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POINT STANDARD MULTIFACTOR INDEX OF SENSITIVITY

It has been considered, characteristic in conditions of uncertainty, multifactor sensitivity of criterion of the plan (project) efficiency towards the set of standard changes of the factors, i.e. corresponding to their identical-percentage changes by the module. For its estimation the point standard multifactor indexes of sensitivity (PSMIS) in a basic point of the factors in the differential form have been received. The connection between PSMIS and one-factor indexes of sensitivity of criterion has been established.

Keywords: *risk, changes, point standard multifactor index of sensitivity, differential, ordered, potential.*

Розглядається характерна в умовах системного багатофакторного підходу чутливість критерію ефективності плану (проекту) щодо сукупності стандартних змін факторів, тобто відповідних до однаково-процентних за модулем їх змін. Для її оцінки одержані точкові стандартні багатофакторні показники чутливості (ТСБПЧ) у базисній точці факторів, що мають диференціальну форму. Встановлений зв'язок між ТСБПЧ і однофакторними показниками чутливості критерію.

Ключові слова: *ризик, зміни, точковий стандартний багатофакторний показник чутливості, диференціальна, упорядкований, потенціал.*

Выявление, возможное управление рисками плана (проекта) и тесно связанное с ними исследование чувствительности его критериев лежат в основе надежного, приспособленного к изменениям конъюнктуры рынка транспортных услуг планирования деятельности предприятий морехозяйственного комплекса. В связи с этим работы, направленные на совершенствование методов исследования чувствительности, представляют научный и практический интерес. Настоящая статья посвящена построению стандартного многофакторного показателя чувствительности, характеризующего ее меру в базисной точке факторов.

Розвиток методів управління
та господарювання на транспорті
№ 1 (54), 2016

Рассматривается характерная в условиях системного многофакторного подхода чувствительность критерия эффективности плана (проекта) к совокупности стандартных изменений факторов, т.е. соответствующих одинаково-процентным по модулю их изменениям. Такой подход целесообразен при неопределенных изменениях, когда объективно нет оснований выделять относительное изменение какого-либо из факторов.

Количественная реакция критерия $y(x_1,...,x_n)$ *на совокупность стандартных изменений факторов называется стандартной чувствительностью критерия, а мера ее изменения – стандартным многофакторным показателем чувствительности (МПЧ) критерия.*

Для оценки чувствительности получены точечные стандартные многофакторные показатели чувствительности (ТСМПЧ) в базисной точке факторов, имеющие дифференциальную форму. Установлена связь между ТСМПЧ и однофакторными показателями чувствительности критерия.

Точечный (предельный) стандартный МПЧ зависит только от структуры критерия и базисных факторов, он представляет собой некоторый системный потенциал меры чувствительности критерия. Поэтому в условиях неопределенности изменений факторов целесообразно использовать стандартную чувствительность критерия.

Ключевые слова: *риск, изменения, точечный стандартный многофакторный показатель чувствительности, дифференциальная, упорядоченный, потенциал.*

Problem statement. Revealing, managing plan risks and corresponding sensitivity research of the plan criteria underlie the reliable planning the sea transport enterprises' activity. Therefore the works concerning the perfection of sensitivity research methods are of scientific and practical interest. This article is dedicated to working out the standard multifactor index of sensitivity, characterizing its measure in the basic point of factors.

Recent research and publications review. In the work [1] it has been represented the bases of one-factor sensitivity analysis of the criterion $y(x_1,...,x_n)$. It has been represented the expressions of one-factor sensitivity indexes on the base of finite (discrete) changing one factor Δx_i

$$sens\,(y,x_i)=\frac{\frac{y(x_1,x_2,...,x_{i-1},x_i+\Delta x_i,x_{i+1},...,x_n)-y(x_1,...,x_n)}{y(x_1,...,x_n)}}{\frac{\Delta x_i}{x_i}},(i=1,...,n), \quad (1)$$

and also point (limiting) one-factor sensitivity indexes in the differential form

$$sens\ *\,(y,x_i)=\lim_{\Delta x_i\to 0} sens\,(y,x_i)=\frac{\partial y}{\partial x_i}:\frac{y(x_1,...,x_n)}{x_i};\quad (i=1,...,n). \quad (2)$$

In the work [1] it has been noted that along with advantages, the one-factor sensitivity analysis has the essential disadvantage – it doesn't allow taking into account the cumulative effect of the changes.

