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INVESTIGATION OF MULTI-STATE SYSTEM AVAILABILITY CHANGE DEPENDING ON SOME COMPONENTS STATES

Importance analysis is one of part of reliability engineering. Methods of importance analysis allow investigating the influence of the system component change to the reliability/availability of the system. In this paper new method for the estimation a Multi-State System(MSS) availability change depending on some (more than one) components states changes is proposed. In this case MSS is defined as system with some (more than two performance level). The method is based on the Direct Partial Logic Derivative (it is part of Logic Differential Calculus of Multiple-Valued Logic).

Key words: *availability, importance measure, multi-state system, logical differential calculus.*

Introduction

A Multi State System (MSS) is a mathematical model in reliability analysis that represents a system with some (more that two) levels of performance (availability, reliability) [1 – 3]. MSS allows presenting the analyzable system in more detail than traditional Binary-State System with two possible states as working and failing. MSS reliability analysis is a complex problem that includes different directions for estimation of MSS behaviour. One of them is crucial to identify the weakness of the system and how state change of each individual component affects the system reliability/availability. Such analysis is named as importance analysis [4 – 6].

These methods allow examining different aspects of MSS performance level change caused by the change of a component states. In particular importance analysis is used for MSS estimation depending on the system structure and its components states. The various evaluations of MSS component importance are called Importance Measure (IM). IM quantifies the criticality of a particular component within MSS. They have been widely used as tools for identifying system weaknesses, and to prioritise reliability improvement activities.

The theoretical aspects of MSS importance analysis have been extensively investigated. Different methods and algorithms are considered. Authors of the paper [2, 5] have considered basic IM for system with two performance level and multi-state components and their definitions by output performance measure. The principal approach for calculation in [5] is universal generating function methods. Authors in paper [6] have generalized this result for MSS. Markov process has been used in [7] for importance analysis both for Binary-State System and MSS. New methods based on Logical Differential Calculus for importance analysis of MSS have been considered in [8, 9]. All methods in papers [4 – 9] have been proposed for analysis of MSS performance level change depending on the change of one component state. But in real application the system performance level can be changed if some of the system components states change. In papers [10, 11] have been proposed joint IMs, but proposed methods have been considered for fixed types of IMs. In this paper we propose the definition and new method for the computation of joint IMs based on Logical Differential Calculus, in particular Direct Partial logic Derivatives. This method allows calculating the joint IMs for any IMs that is defined in terms of Direct Partial logic Derivatives. Therefore the proposed method for joint IMs calculation is universal and can be used for most of IMs [9].

This paper has next structure. In the section 1 the basic conception as structure function, Direct Partial logic Derivatives with respect the i-th variable or the vector of variables are considered. The correlation of these derivatives and MSS availability are investigated in this section too. The section 2 includes definition of IMs and joint IMs for MSS in terms of Logical Differential Calculus. The example of the calculation of joint IMs is shown in the section 3.

1. Basic Conception

1.1. Structure Function of MSS

A MSS consists of n components. The correlation of the system availability and components states is represented by the structure function [2, 8]:

$$\phi(x_1, \ldots, x_n) = \phi(\mathbf{x}):$$
$$\{0, \ldots, m_1\text{-}1\}\times\ldots\times\{0, \ldots, m_n\text{-}1\} \rightarrow \{0, \ldots, M\text{-}1\}, \quad (1)$$

where $\phi(\mathbf{x})$ is the MSS availability; x_i is components state (i = 1, …, n); m_i is number of the i-th component states; M is number of MSS availability level.

The component probability characterizes every system component state x_i from zero to (m_i-1):

$$p_{i,s_i} = \Pr\{x_i = s\}\,, \qquad (2)$$

where i = 1, …, n and s = 0, …, m_i-1.

In paper [9] the Logic Differential Calculus has been proposed for reliability analysis of MSS. In particular, we propose to use Direct Partial Logic Derivative that reflects the change in the value of the structure function when the values of variables change. Therefore this mathematical approach allows to investigate the system availability change depending on the change of component state.

These assumptions for structure function (1) in reliability analysis of MSS are used [2, 3]:

(a) the structure function is monotone and $\phi(\mathbf{s})$=s (s∈{0, …, m-1});

(b) all components are s-independent and are relevant to the system.

1.2. Direct Partial Logic Derivative

There are two types of Direct Partial Logic Derivatives in Logic Differential Calculus: with respect to one variable and with respect to variables vector. The first type permits to examine the influence of one component change to modification the MSS availability. The second type of derivative reveals the MSS availability change depending on changes of fixed system components.

