e-ISSN 1607-3274 Радіоелектроніка, інформатика, управління. 2019. № 2
p-ISSN 2313-688X Radio Electronics, Computer Science, Control. 2019. № 2

МАТЕМАТИЧНЕ ТА КОМП'ЮТЕРНЕ МОДЕЛЮВАННЯ

МАТЕМАТИЧЕСКОЕ И КОМПЬЮТЕРНОЕ МОДЕЛИРОВАНИЕ

MATHEMATICAL AND COMPUTER MODELING

UDC 517.968.21

POLYNOMIAL SOLUTIONS FOR THE KOLMOGOROV-WIENER FILTER WEIGHT FUNCTION FOR FRACTAL PROCESSES

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ABSTRACT

Context. We consider a Kolmogorov-Wiener filter for fractal random processes, which, for example, may take place in modern information-telecommunication systems and in control of complex technological processes. The weight function of the considered filter may be applied to data forecast in the corresponding systems.

Objective. As is known, in the continuous case the Kolmogorov-Wiener filter weight function obeys the Fredholm integral equation of the first kind. The aim of the work is to obtain the Kolmogorov-Wiener filter weight function as an approximate solution of the corresponding integral equation.

Method. The truncated orthogonal polynomial expansion method for approximate solution of the Fredholm integral equation of the first kind is used. A set of orthonormal polynomials is used.

Results. We obtained approximate results for the Kolmogorov-Wiener weight function for fractal processes with a power-law structure function. The weight function is found as an approximate solution of the Fredholm integral equation of the first kind the kernel of which is the correlation function of the corresponding fractal random process. Analytical results for the one-, two-, three-, four- and five-polynomial approximations are obtained. A numerical comparison of the left-hand and right-hand sides of the integral equation for the obtained weight functions is given for different values of the parameters. The corresponding numerical investigation is made up to the 18-polynomial approximation on the basis of the Wolfram Mathematica package. The applicability of the obtained solutions is discussed.

Conclusions. The Kolmogorov-Wiener weight function for fractal processes is obtained approximately in the form of a truncated orthogonal polynomial series. The validity of the obtained weight functions is discussed. The obtained results may be applied to the data forecast in a wide variety of different systems where fractal random processes take place.

KEYWORDS: Kolmogorov-Wiener filter weight function, truncated orthogonal polynomial expansion, Fredholm integral equation of the first kind, approximate solution.

NOMENCLATURE

T – time interval along which the input data are observed

k – time interval for which the forecast should be made

$h(t)$ – Kolmogorov-Wiener filter weight function

H – Hurst exponent

$S_n(t)$ – set of orthonormal polynomials in $t \in [0, T]$.

$R(t)$ correlation function of the random fractal process

σ^2 process variance

INTRODUCTION

Nowadays fractal processes take place in a huge variety of different systems (see, for example, [1–4] and various references in [4]). This paper is devoted to the obtaining of the Kolmogorov–Wiener filter weight function for continuous fractal processes. The structure function of the corresponding random fractal process is supposed to be a power-law one. Such a model is widely used for description of different systems in different fields of knowledge; see, for example, [5] and references therein.

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DOI 10.15588/1607-3274-2019-2-5

In paper [4] the problem of data forecast for fractal processes in telecommunication systems was considered on the basis of Kolmogorov-Wiener filter. The results of paper [4] were refined in [5], but the Volterra integral equation was considered in [4, 5] rather than the Fredholm one. As is known [6], in the general case for such a problem the Fredholm integral equation of the first kind should be used. The exact analytical solution for such equation meets difficulties, so an approximate solution of the corresponding integral equation is obtained in this paper.

The object of study is the Kolmogorov-Wiener filter for continuous fractal processes.

The subject of study is the weight function of the corresponding filter.

The aim of the work is to obtain the corresponding weight function as an approximate solution of the Fredholm integral equation of the first kind.

1 PROBLEM STATEMENT

We consider the Kolmogorov-Wiener filter for continuous fractal processes. As is known, the weight function of the considered filter obeys the following integral equation

$$\int_0^T d\tau h(\tau) R(t-\tau) = R(t+k), \qquad (1)$$

where T is the time interval along which the input data are observed, $k \ll T$ the time interval for which the forecast should be made, $h(\tau)$ is the Kolmogorov-Wiener filter weight function and $R(t)$ is the correlation function of the corresponding fractal process, the noiseless case is considered. Here we consider a random fractal process with the power-law structure function which leads to the following correlation function [5]

$$R(t) = \sigma^2 - \frac{\alpha}{2}|t|^{2H}, \qquad (2)$$

where σ^2 is the process variance, α is a constant and H is the Hurst exponent. The statement of the problem is to obtain the weight function $h(\tau)$ as an approximate solution to the integral equation (1).

2 REVIEW OF THE LITERATURE

Nowadays fractal processes are used for the description of a huge variety of different systems, and a model with a power-law structure function is a popular model of the fractal process, (see, for example, [1–5] and references therein).

In paper [4] the Kolmogorov-Wiener filter is proposed in order to make the forecast for the fractal traffic which takes place in some telecommunication systems. Such a traffic is rather data-intensive, that is why for convenience it is described as a continuous process in [4].

