Розвиток методів управління
та господарювання на транспорті
№ 2 (59), 2017

S.V. Melnikov

Odessa national maritime university

PRICE COMPETITION BETWEEN RETAILERS UNDER SYMMETRIC COSTS

Integrating operations and marketing decisions are an important objective for retailers in today's competitive environment. An obvious problem common to the retail industry is the joint optimization of the lot size of a product to be stocked and the selling price in order to maximize profit. Pricing and inventory control strategies are closely related. Pricing decisions alter demand forecasts, which are used by inventory control systems. We note that in the classical economic order quantity (EOQ) model the demand is assumed as constant. A lot of important results have been obtained in this field of study. There are many studies which modify this condition, in particular with linear demand function. Further studies are related to the complexity of market structures, in particular, considering the horizontal competition between retailers.

In this paper the Bertrand's price competition in the retail duopoly with symmetric costs is analyzed. Retailers sell substitute products in the framework of the classical EOQ model with linear demand function. Products are stored in a warehouse and are evenly sold in the retail network. We assume that every retailer maximizes profit per unit of time on the order size and the price at a current price competitor. The market potential is considered to be the bifurcation parameter of retailers. Level of the barrier to market penetration depending on the bifurcation parameter is analyzed. The conditions of Bertrand-Nash equilibrium in parametric and trigonometric forms are found. The theoretical results are illustrated by a numerical example. Based on the results obtained, retailers can plan the level of profitability of logistics costs and assess the level of the entry barrier to the market.

_____

Розвиток методів управління
та господарювання на транспорті
№ 2 (59), 2017

Мельников С.В.

ЦІНОВА КОНКУРЕНЦІЯ МІЖ РІТЕЙЛЕРАМИ ПРИ СИМЕТРИЧНИХ ВИТРАТАХ

Одеський національний морський університет

У даній роботі аналізується цінова конкуренція за Бертраном у моделі дуополії рітейлерів в умовах симетрії витрат. Рітейлери продають взаємозамінні продукти в умовах класичної EOQ моделі за лінійної функції попиту. Потенціал ринку розглянуто в якості біфуркаційного параметра рітейлерів. Проаналізовано рівень бар'єру входу на ринок у залежності від пара-метра біфуркації. Знайдено умови стану рівноваги за Бертраном-Нешем в параметричній та тригонометричній формах.

***Ключові слова**: EOQ модель, модель дуополії рітейлерів, рівновага Бертрана-Неша, потенціал ринку, біфуркаційний параметр, рентабельність логістичних витрат, бар'єр входу.*

Мельников С.В.

ЦЕНОВАЯ КОНКУРЕНЦИЯ МЕЖДУ РИТЕЙЛЕРАМИ ПРИ СИММЕТРИЧНЫХ ЗАТРАТАХ

Одесский национальный морской университет

В данной работе анализируется ценовая конкуренция по Бертрану в модели дуополии ритейлеров при симметрии затрат. Ритейлеры продают взаимозаменяемые продукты в условиях классической EOQ модели при линейной функции спроса. Потенциал рынка рассмотрен в качестве бифуркационного параметра ритейлеров. Проанализирован уровень барьера входа на рынок в зависимости от параметра бифуркации. Найдены

Розвиток методів управління
та господарювання на транспорті
№ 2 (59), 2017

условия состояния равновесия Бертрана-Нэша в параметрической и тригонометрической формах.

Ключевые слова: *ЕОQ модель, модель дуополии ритейлеров, равновесие Бертрана-Нэша, потенциал рынка, бифуркационный параметр, рентабельность логистических затрат, барьер входа.*

Problem statement. Integrating operations and marketing decisions are an important objective for retailers in today's competitive environment. An obvious problem common to the retail industry is the joint optimization of the lot size of a product to be stocked and the selling price in order to maximize profit. Pricing and inventory control strategies are closely related. Pricing decisions alter demand forecasts, which are used by inventory control systems.

We note that in the classical economic order quantity (EOQ) model the demand is assumed as constant. A lot of important results have been obtained in this field of study. There are many studies which modify this condition [1], in particular with linear demand function [2; 3]. Further studies are related to the complexity of market structures, in particular, considering the horizontal competition between retailers [4].