A Direct Partial Logic Derivative $\partial\phi(j\to h)/\partial x_i(a\to b)$ of a structure function $\phi(\mathbf{x})$ of n variables with respect to variable x_i reflects the fact of changing of function from j to h when the value of variable x_i is changing from a to b [8, 9]:

$$\partial\phi(j\to h)/\partial x_i(a\to b) = \begin{cases} m-1, & \text{if } \phi(a_i, x) = j \;\&\; \phi(b_i, x) = h \\ 0, & \text{in the other case} \end{cases} \qquad (3)$$

where $\phi(a_i, \mathbf{x}) = \phi(x_1,\ldots, x_{i-1}, a_i, x_{i+1},\ldots, x_n)$ and $\phi(b_i, \mathbf{x}) = \phi(x_1,\ldots, x_{i-1}, b_i, x_{i+1},\ldots, x_n)$, $a_i, b_i \in \{0, \ldots, m_i-1\}$.

A Direct Partial Logic Derivatives with respect to variables vector is generalization of a Direct Partial Logic Derivative (3).

A Direct Partial Logic Derivatives of a structure function $\phi(\mathbf{x})$ of n variables with respect to variables vector $\mathbf{x}^{(\mathbf{p})} = (x_{i_1}, x_{i_2}, \ldots, x_{i_p})$ reflects the fact of changing of function from j to h when the value of every variable of vector $\mathbf{x}^{(\mathbf{p})}$ is changing from a to b [12, 13]:

$$\partial\phi(j\to h)/\partial \mathbf{x}^{(\mathbf{p})}(\mathbf{a}^{(\mathbf{p})}\to\mathbf{b}^{(\mathbf{p})}) = \begin{cases} m-1, & \text{if } \phi(a_{i_1},\ldots,a_{i_p}, x) = j \;\&\; \phi(b_{i_1},\ldots,b_{i_p}, x) = h \\ 0, & \text{in the other case} \end{cases} \qquad (4)$$

Every variable values of the vector $\mathbf{x}^{(\mathbf{p})} = (x_{i_1}, x_{i_2}, \ldots, x_{i_p})$ changes form a to b. So, vector $\mathbf{x}^{(\mathbf{p})}$ can be interpreted as components states vector or components efficiencies vector. For example, a MSS availability decrease from value j to value h is caused by deterioration of efficiency of the first component form value "four" to "three" and breakdown of the fifth component. This behaviour of this MSS is declared by (4) as: $\partial\phi(j\to h)/\partial \mathbf{x}^{(2)}(\mathbf{a}^{(2)}\to\mathbf{b}^{(2)}) = \partial\phi(j\to h)/\partial x_1(4\to 3)x_5(1\to 0)$.

2. MSS measure

2.1. Measures of a MSS states

The Reliability Function R(t), is one of best known reliability measures. For the Binary-state System this measure is defined as the probability of the system functioning: $R(t) = \Pr\{\phi(\mathbf{x}) = 1\} = 1 - U(t)$ (where U(t) is a system unreliability). In paper [10, 11] some generalizations of this measure for a MSS have been proposed. In particular, the Reliability Function of a MSS for the performance level j is defined as:

$$R_j(t) = \Pr\{\phi(\mathbf{x}) \ge j\},\; j \in (1, \ldots, m\text{-}1\}. \qquad (5)$$

The system availability or probability of a MSS state is defined as [2, 9]

$$A_j = \Pr\{\phi(\mathbf{x}) = j\},\; j \in (1, \ldots, m\text{-}1\}. \qquad (6)$$

But measures (5) and (6) don't permit the analysis of the availability change depending change of component state or some components states. At the same time, there are indices for the estimation of the influence of component states changes to MSS reliability/availability. These indices are importance measures.

2.2. Importance Measures

Importance measures estimate the probability of a system reliability/availability change caused by the fixed system component state change. The most used in engineering practice importance measures are [4 – 6]:

- Structural Importance (SI) concentrates on the topological structure of the system and determines the proportion of working states of the system in which the working of the i-th component makes the difference between system failure and working state
- Birnbaum Importance (BI) of a given component is defined as the probability that such a component is critical to MSS functioning and represents loss in MSS when the i-th component fails.

SI. In papers [8, 9] the method for the calculation of SI and BI based on Direct Partial Logic Derivatives with respect to the one variable has been considered. In paper [13] Direct Partial Logic Derivatives with respect to variables vector has been defined. The development of these derivatives for the application in the analysis and calculation of IMs (SI and BI) is considered.

The SI takes into account the topological specifics of the system. It is used for analyzing such systems, which are in designs and this measure is calculated by the next equation:

$$IS_i^{s,j} = \frac{\rho_i^{s,j}}{m_1 \ldots m_{i-1} m_{i+1} \ldots m_n}, \qquad (7)$$

where $\rho_i^{s,j}$ is number of system states when the change component state from a to a-1 results the system performance level decrement and this number is calculated as numbers of nonzero values of DPLDs (3).