But in [4] the Volterra integral equation of the first kind is used rather than the Fredholm one. In [4] the method of solution of the corresponding Volterra integral equation is described and finally an exact analytical solution of this equation was obtained in [5]. It should be stressed that the Volterra integral equation is not so complicated as the Fredholm one and it admits an exact analytical solution. Maybe, in some cases the use of the Volterra equation instead of the Fredholm one is a reasonable simplification. But definitely in the general case one should use the Fredholm integral equation of the first kind rather than the Volterra one, see, for example, [6]. So the aim of the paper is to solve the corresponding Fredholm equation.

But the exact analytical solution of the Fredholm integral equation (1) meets difficulties, so here we restrict ourselves only to a search for an approximate solution of (1).

It should be stressed that Fredholm integral equations of the first kind take place in various fields of knowledge. One of the most popular methods of their approximate solution, which is used in this paper, is the expansion of the unknown function into a truncated orthogonal polynomial series, see the corresponding solution of the kinetic equation in the framework of statistical physics [7, 8]. In fact, this method is a special case of the Galerkin's method described in [9].

3 MATERIALS AND METHODS

The solution of eq, (1) is sought as the orthogonal polynomials series

$$h(t) = \sum_n g_n S_n(t) \qquad (3)$$

where $S_n(t)$ a set of polynomials which are orthonormal in $t \in [0,T]$ and g_n are unknown coefficients.

The polynomials $S_n(t)$ are constructed as follows. As is known [10], the polynomials $S_n'(t)$ orthogonal in $t \in [0,T]$ can be constructed as follows:

$$S_n'(\tau) = \begin{vmatrix} \mu_0 & \mu_1 & \mu_2 & \cdots & \mu_n \\ \mu_1 & \mu_2 & \mu_3 & \cdots & \mu_{n+1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mu_{n-1} & \mu_n & \mu_{n+1} & \cdots & \mu_{2n-1} \\ 1 & \tau & \tau^2 & \cdots & \tau^n \end{vmatrix}, \quad \tau \in [0,T] \qquad (4)$$

where

$$\mu_n = \int_0^T x^n dx = \frac{T^{n+1}}{n+1}. \qquad (5)$$

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DOI 10.15588/1607-3274-2019-2-5

The numerical values of $S_n'(t)$ for $t \approx T$ may be rather large, that is why the use of the orthonormal polynomials $S_n(t)$ may be convenient:

$$S_n(\tau) = \frac{S_n'(\tau)}{\sqrt{\int_0^T dt \left(S_n'(t)\right)^2}}, \tag{6}$$

the polynomials (6) obey the property

$$\int_0^T dt S_n(t) S_m(t) = \delta_{mn} \tag{7}$$

where δ_{mn} is the Kronecker delta.

By a straightforward calculation on the basis of (4)–(6) one can obtain explicit expressions for the first 5 polynomials:

$$\begin{gathered} S_0(t) = \frac{1}{\sqrt{T}},\ S_1(t) = \frac{\sqrt{3}}{\sqrt{T}}\left(\frac{2}{T}t - 1\right), \\ S_2(t) = \frac{\sqrt{5}}{T^{1/2}}\left(\frac{6}{T^2}t^2 - \frac{6}{T}t + 1\right), \\ S_3(t) = \frac{\sqrt{7}}{T^{1/2}}\left(-1 + \frac{12}{T}t - \frac{30}{T^2}t^2 + \frac{20}{T^3}t^3\right), \\ S_4(t) = \frac{3}{T^{1/2}}\left(1 - \frac{20}{T}t + \frac{90}{T^2}t^2 - \frac{140}{T^3}t^3 + \frac{70}{T^4}t^4\right). \end{gathered} \tag{8}$$

On the basis of (3) one can rewrite (1) as

$$\sum_n g_n \int_0^T d\tau S_n(\tau) R(t-\tau) = R(t+k). \tag{9}$$

After multiplying (9) by $S_m(t)$ and integrating one can obtain

$$\sum_n g_n G_{mn} = b_m, \tag{10}$$

where

$$\begin{gathered} G_{mn} = \int_0^T dt \int_0^T d\tau S_n(\tau) S_m(t) R(t-\tau), \\ b_m = \int_0^T dt S_m(t) R(t+k), \end{gathered} \tag{11}$$

the quantities G_{mn} are called the integral brackets. The obtained set of linear equations (10) is infinite, and solving (10) meets difficulties. So one should artificially truncate (10):

$$\sum_{n=0}^{l-1} g_n G_{mn} = b_m,\ m = \overline{0, l-1}. \tag{12}$$

The set (12) is finite, and one can obtain the coefficients g_n as the solution of the set (12). The corresponding approximate solution of the integral equation (1)

$$h(\tau) = \sum_{n=0}^{l-1} g_n S_n(\tau) \tag{13}$$

is called the solution in the l-polynomial approximation. In matrix form (12) can be rewritten as