In this article, we develop the obtained results herein [4] for model duopoly retailers with the Bertrand's price competition in the case of substitute products. Along with trigonometric solution, parametric solution is received as well, where the parameter is return on logistics costs. Parametric solution allows to determine the sufficient conditions for the existence of equilibrium with the symmetry retailers cost. The market potential is considered to be bifurcation parameters of retailers.

Recent research and publications analysis. Whitin [5] was the first researcher who indicated the fundamental connection between price theory and inventory control. He extended the basic EOQ model by considering the selling price in addition to the order quantity as the decision variables.

Optimization solutions for pricing and inventory management for the retailer's monopoly are represented in the works [2; 6-10].

The paper of Abad [2] is concerned with finding the optimal price and lot size for a retailer purchasing a product for which the supplier offers all-unit quantity discounts. Demand for the product is assumed to be a decreasing function of price, and a procedure is developed for finding the optimal price and lot size for a class of demand functions. Thomas [6] considers the problem of simultaneously making price and production decisions in dynamic for a single product with a known deterministic demand function. To maximise profit, an efficient algorithm is developed.

Kunreuther and Richard [7] have investigated the interrelationship between the pricing and inventory decisions for a retailer who orders his goods from an outside distributor. Smith *et al.* [8] have formulated and solved a single-item joint pricing and master planning optimization problem with capacity and inventory constrains.

Tripathi [9] develops an inventory model for deteriorating items with linearly time dependent demand rate under inflation and time discounting over a finite planning horizon. In paper [10] have analyzed the problem of determining the optimal price and lot size of retailer when demand is a linear function of price.

Study on retailers' competition in the vertical market conditions is presented in [11-16].

Sinha and Sarmah [11] have analyzed the coordination and competition issues in a two-stage supply-chain distribution system where two vendors compete to sell differentiated products through a common retailer in the same market. Huang *et al.* [12] analyze the coordination of enterprise decisions such as supplier and component selection, pricing and inventory in a three-level supply chain composed of multiple suppliers, a single manufacturer and multiple retailers. The problem is modelled as a dynamic non-cooperative game.

Розвиток методів управління
та господарювання на транспорті
№ 2 (59), 2017

Розвиток методів управління
та господарювання на транспорті
№ 2 (59), 2017

Feng and Lu [13] have analyzed the contracting behaviors in a two-tier supply chain system consisting of two competing manufacturers selling to two competing retailers. Alaei *et al.* [14] have analyzed production – inventory decisions in a decentralized supply chain. A production inventory problem is considered in a two-level supply chain. Modak *et al.* [15] have explored channel coordination and profit distribution in a two-layer socially responsible supply chain that consists of a manufacturer and two competitive retailers. In paper [16] the logistical system of manufacture and delivery of production on the consumer market, including suppliers, the manufacturer and the transport enterprises is considered. The paper [17] presents the effects arising at horizontal merger of firms, operating under conditions of the vertical market.

Study on retailers' competition in the horizontal market conditions is presented in [4; 18; 19].

Otake and Min [4] have analyzed inventory and pricing policies for a duopoly of substitute products. Min [18] has extended the profit maximizing EOQ model to the case of a symmetric oligopoly consisting of sellers of a homogeneous product who complete with each other for the same potential buyers. Sadjadi and Fathollah Bayati [19] develop generalized network data environment analysis models to examine the efficiency of two-tier suppliers under cooperative and non-cooperative strategies where each tier has its own inputs/outputs and some outputs of the first tier can be fed back to the second tier.

The **goal of this article** is the optimization of price-inventory decisions of retailers in duopoly model under symmetric costs.

The basic results and their justification. Two retailers periodically buy the finished product from a wholesaler. Products are stored in a warehouse and are evenly sold in the retail network. We assume that every retailer maximizes profit per unit of time on the order size and the price at a current price competitor.

Linear demand function of i-th retailer (1)

$$D_i = b - k \cdot p_i + \gamma \cdot (p_j - p_i), \qquad (1)$$

where D_i – product demand of i-th retailer per unit time, $i = 1,2$;

p_i – price of product of i-th retailer per unit;

p_j – unit price of i-th retailer's competitor product, $j = 3 - i$;

b – market potential (maximal demand) per unit time;

$k > 0$ – the own price effect;

$\gamma > 0$ – the cross price effect.