BI of a given component is defined as the probability that such component is critical to MSS functioning. The mathematical generalization of this measure for MSS in terms of Logical Differential Calculus can be interpreted as:

$$IB_i^{s,j} = \Pr\left(\partial\phi(j \to h) / \partial x_i(a \to a-1) \neq 0\right). \qquad (8)$$

2.3. Joint Importance Measures

In paper [10] the joint BI has been considered. This measure is defined for the investigation of the influence of some components states changes to a MSS performance level conversion. But for this measure calculation new algorithm has been developed.

The generation and definition of joint SI and BI based on the (7) and (8) don't need special and new algorithms. In this case the Direct Partial Logic Derivatives with respect to the variables vector inside the Direct Partial Logic Derivatives with respect to the variable is used only.

Therefore the joint SI is defined as:

$$IS_{i_1 \ldots i_p}^{s,j} = \frac{\rho_{i_1 \ldots i_p}^{s,j}}{m_1 \ldots m_{i_1 - 1} m_{i_p + 1} \ldots m_n}, \qquad (9)$$

where $\rho_{i_1 \ldots i_p}^{s,j}$ is number of system states when the changes of components states from s to s-1 results the system performance level decrement and this number is calculated as numbers of nonzero values of the Direct Partial Logic Derivative (4).

The joint BI based on mathematical approach of the Direct Partial Logic Derivatives is calculated as:

$$IB_{i_1 \ldots i_p}^{s,j} = \Pr\left(\partial\phi(j \to h) / \partial x^{(p)}(a^{(p)} \to \tilde{a}^{(p)}) \neq 0\right). \qquad (10)$$

3. Application of Importance Measures

In paper [2] two type of MSS simple configuration are proposed: series and parallel systems. And different types of systems can be formed by these configurations. But typical configuration of MSS can have some interpretation. For example, in Fig.1 the parallel structure of two components (n = 2) for $m_1 = 2$, $m_2 = 4$ and M = 3 with different structure function is shown.

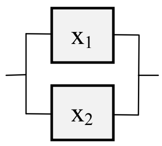
Therefore important step in analysis of typical configuration of a MSS is definition the structure function of such system.

We will use the parallel MSS with structure function that is defined:

$$\phi_s(\mathbf{x}) = AND(x_1, x_2, \ldots, x_n) = \min(x_1, x_2, \ldots, x_n), \qquad (11)$$

and the series MSS with structure function as:

$$\phi_p(\mathbf{x}) = OR(x_1, x_2, \ldots, x_n) = \max(x_1, x_2, \ldots, x_n). \qquad (12)$$



x_1	x_2	$\phi_1(\mathbf{x})$	$\phi_2(\mathbf{x})$	$\phi_3(\mathbf{x})$
0	0	0	0	0
0	1	1	1	1
0	2	2	1	1
0	3	2	1	2
1	0	1	1	1
1	1	1	2	1
1	2	2	2	1
1	3	2	2	2

Fig. 1 Example of different interpretation of MSS

Consider example of a MSS analysis by Integrate IMs. It is "bridge" MSS [2], that is presented in Fig.2. This MSS consists of five components (n = 5) and m_i = M = 4 (i = 1, …, n) and the structure function is defined as:

$$\phi(\mathbf{x}) = OR(AND(x_1,x_2),AND(x_1,x_3,x_5),AND \; AND(x_4,x_5),AND(x_4,x_3,x_2))$$

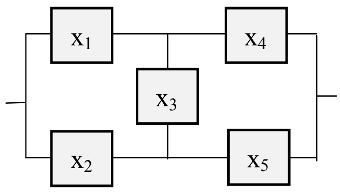


Figure 2. The bridge MSS

Consider the calculation of SI for this system break skip caused by the failures of two components. These measures is defined according to (9) based on the Direct Partial Logic Derivatives

$$\partial\phi(0 \to 1)/\partial x_i(1 \to 0) x_j(1 \to 0).$$

These values of SI measures are shown in Table 1.

The MSS failure is most possible if the first and the fourth components or the second and fifth components break down, because the SI measures for these variables have maximum values $IS_{14}^{1,1} = IS_{14}^{1,1} = 0.9375$.