$$Gg = b \tag{14}$$

where

$$\begin{gathered} G = \begin{pmatrix} G_{00} & G_{10} & \cdots & G_{0j} \\ G_{01} & G_{11} & \cdots & G_{1j} \\ \vdots & \vdots & \ddots & \vdots \\ G_{0j} & G_{1j} & \cdots & G_{jj} \end{pmatrix},\ j = l-1, \\ g = \begin{pmatrix} g_0 \\ g_1 \\ \vdots \\ g_j \end{pmatrix},\ b = \begin{pmatrix} b_0 \\ b_1 \\ \vdots \\ b_j \end{pmatrix}, \end{gathered} \tag{15}$$

so the coefficients g_n can the obtained in matrix form as

$$g = G^{-1} b. \tag{16}$$

Now let us consider the properties of the matrix G. First of all,

$$\begin{gathered} G_{mn} = \int_0^T dt \int_0^T d\tau S_n(\tau) S_m(t) R(t-\tau) = \{t \leftrightarrow \tau\} = \\ = \int_0^T d\tau \int_0^T dt S_n(t) S_m(\tau) R(\tau - t). \end{gathered} \tag{17}$$

The correlation function is an even one:

$$R(\tau - t) = R(t - \tau), \tag{18}$$

and on the basis of (17) we have

$$G_{mn} = \int_0^T d\tau \int_0^T dt S_m(\tau) S_n(t) R(\tau - t) = G_{nm}. \tag{19}$$

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DOI 10.15588/1607-3274-2019-2-5

Also the polynomials (6) obey the property

$$S_n\left(\frac{T}{2}+t\right)=\begin{cases}S_n\left(\frac{T}{2}-t\right), n\vdots 2\\ -S_n\left(\frac{T}{2}-t\right), n\not\vdots 2\end{cases},\ t\in\left[0,\frac{T}{2}\right]. \qquad (20)$$

Let us consider the quantity G_{nm} where m and n are of different evenness. On the basis of (11) and (18) we have

$$\begin{aligned}G_{mn}&=\int_0^T dt\int_0^T d\tau S_m(t)S_n(\tau)R(t-\tau)=\\&=\left\{x=\frac{T}{2}-t, y=\frac{T}{2}-\tau\right\}=\\&=\int_{-\frac{T}{2}}^{\frac{T}{2}}dx\int_{-\frac{T}{2}}^{\frac{T}{2}}dyS_m\left(\frac{T}{2}-x\right)S_n\left(\frac{T}{2}-y\right)R(y-x)=\\&=\{y\to -y, x\to -x\}=\\&=\int_{-\frac{T}{2}}^{\frac{T}{2}}dx\int_{-\frac{T}{2}}^{\frac{T}{2}}dyS_m\left(\frac{T}{2}+x\right)S_n\left(\frac{T}{2}+y\right)R(y-x),\end{aligned} \qquad (21)$$

here the fact that the correlation function is even is used. On the basis of (20) and the fact that m and n are of different evenness, we have

$$S_m\left(\frac{T}{2}+x\right)S_n\left(\frac{T}{2}+y\right)=-S_m\left(\frac{T}{2}-x\right)S_n\left(\frac{T}{2}-y\right) \qquad (22)$$

and with account for (21) we have

$$\begin{aligned}G_{mn}&=\int_{-\frac{T}{2}}^{\frac{T}{2}}dx\int_{-\frac{T}{2}}^{\frac{T}{2}}dyS_m\left(\frac{T}{2}-x\right)S_n\left(\frac{T}{2}-y\right)R(y-x)=\\&=-G_{mn}\Rightarrow G_{mn}=0.\end{aligned} \qquad (23)$$

So the matrix G obeys the following properties:

1. G is a symmetrical matrix: $G_{mn}=G_{nm}$
2. $G_{mn}=0$ if m and n are of different evenness.

These properties allow one not to calculate all the matrix G by a straightforward calculation. It is enough to calculate by straightforward calculation only those components G_{mn} for which $m\ge n$ and m and n are of the same evenness.

By a straightforward calculation on the basis of (8) and (11) one can obtain the components of the matrix G up to G_{44}:

$$\begin{aligned}G_{00}&=\sigma^2T-\frac{\alpha}{2}\frac{1}{(2H+1)(H+1)}T^{2H+1},\\G_{11}&=\frac{\alpha}{2}\frac{3H}{(2H+1)(H+1)(H+2)}T^{2H+1},\\G_{22}&=-\frac{5}{3}\frac{H-1}{H+3}G_{11},\ G_{33}=-\frac{7}{5}\frac{H-2}{H+4}G_{22},\\G_{44}&=-\frac{9}{7}\frac{H-3}{H+5}G_{33},\\G_{20}&=-\frac{2\sqrt{5}\alpha H}{(2H+2)(2H+3)(2H+4)}T^{2H+1},\\G_{40}&=-\frac{\alpha}{2}\frac{24H(H-1)(2H-1)}{(2H+6)(2H+5)(2H+4)}\times\\&\times\frac{1}{(2H+3)(2H+2)}T^{2H+1},\\G_{42}&=-\frac{\alpha}{2}\frac{3\sqrt{5}H(H-1)(H-2)}{(H+1)(H+2)(H+3)}\times\\&\times\frac{1}{(H+4)(2H+3)}T^{2H+1},\\G_{31}&=\frac{\alpha}{2}\frac{\sqrt{21}H(H-1)}{(H+1)(H+2)(H+3)(2H+3)}T^{2H+1}.\end{aligned} \qquad (24)$$