In the works [2; 3; 4] it has been grounded the problem concerning the reasonability of a system multifactor sensitivity research of the plan (project) criterion of efficiency, that allow estimating the reaction of the plan (project) to cumulative change of factors $\{\Delta x_1,...,\Delta x_n\}$, including the conditions of uncertainty. It has been noted the necessity of ordering the totality $\{\Delta x_1,...,\Delta x_n\}$, and orde-ring should be rational.

In the works [2,5] it has been determined as the rational the totality of *standard* changes $\{\Delta x_i\}$ of all *n* factors x_i or their groups, i.e. satisfying the following conditions:

$$\frac{\pm\Delta x_i}{x_i}=\alpha=const>0,\quad (i=1,...,n) \text{ или } \quad (i\in I\subseteq\{1,...,n\}); \quad (3)$$

the sign of changing Δx_i depends on the economic reasons, the quantity α is set by the researcher. The standard changes Δx_i correspond to the identical-percentage changes of the factors x_i, when all factors appear equal concerning their absolute changes. The conditions (3) imposed on changes of factors $\{\Delta x_i\}$, and such changes we'll call ***standard***. Such approach is reasonable at uncertain changes when objectively there are no bases to allocate the relative changing any of factors.

The quantitative reaction of the criterion $y(x_1,...,x_n)$ to the totality of standard changes of factors is called as standard sensitivity of criterion, and a measure of its change – a standard multifactor index of sensitivity (MIS) of the criterion.

Tasks of research. The tasks of research are: 1) working out the point (limiting) standard MIS (PSMIS) of the criterion in a basic point of factors in the differential form; 2) determining the connection between the point standard multifactor indexes of sensitivity of the criterion and the point standard one-factor indexes of sensitivity of the criterion.

The basic material of research. We'll consider sensitivity of the criterion $y(x_1,...,x_n)$, carrying out the system multifactor approach. We'll note a number of structural elements of this process. The criterion $y(x_1,...,x_n)$ is considered as undimensioned function of many undimensioned variables, set in the area containing a basic point of factors $M(x_1,...,x_n)$. The value of criterion in basic point is accepted as the initial value of criterion. Criterion's disturbance

occurs under the influence of the generalized argument consisting of the standard totality of the factor's changes $\{\Delta x_1,...,\Delta x_n\}$. The totality of the relative factor's changes $\{\Delta x_1/x_1,...,\Delta x_n/x_n\}$ plays the essential role in estimating sensitivity. It is represented by the ave-rage square-law

$$\sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(\frac{\Delta x_i}{x_i}\right)^2}=\alpha\,, \qquad (4)$$

which is the absolute standard relative change of one (every) factor. The estimated quantity includes relative n-factor change of the criterion

$$\frac{1}{n}\frac{\Delta y}{y}=\frac{1}{n}\frac{y(x_1+\Delta x_1,...,x_n+\Delta x_n)-y(x_1,...,x_n)}{y(x_1,...,x_n)}. \qquad (5)$$

Both these expressions represent the n-factor quantities reduced to one factor, that is why they are comparable.

