imposed on the coefficients of the numerator polynomial of the transfer function of a discrete system synthesized in a closed condition. Using basic properties of z-transform has been proved that the sum of all coefficients of the numerator polynomial of the transfer function of a discrete system, which is synthesized in a closed state in nonminimum and finite duration of the transition process at the output of the system, must be equal to the size staircase steps, specified.

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ПОВЫШЕНИЕ ОБЩЕЙ СТОЙКОСТИ И КОРРЕКЦИЯ ПРЯМЫХ ПОКАЗАТЕЛЕЙ КАЧЕСТВА ПЕРЕХОДНЫХ ПРОЦЕССОВ НЕМИНИМАЛЬНОЙ КОНЕЧНОЙ ДЛИТЕЛЬНОСТИ В ЦИФРОВЫХ СИСТЕМАХ УПРАВЛЕНИЯ

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Предложены методы, дополняющие общую методику синтеза дискретных (цифровых) систем управления при решении задач, связанных с переходом от дискретных переходных процессов минимальной конечной длительности к переходным процессам неминимальной конечной длительности. Это позволяет улучшить прямые показатели качества соответствующих процессов. Решение данного типа задач связано с определенными сложностями вследствие несоответствия количества коэффициентов характеристического уравнения синтезируемой системы числу исходных расчетных уравнений. Использование предложенных методов позволяет во многих случаях обойти эту проблему путем использования специальных соотношений расчетных коэффициентов, получаемых из анализа желаемых дискретных переходных характеристик или путем угрубления синтезируемой системы после повышения ее общей устойчивости. Это осуществляется за счет формального причисления к неустойчивым устойчивых корней, находящихся в непосредственной близости к границе устойчивости. Кроме этого, предложенные методы в совокупности позволяют настраивать на конечную длительность переходных процессов даже те системы, для которых изначально это было невозможно вследствие их малой размерности или по подобной причине.

Ключевые слова: проектирование, синтез, цифровая система управления, устойчивость, грубость системы, факторизация, переходной процесс.

ПІДВИЩЕННЯ ЗАГАЛЬНОЇ СТІЙКОСТІ І КОРЕКЦІЯ ПРЯМИХ ПОКАЗНИКІВ ЯКОСТІ ПЕРЕХІДНИХ ПРОЦЕСІВ НЕМІНІМАЛЬНОЇ КІНЦЕВОЇ ТРИВАЛОСТІ У ЦИФРОВИХ СИСТЕМАХ УПРАВЛІННЯ

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Запропоновані методи, що доповнюють загальну методику синтезу дискретних (цифрових) систем управління при рішенні задач, пов'язаних з переходом від дискретних перехідних процесів мінімальної кінцевої тривалості до перехідних процесів немінімальної кінцевої тривалості. Це дозволяє покращити прямі показники якості відповідних процесів. Рішення даного типу задач пов'язано з конкретними складнощами у зв'язку з невідповідностями кількості коефіцієнтів характеристичного рівняння синтезуючої системи числу первинних розрахункових рівнянь. Використання запропонованих методів дозволяє в багатьох випадках обійти цю проблему шляхом використання спеціальних співвідношень розрахункових коефіцієнтів, які отримуються з аналізу бажаних дискретних перехідних характеристик або шляхом загрублення синтезованої системи після підвищення її загальної стійкості. Крім цього, запропоновані методи в сукупності дозволяють налаштовувати на кінцеву тривалість перехідних процесів навіть ті системи, для яких на початку це було неможливо, враховуючи їх малу розмірність чи по іншій подібній причині.

Ключові слова: проектування, синтез, цифрова система управління, стійкість, грубість системи, факторизація, перехідний процес.

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