On the basis of (11) and 8 the coefficients b_m are calculated up to b_4:

$$\begin{aligned}b_0&=\frac{1}{T^{1/2}}\left(\sigma^2T-\frac{\alpha}{2}J_1\right),\\b_1&=-\frac{\alpha}{2}\frac{\sqrt{3}}{T^{3/2}}\left(2J_2-(2k+T)J_1\right),\\b_2&=-\frac{\alpha}{2}\frac{\sqrt{5}}{T^{5/2}}\left[6J_3-(12k+6T)J_2+\right.\\&\left.+\left(6k^2+6kT+T^2\right)J_1\right],\\b_3&=-\frac{\alpha}{2}\frac{\sqrt{7}}{T^{7/2}}\left[-\left(T^3+12T^2k+30Tk^2+20k^3\right)J_1+\right.\\&\left.+12\left(T^2+5kT+5k^2\right)J_2-30(T+2k)J_3+20J_4\right],\\b_4&=-\frac{\alpha}{2}\frac{3}{T^{9/2}}\left[J_1\left(T^4+20kT^3+90k^2T^2+\right.\right.\\&\left.+140k^3T+70k^4\right)-20J_2\left(T^3+9kT^2+21k^2T+\right.\\&\left.+14k^3\right)+10J_3\left(9T^2+42kT+42k^2\right)-\\&\left.-140(T+2k)J_4+70J_5\right],\\J_n&=\frac{(T+k)^{2H+n}-k^{2H+n}}{2H+n}.\end{aligned} \qquad (25)$$

On the basis of (15), (16) and the above-mentioned properties of the matrix G one can obtain the following results:

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DOI 10.15588/1607-3274-2019-2-5

$$h^{[1]}(t)=g_0^{[1]}S_0(t),\ g_0^{[1]}=g_0^{[2]}=\frac{b_0}{G_{00}},$$

$$h^{[2]}(t)=g_0^{[1]}S_0(t)+g_1^{[2]}S_1(t),\ \ g_1^{[2]}=g_1^{[3]}=\frac{b_1}{G_{11}},$$

$$h^{[3]}(t)=g_0^{[3]}S_0(t)+g_1^{[3]}S_1(t)+g_2^{[3]}S_2(t),$$

$$g_0^{[3]}=g_0^{[4]}=\frac{b_0G_{22}-b_2G_{20}}{G_{00}G_{22}-G_{20}^2},$$

$$g_2^{[3]}=g_2^{[4]}=\frac{b_0G_{20}-b_2G_{00}}{G_{20}^2-G_{00}G_{22}},$$

$$h^{[4]}(t)=g_0^{[4]}S_0(t)+g_1^{[4]}S_1(t)+g_2^{[4]}S_2(t)+$$
$$+g_3^{[4]}S_3(t),$$

$$g_1^{[4]}=g_1^{[5]}=\frac{b_1G_{33}-b_3G_{31}}{G_{11}G_{33}-G_{31}^2},$$

$$g_3^{[4]}=g_3^{[5]}=\frac{b_1G_{31}-b_3G_{11}}{G_{31}^2-G_{11}G_{33}},\qquad(26)$$

$$h^{[5]}(t)=g_0^{[4]}S_0(t)+g_1^{[4]}S_1(t)+g_2^{[4]}S_2(t)+$$
$$+g_3^{[4]}S_3(t)+g_4^{[5]}S_4(t),$$

$$g_4^{[5]}=\frac{\xi}{\eta},$$

$$\xi=\left(G_{22}G_{40}-G_{20}G_{42}\right)\left(b_0G_{40}-G_{00}b_4\right)-$$
$$-\left(b_2G_{40}-G_{20}b_4\right)\left(G_{20}G_{40}-G_{00}G_{42}\right),$$

$$\eta=\left(G_{22}G_{40}-G_{20}G_{42}\right)\left(G_{40}^2-G_{00}G_{44}\right)-$$
$$-\left(G_{20}G_{40}-G_{00}G_{42}\right)\left(G_{42}G_{40}-G_{20}G_{44}\right),$$

$$g_2^{[5]}=\frac{G_{40}b_2-G_{20}b_4-\left(G_{40}G_{42}-G_{20}G_{44}\right)g_4^{[5]}}{G_{40}G_{22}-G_{20}G_{42}},$$

$$g_0^{[5]}=\frac{b_4}{G_{40}}-\frac{G_{42}}{G_{40}}g_2^{[5]}-\frac{G_{44}}{G_{40}}g_4^{[5]},$$

where explicit expressions for G_{mn}, b_m and $S_n(t)$ are given in (24), (25) and (8), respectively, and the superscript $[n]$ denotes that the corresponding quantity is taken in the n-polynomial approximation. The approximations of a larger number of polynomials are investigated in this paper only numerically because the corresponding analytical expressions are too cumbersome.

4 EXPERIMENTS

As is known, the above-mentioned method of truncated polynomial expansion is convergent if the kernel of the corresponding integral equation is positively defined function (see, for example, a similar discussion for the solution of kinetic equations in electron-phonon systems in [11]). The kernel of the integral equation (1) is the correlation function (2), which is not a positively defined function, so the proposed method is not necessarily convergent for all the parameters. So the obtained solutions should be checked at different numerical values of the parameters, and the aim of this section is to answer the question for which parameters the proposed method is reliable.