By generalizing formula (1) of the one-factor index of sensitivity, as ratios of the relative changes of the criterion and the factor, by means of the reduced n-factor quantities, we'll define the average standard MIS of the plan (project) concerning the criterion y towards the totality of finite standard changes $\{\Delta x_i\}$ (3) of all n factors or of a group n_1 of factors $(x_i, i\in I\subseteq\{1,...,n\})$:

$$sens(y,(x_1,...,x_n))=\frac{\dfrac{y(M')-y(M)}{ny(M)}}{\sqrt{\dfrac{1}{n}\sum_{i=1}^{n}\left(\dfrac{\Delta x_i}{x_i}\right)^2}}\,, \qquad (6)$$

$$sens(y,(x_i,i\in I\subseteq\{1,...,n\}))=\frac{\dfrac{y(M_I')-y(M)}{n_1y(M)}}{\sqrt{\dfrac{1}{n_1}\sum_{i\in I}^{n}\left(\dfrac{\Delta x_i}{x_i}\right)^2}}\,; \qquad (7)$$

basic and displaced n-measured points have been used here

$$M(x_1,...,x_n);\;\; M'(x_1+\Delta x_1,...,x_n+\Delta x_n)\,, \qquad (8)$$

$$M_I'(x_i+\Delta x_i,x_\nu),\quad (i\in I\subseteq\{1,...,n\};\nu\in\{\{1,...,n\}\setminus I\})\,, \qquad (9)$$

n_1 – number of indexes in the ordered subset I.

The formula (6) expresses MIS as the ratio of the reduced to one factor quantities of the relative standard n-factor criterion's change and of the average square-law standard relative changes of all n factors.

Let's carry out the approbation of the formula (6). Using the formula (6), we'll make

$$sens(y,(x_1,x_2))=\frac{\dfrac{y(x_1+\Delta x_1,x_2+\Delta x_2,x_3,...,x_n)-y(x_1,...,x_n)}{2\cdot y(x_1,...,x_n)}}{\sqrt{\dfrac{1}{2}\left[\left(\dfrac{\Delta x_1}{x_1}\right)^2+\left(\dfrac{\Delta x_2}{x_2}\right)^2\right]}}\,, \qquad (10)$$

$$sens(y,(x_3,x_4))=\frac{\dfrac{y(x_1,x_2,x_3+\Delta x_3,x_4+\Delta x_4,x_5,...,x_n)-y(x_1,...,x_n)}{2\cdot y(x_1,...,x_n)}}{\sqrt{\dfrac{1}{2}\left[\left(\dfrac{\Delta x_3}{x_3}\right)^2+\left(\dfrac{\Delta x_4}{x_4}\right)^2\right]}}\,. \qquad (11)$$

At any, disordered changes Δx_1, Δx_2, Δx_3, Δx_4, ratios between the fractions (10) and (11) are also disordered, it is impossible to see in expressions (10) and (11) any structure of the general content which could be used for their comparison. However we consider the case when all changes of factors are ordered by equalities (3), i.e. they are standard. Therefore we can represent the expressions (10) and (11) in such form

$$sens(y,(x_1,x_2))=\frac{\dfrac{y(x_1+\Delta x_1,x_2+\Delta x_2,x_3,...,x_n)-y(x_1,...,x_n)}{y(x_1,...,x_n)}}{2\alpha}\,, \qquad (12)$$

$$sens(y,(x_3,x_4)) = \frac{\dfrac{y(x_1,x_2,x_3+\Delta x_3,x_4+\Delta x_4,x_5,...,x_n)-y(x_1,...,x_n)}{y(x_1,...,x_n)}}{2\alpha}. \quad (13)$$

Comparing (10) and (11), we can notice that they have the general structure – equal denominators, and at any sets of (standard) changes $\{\Delta x_1, \Delta x_2, \Delta x_3, \Delta x_4\}$. Now the comparison of (10) and (11) is obvious , and they can be considered as standard MIS of the criterion on corresponding connections.

This conclusion is true at any number $n_1 \le n$ of standard changes $(\Delta x_i, i \in I \subseteq \{1,...,n\})$.

Offered $sens(y,(x_1,...,x_n))$ (6) can be represented in the transformed form

$$sens(y,(x_1,...,x_n)) = \frac{\dfrac{y(M')-y(M)}{ny(M)}}{\alpha}, \quad (14)$$

it can be considered as the fraction showing the quantity reduced to one factor of the relative n-factor criterion's change which is necessary per unit of standard relative factor's change.