The values of SI measures for the MSS failure depending on the failures of three components are in Table 1. These measures are calculated based on the Direct Partial Logic Derivatives

$$\partial\phi(0\to1)/\partial x_i(1\to0)x_j(1\to0)x_w(1\to0).$$

The MSS failure in this case has maximal probability according to the next SI measures:

$$IS^{1,1}_{124} = IS^{1,1}_{125} = IS^{1,1}_{135} = IS^{1,1}_{145} = IS^{1,1}_{235} = IS^{1,1}_{245} = 1.$$

Table 1

SI measures for the bridge MSS (m = 3) failure that depending on two (**p** = 2) and three (**p** = 3) components failures

Components x_i	x_j	**p** = 2 Numbers $\rho^{1,1}_{ij}$	$IS^{1,1}_{ij}$	**p** = 3 Numbers $\rho^{1,1}_{ijw}$	$IS^{1,1}_{ijw}$
x_1	x_2	48	0.7500	12	0.7500
x_1	x_3	44	0.6875	16	1
x_1	x_4	60	0.9375	16	1
x_1	x_5	55	0.8594	15	0.9375
x_2	x_3	44	0.6875	16	1
x_2	x_4	55	0.8594	16	1
x_2	x_5	60	0.9375	16	1
x_3	x_4	44	0.6875	15	0.9375
x_3	x_5	44	0.6875	16	1
x_4	x_5	48	0.7500	12	0.7500

Now consider bridge MSS (Fig.2) as system with three performance levels of the system and components. We analyse the influence of two and tree system components breakdowns to this MSS availability. SI measures of this MSS failure are shown in Table 2 depending on two and three components failures accordantly.

Table 2

SI measures for the bridge MSS (m = 3) failure that depending on two (**p** = 2) and three (**p** = 3) components failures

Components x_i	x_j	**p** = 2 Numbers $\rho^{1,1}_{ij}$	$IS^{1,1}_{ij}$	**p** = 3 Numbers $\rho^{1,1}_{ijw}$	$IS^{1,1}_{ijw}$
x_1	x_2	24	0.2963	8	0.8889
x_1	x_3	21	0.2593	9	1
x_1	x_4	24	0.2963	9	1
x_1	x_5	25	0.3086	8	0.8889
x_2	x_3	21	0.2593	9	1
x_2	x_4	24	0.2963	9	1
x_2	x_5	24	0.2963	9	1
x_3	x_4	21	0.2593	8	0.8889
x_3	x_5	21	0.2593	9	1
x_4	x_5	24	0.2963	8	0.8889

The values of SI measures for the MSS with m = 4 and m = 3 are similar: the equal components have maximal influence to the system failure according to the Tables 1 and 2. It is clear result because the SI analyses the topological aspect of a MSS only and doesn't consider the probabilities of the components states (2).

Conclusion

In this paper two IMs and joint IMs as Si and BI for MSS analysis are considered in terms of Logical Differential Calculation (Direct Partial Logic Derivatives). System components with maximal and minimal influence for MSS performance level changes are revealed based on these measures. This information is principal for reliability analysis of real-world systems design and their behaviour in time of exploitation. There are applications of IM in reliability analysis of nuclear power engineering [14] or transport system [15]. Therefore development of these measures mathematical approach calculation has important influence on reliability engineering.

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ИССЛЕДОВАНИЕ РАБОТОСПОСОБНОСТИ СИСТЕМЫ С НЕСКОЛЬКИМИ УРОВНЯМИ РАБОТОСПОСОБНОСТИ В ЗАВИСИМОСТИ ОТ ИЗМЕНЕНИЯ СОСТОЯНИЯ НЕСКОЛЬКИХ ЭЛЕМЕНТОВ

Е. Зайцева, В. Левашенко

Анализ значимости представляет собой одно из направлений исследований в теории надежности. Методы анализа значимости позволяют исследовать влияние изменение состояния одного элемента на надежность/работоспособность системы. В данной работе предложен новый метод оценки изменения работоспособности системы с несколькими уровнями работоспособности в зависимости от изменения состояний ее элементов (одного и более). В основу предлагаемого метода положены направленные логические производные, являющиеся предметом логического дифференциального исчисление многозначной логики.

Ключевые слова: работоспособность, индексы значимости, системы с несколькими уровнями работоспособности, логические направленные производные.

ДОСЛІДЖЕННЯ ПРАЦЕЗДАТНОСТІ СИСТЕМИ З ДЕКІЛЬКОМА РІВНЯМИ ПРАЦЕЗДАТНОСТІ ЗАЛЕЖНО ВІД ЗМІНИ СТАНУ ДЕКІЛЬКОХ ЕЛЕМЕНТІВ

О. Зайцева, В. Левашенко

Аналіз значущості являє собою один з напрямків досліджень в теорії надійності. Методи аналізу значущості дозволяють досліджувати вплив зміна стану одного елемента на надійність/працездатність системи. У даній роботі запропоновано новий метод оцінки зміни працездатності системи з кількома рівнями працездатності залежно від зміни станів її елементів (одного і більше). В основу пропонованого методу покладені спрямовані логічні похідні, які є предметом логічного диференціального числення багатозначної логіки.

Ключові слова: працездатність, індекси значимості, системи з декількома рівнями працездатності, логічні спрямовані похідні.

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