Let us take the following parameters:

$$T=100,\ k=3,\ \sigma=1.2,\ H=0.8,\ \alpha=\pi/2\qquad(27)$$

and compare numerically the left-hand and right-hand sides of eq. (1) for the obtained weight functions (26). The corresponding numerical calculation is made in Wolfram Mathematica 11 package. The corresponding graphs for one-, two-, three-, four- and five-polynomial approximations are given in Fig. 1, Fig. 2, Fig. 3, Fig. 4 and Fig. 5, respectively:

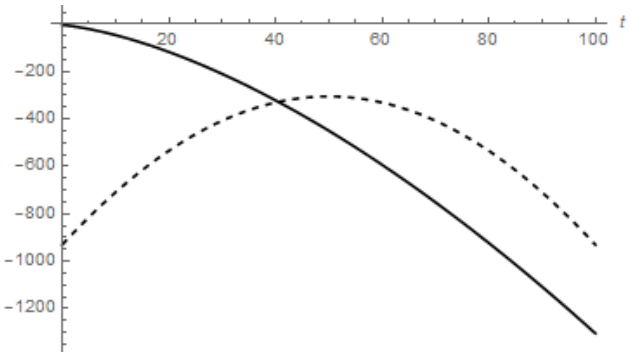

Figure 1 – Comparison the left-hand and right-hand sides of eq. (1) for parameters (27) for the one-polynomial approximation

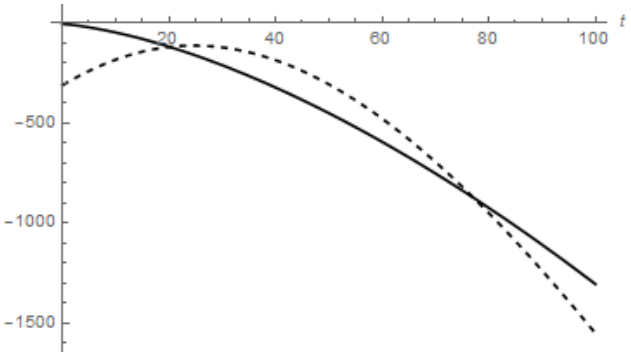

Figure 2 – Comparison the left-hand and right-hand sides of eq. (1) for parameters (27) for the two-polynomial approximation

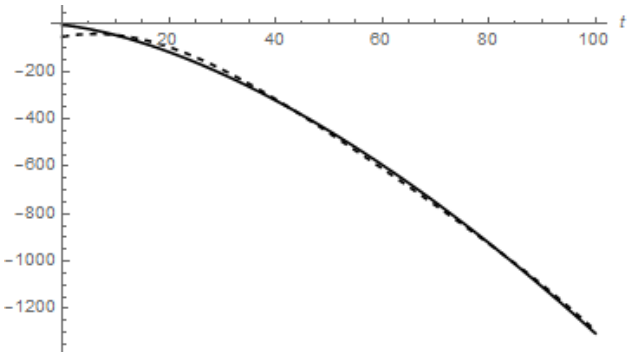

Figure 3 – Comparison the left-hand and right-hand sides of eq. (1) for parameters (27) for the three-polynomial approximation

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DOI 10.15588/1607-3274-2019-2-5

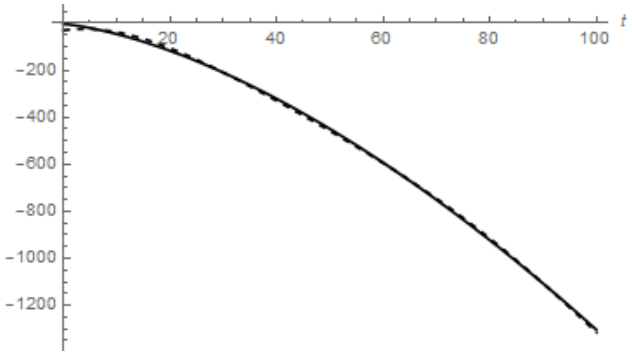

Figure 4 – Comparison the left-hand and right-hand sides of eq. (1) for parameters (27) for the three-polynomial approximation

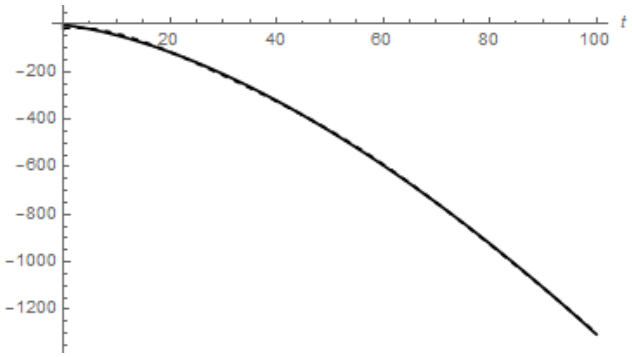

Figure 5 – Comparison the left-hand and right-hand sides of eq. (1) for parameters (27) for the three-polynomial approximation