The average standard MIS (6) depends on choosing the set $\{\Delta x_i\}$. Naturally we have the problem concerning the point (limiting) multifactor index of standard sensitivity of the criterion in the basic point $M(x_1,...,x_n)$ $sens^*(y,(x_1,...,x_n))$ that is free of operations with the finite changes of the factors $\{\Delta x_i\}$.

Assuming that $y(M)$ is the differentiable function [6; 7], we'll represent the MIS (6) in the following form:

$$sens(y,(x_1,...,x_n)) = \frac{\sum_{i=1}^{n}\frac{\partial y(M)}{\partial x_i}\Delta x_i + \varepsilon\rho}{n\alpha y(M)}, \quad (15)$$

where

$$\rho = \rho(MM') = \sqrt{\sum_{i=1}^{n}\Delta x_i^2} = \sqrt{\sum_{i=1}^{n}\alpha^2 x_i^2} = \alpha\sqrt{\sum_{i=1}^{n}x_i^2},$$

ρ is a generalized distance between the n-measured basic point $M(x_1,...,x_n)$ and n-measured displaced point $M'(x_1+\Delta x_1,...,x_n+\Delta x_n)$, and $\rho \to 0$ $\varepsilon \to 0$.

Using standardness Δx_i (3), we have received the following equality:

$$sens(y,(x_1,...,x_n)) = \frac{\sum_{i=1}^{n}\frac{\partial y(M)}{\partial x_i}(\pm\alpha x_i) + \varepsilon\rho}{n\alpha}\frac{1}{y(M)}. \quad (16)$$

We'll define the point (limiting) standard MIS (PSMIS)

$$sens^*(y,(x_1,...,x_n)) = \lim_{\rho\to 0} sens(y,(x_1,...,x_n)) = \lim_{\rho\to 0}\frac{\sum_{i=1}^{n}\frac{\partial y(M)}{\partial x_i}(\pm\alpha x_i)+\varepsilon\rho}{n\alpha y(M)}. \quad (17)$$

By means of the limit transition it has been received the following point standard MIS of the criterion on the connections with all n factors, $sens^*(y,(x_1,...,x_n))$

$$sens^*(y,(x_1,...,x_n)) = \frac{1}{n}\sum_{i=1}^{n}\frac{\partial y(M)}{\partial x_i}\frac{\pm x_i}{y(M)}. \quad (18)$$

Concerning the criterion's connections with the group n_1 of factors, on the basis of equality (7), we have the following formula:

$$sens^*(y,(x_i, i \in I \subseteq \{1,...,n\})) = \frac{1}{n_1}\sum_{i\in I}\frac{\partial y(M)}{\partial x_i}\frac{\pm x_i}{y(M)}. \quad (19)$$

Conclusions. The point standard MIS on connections with all n factors (18) is equal to an arithmetical mean of all one-factor point indexes of sensitivity of the criterion (2). We can analogically interpret $sens^*(y,(x_i, i \in I \subseteq \{1,...,n\}))$ (19). This fact is of interest

from the methodological point of view. It shows that the entered new quantity of point multifactor index of standard sensitivity of the criterion (18), (19) is connected with already known quantities, and therefore we should consider it effective at the multifactor sensitivity analysis.

The point (limiting) standard multifactor index of sensitivity (PSMIS) depends just on the criterion's structure and basic factors, it represents some system potential of the measure of criterion's sensitivity. Therefore it is reasonable to use standard criterion's sensitivity in conditions of uncertainty of the factor's changes.

The developed method of multifactor sensitivity research on the basis of suggested point standard MIS should promote increasing the efficiency of the ports', of the shipping companies' management decisions in conditions of functioning in changeable market environment of the sea transport services.

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Розвиток методів управління
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№ 1 (54), 2016

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