It should be stressed that the left-hand side of (1) is calculated numerically on the basis of Wolfram Mathematica as

$$\int_0^T d\tau h(\tau) R(t-\tau) = \int_0^T d\tau h(\tau)\left(\sigma^2 - \frac{\alpha}{2}|t-\tau|^{2H}\right) = \\ = \int_0^t d\tau h(\tau)\left(\sigma^2 - \frac{\alpha}{2}(t-\tau)^{2H}\right) + \\ + \int_t^T d\tau h(\tau)\left(\sigma^2 - \frac{\alpha}{2}(\tau-t)^{2H}\right). \qquad (28)$$

As can be seen from the figures, the one-polynomial approximation is not quite accurate, but the accuracy of the obtained solution increases with the number of polynomials, and the three-polynomial approximation is already rather accurate. For the five-polynomial approximation the obtained curves are very close to each other. Approximations of a larger number of polynomials are made numerically in Wolfram Mathematica up to the 18-polynomial approximation. It should be stressed that Mathematica is not able to calculate the approximation of higher-than-18 polynomials adequately due to machine errors (the corresponding «ripple» can be seen on the graphs). A rather strange behavior of n-polynomial approximation solutions is obtained: for $1 \le n \le 8$ the accuracy increases, and for $n = 7$ and $n = 8$ the curves are in fact ideally identical. For $9 \le n \le 15$ the method fails – the left-hand and right-hand sides of (1) are totally different. But for $16 \le n \le 18$ the method is again very good and the curves are in fact ideally identical. Such a strange behavior can hardly be explained. Maybe the reason is that the kernel of the integral equation (1) is not a positively defined function and the convergence of the method is not guaranteed. But nevertheless it should be stressed that for parameters (27) and for $3 \le n \le 8$ the method works really good and the obtained solutions for the weight function give the good coincidence of the left-hand and right-hand sides of eq. (1).

Then let us change the parameters. The most interesting change is the change of the parameter T because this parameter may vary most significantly in real systems. So let us take the parameters

$$T = 10,\ k = 3,\ \sigma = 1.2,\ H = 0.8,\ \alpha = \pi/2. \qquad (29)$$

For parameters (29) the corresponding investigation is also made up to the 18-polynomial approximation, and it seems that for parameters (29) the method is really convergent. The accuracy increases with the number of polynomials, the three-polynomial approximation is already rather accurate, for the five-polynomial approximation the obtained curves are very close to each other and starting from $n = 8$ the curves are in fact ideally identical.

Another set of parameters which is investigated in the paper is the following:

$$T = 1000,\ k = 3,\ \sigma = 1.2,\ H = 0.8,\ \alpha = \pi/2. \qquad (30)$$

For this set of parameters the method is not convergent. For $1 \le n \le 4$ the accuracy increases and for $n = 3$ and $n = 4$ the coincidence of the curves is rather good. But starting from $n = 5$ the method begins to fail, and the accuracy decreases with the number of polynomials for $n \ge 5$. The corresponding comparisons of the left-hand and right-hand sides of (1) for the four- and five-polynomial approximation are given in Fig. 6 and Fig. 7, respectively.

But, as can be seen, the four-polynomial approximation gives a really good coincidence of the curves. So, although the method is not convergent for parameters (30), the four-polynomial approximation gives a good approximate solution for the weight function.

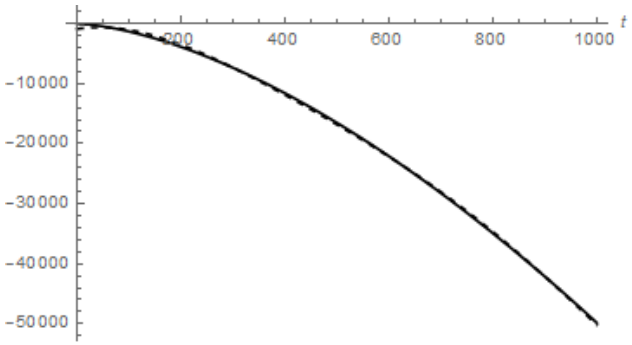

Figure 6 – Comparison the left-hand and right-hand sides of eq. (1) for parameters (30) for the four-polynomial approximation

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DOI 10.15588/1607-3274-2019-2-5

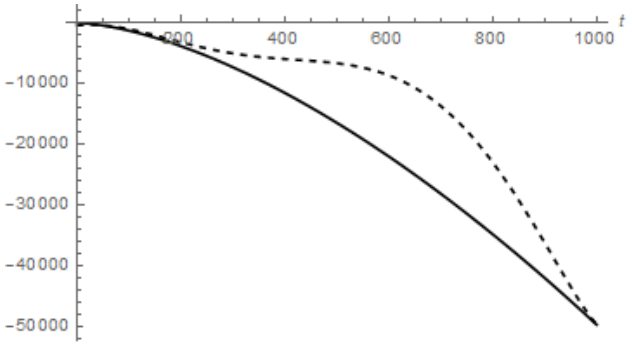

Figure 7 – Comparison the left-hand and right-hand sides of eq. (1) for parameters (30) for the five-polynomial approximation

It should be stressed that, strictly speaking, the correlation function should obey the property

$$\left|R(t)\right| \le R(0), \qquad (31)$$

so the considered parameters may not be physical. But if, for example, we change the parameter α in such a way that $R(t)$ obeys (31), the situation does not change significantly; anyway, the situation does not become better.

If, for example, we take $\alpha = 3\cdot 10^{-3}$ rather than $\alpha = \pi/2$ in (27), we have the following behavior of n-polynomial approximations. The one-polynomial approximation is not accurate, for the two-polynomial approximation the curves on the corresponding graphs are very close to each other. The three and four-polynomial approximation give approximately the same pictures. They are worse than the two-polynomial approximation, but better than the one-polynomial approximation. But the five-polynomial approximation gives almost identical curves, and for $5 \le n \le 18$ the qualitative behavior of the solutions is the same as for parameters (27).

If, for example, we take $\alpha = 10^{-1}$ rather than $\alpha = \pi/2$ in (29), we have the following behavior of n-polynomial approximations. The one-polynomial approximation is not accurate, the two-polynomial approximation is much better than the one-polynomial one, and for $2 \le n \le 6$ the accuracy slowly increases with the number of polynomials. The accuracy of the 7-polynomial approximation is lower than that of 6-polynomial approximation, but for $7 \le n \le 10$ the accuracy slowly increases with the number of polynomials. For $11 \le n \le 17$ the accuracy increases with the number of polynomials and the curves on the corresponding graphs are very close to each other. Mathematica is not able to calculate the approximations of more than 17 polynomials adequately due to machine errors (the corresponding «ripple» can be seen on the graphs).

If, for example, we take $\alpha = 8\cdot 10^{-5}$ rather than $\alpha = \pi/2$ in (30), we have the following behavior of n-polynomial approximations. The one-polynomial approximation is not accurate, but the two-polynomial approximation is rather accurate: the curves in the corresponding graphs are very close to each other. For $n \ge 3$ the accuracy of the result decreases with the number of polynomials, starting from $n = 5$ the curves are far from each other.

5 RESULTS

The method of truncated orthogonal polynomial expansion is proposed in order to solve the integral equation (1). Analytical expressions for a one-, two-, three-, four- and five-polynomial approximation solutions are obtained.

The kernel of this integral equation is not a positively defined function, so the method may not be convergent, in other words, the accuracy of the obtained solution may not increase with the number of polynomials. But in a rather wide range of parameters the approximations of rather small number of polynomials are rather accurate and may be applied to the following investigation of the data forecast. Moreover, for some parameters the method is convergent.

6 DISCUSSION

We propose the method of truncated orthogonal polynomial expansion in order to obtain the Kolmogorov–Wiener filter weight function on the basis of the Fredholm integral equation of the first kind (1). A set of polynomials orthogonal for $t \in [0, T]$ is built (6), and this set is convenient because of the above-mentioned properties of the integral brackets. The analytical expressions for the approximate solutions for the integral equation (1) are obtained in the one-, two-, three-, four- and five-polynomial approximation.

The kernel of the integral equation (1) is not a positively defined function, so the proposed method is not necessarily convergent for all the parameters. The sets of parameters (27), (29), (30) are chosen to check the convergence and the accuracy of the proposed method. The investigation is numerically made up to the 18-polynomial approximation.

For rather small T ($T = 10$) the method is convergent, and starting from the three-polynomial approximation the left-hand and the right-hand sides of (1) are rather close, starting from the eight-polynomial approximation they are almost ideally identical.

For $T = 100$ a rather strange behavior of n-polynomial approximations is seen. The accuracy increases up to the eight-polynomial approximation, starting from the three-polynomial approximation the left-hand and the right-hand sides of (1) are rather close, for the seven- and eight-polynomial approximation they are almost ideally identical. The corresponding graphs for the one-, two-, three-, four and five-polynomial approximations are given. Then for $9 \le n \le 15$ the method fails, and for $16 \le n \le 18$ the method again works very well. Such behavior can hardly be explained. But it should be stressed that the approximation of 3–8 polynomials works well, and the corresponding obtained

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DOI 10.15588/1607-3274-2019-2-5

weight functions may be applied to a further investigation of the data forecast.

For $T = 1000$ the method is not convergent, the five-polynomial approximation is not good, and the accuracy decreases with the number n of polynomials for $n \geq 5$. But for $n = 1, 2, 3, 4$ the accuracy of the obtained solutions increases, and three- and four-polynomial approximations are rather accurate.

To summarize the above-mentioned, we should stress that the proposed method is not necessarily convergent for all the parameters. It works well for rather small values of the parameter T, but for high values of this parameter the method may not be convergent. Nevertheless, the approximation of rather small number of polynomials may be rather accurate in a wide range of parameters. But it should be stressed that generally speaking, each of the approximations obtained by the proposed method should be checked numerically before it is applied in further investigation of data forecast.

CONCLUSIONS

Approximate solutions for the Kolmogorov-Wiener filter weight function for fractal processes are obtained and the applicability of the proposed method is discussed.

The scientific novelty of the obtained results is that the approximate solutions for the weight function in the problem under consideration are obtained on the basis of the truncated orthogonal polynomial expansion method. The applicability of the proposed solutions is discussed.

The practical significance is that the obtained results may be applied to further investigation of data forecast for continuous fractal processes.

Prospects for further research are to apply the obtained results to the corresponding investigation of data forecast.

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Received 21.02.2019.
Accepted 24.03.2019.

УДК 517.968.21

ПОЛІНОМНІ РОЗВ'ЯЗКИ ДЛЯ ВАГОВОЇ ФУНКЦІЇ ФІЛЬТРА КОЛМОГОРОВА-ВІНЕРА ДЛЯ ФРАКТАЛЬНИХ ПРОЦЕСІВ

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АНОТАЦІЯ

Актуальність. Ми розглядаємо фільтр Колмогорова-Вінера для випадкових фрактальних процесів, які, наприклад, можуть мати місце в сучасних інформаційно-телекомунікційних системах та у керуванні складними технологічними процесами. Вагова функція фільтру, що розглядається, може бути застосована до прогнозу даних у відповідних системах.

Мета роботи. Як відомо, у неперервному випадку рівняння на вагову функцію фільтра Колмогорова-Вінера є рівнянням Фредгольма першого роду. Метою роботи є отримати вагову функцію фільтра Колмогорова-Вінера як наближений розв'язок відповідного інтегрального рівняння.

Метод. Використано метод обірваного розвинення за ортогональними поліномами для наближеного розв'язання інтегрального рівняння Фредгольма першого роду. Використано систему ортонормованих поліномів.

Результати. Нами отримано наближені результати для вагової функції фільтру Колмогорова-Вінера для фрактальних процесів з степеневою структурною функцією. Вагову функцію знайдено як наближений розв'язок інтегрального рівняння Фредгольма першого роду, ядром якого є кореляційна функція відповідного випадкового фрактального процесу. Аналітичні результати отримано для наближень одного, двох, трьох, чотирьох та п'яти поліномів. Для різних значень параметрів зроблене чисельне порівняння лівої та правої частин інтегрального рівняння для отриманих вагових функцій. Відповідне

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DOI 10.15588/1607-3274-2019-2-5

чисельне дослідження зроблене у математичному пакеті Wolfram Mathematica до наближення 18 поліномів включно. Обговорюється застосовність отриманих результатів.

Висновки. Наближено отримано вагову функцію фільтра Колмогорова-Вінера для фрактальних процесів у вигляді обірваного ряду за ортогональними поліномами. Обговорено застосовність отриманих вагових функцій. Отримані результати можуть бути застосовними до прогнозування данних для багатьох різних систем, де мають місце фрактальні процеси.

КЛЮЧОВІ СЛОВА: вагова функція фільтру Колмогорова-Вінера, обірване розвинення за ортогональними поліномами, інтегральне рівняння Фредгольма першого роду, наближений розв'язок.

УДК 517.968.21

ПОЛИНОМНЫЕ РЕШЕНИЯ ДЛЯ ВЕСОВОЙ ФУНКЦИИ ФИЛЬТРА КОЛМОГОРОВА-ВИНЕРА ДЛЯ ФРАКТАЛЬНЫХ ПРОЦЕССОВ

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АННОТАЦИЯ

Актуальность. Мы рассматриваем фильтр Колмогорова-Винера для фрактальных процессов, которые, например, могут иметь место в современных информационно-телекоммуникационных системах и в управлении сложными технологическими процессами. Весовая функция рассматриваемого фильтра может быть применена для прогноза данных в соответствующих системах.

Цель работы. Как известно, в непрерывном случае уравнение на весовую функцию фильтра Колмогорова–Винера есть интегральным уравнением Фредгольма первого рода. Цель работы – получить весовую функцию фильтра Колмогорова–Винера как приближенное решение соответствующего интегрального уравнения.

Метод. Использован метод оборванного разложения по ортогональным полиномам для получения приближенного решения интегрального уравнения Фредгольма первого рода. Использовалась система ортонормированных полиномов.

Результаты. Нами получены приближенные результаты для весовой функции фильтра Колмогорова-Винера для фрактальных процессов, структурная функция которых есть степенной функцией. Весовая функция найдена как приближенное решение интегрального уравнения Фредгольма первого рода, ядром которого есть корреляционная функция соответствующего фрактального процесса. Аналитические результаты получены для приближений одного, двух, трех, четырех и пяти полиномов. Для разных значений параметров сделано численное сравнение левой и правой частей интегрального уравнения для полученных весовых функций. Соответствующее численное исследование проводилось с помощью математического пакета Wolfram Mathematica вплоть до приближения 18 полиномов. Обговаривается применимость полученных решений.

Выводы. Весовая функция фильтра Колмогорова-Винера для фрактальных процессов приближенно получена в виде оборванного ряда по ортогональным полиномам. Обсуждается применимость полученных весовых функций. Полученные результаты могут быть применены к прогнозированию данных в широком разнообразии различных систем, в которых имеют место случайные фрактальные процессы.

КЛЮЧЕВЫЕ СЛОВА: весовая функция фильтра Колмогорова-Винера, оборванное разложение по ортогональным полиномам, интегральное уравнение Фредгольма первого рода, приближенное решение.

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DOI 10.15588/1607-3274-2019-2-5