From condition $0 < D_i < b$ we obtain the range of acceptable prices $p_i < \dfrac{b + \gamma \cdot p_j}{k + \gamma}$, $p_i < \dfrac{p_j \cdot (k + \gamma)}{\gamma}$ (Figure 1).

Note that in paper [4] the demand function was used in the form $D_i = b - k \cdot p_i + \gamma \cdot p_j$. In our opinion, this function has a disadvantage – at any ratio between prices, the final demand of i-th retailer is always higher than the base demand: $D_i > b - k \cdot p_i$. The increase in the basic demand of i-th retailer is due to the influx of new consumers from the competitor at $p_i < p_j$. Obviously, with $p_i > p_j$, the basic demand of i-th retailer should not increase, but decrease. Therefore, it is necessary to take into account the relationship between prices, as in (1).

Variables and parameters should be defined as follows:

Q_i – the order size of i-th retailer;

$d_i = d_j = d$ – the ordering cost of retailers;

$w_i = w_j = w$ – the variable cost per unit time of retailers;

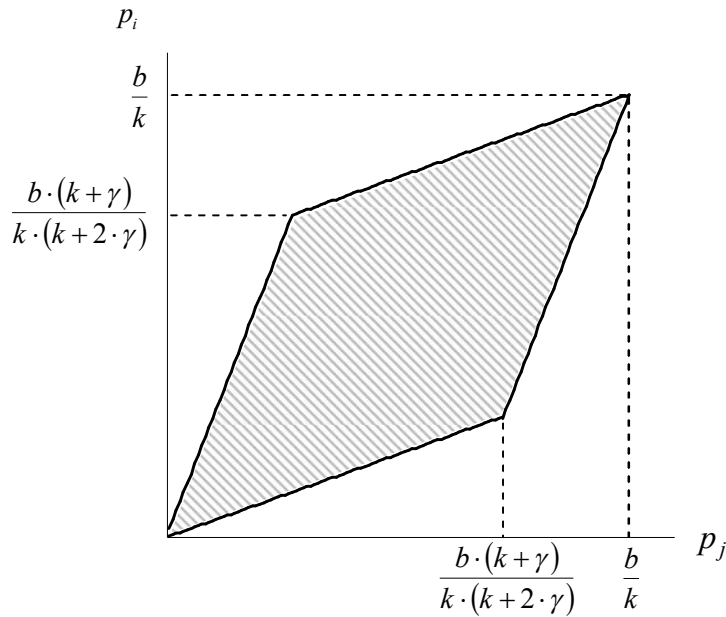$l_i = l_j = l$ – the holding cost per unit per unit time of retailers.

Розвиток методів управління
та господарювання на транспорті
№ 2 (59), 2017

$$p_i$$

$$\frac{b}{k}$$

$$\frac{b \cdot (k + \gamma)}{k \cdot (k + 2 \cdot \gamma)}$$

$$\frac{b \cdot (k + \gamma)}{k \cdot (k + 2 \cdot \gamma)} \qquad \frac{b}{k} \qquad p_j$$

Fig. 1. The range of acceptable prices

The basic assumptions for traditional EOQ model applied in this article are the following:

– buyer's demand does not change;
– unlimited supply volume;
– no shortage;
– instant delivery.

With reference to the above mentioned, the price competition between retailers on the Bertrand model is analyzed. The objective function of i-th retailer (2)

$$F_i = GP_i - LC_i = (p_i - w) \cdot D_i - \frac{d \cdot D_i}{Q_i} - \frac{Q_i \cdot l}{2} \to \max_{p_i, Q_i}. \qquad (2)$$

Розвиток методів управління
та господарювання на транспорті
№ 2 (59), 2017

where $\quad GP_i = (p_i - w) \cdot D_i \quad$ – gross profit per unit time, $LC_i = \dfrac{d \cdot D_i}{Q_i} + \dfrac{Q_i \cdot l}{2}$ – logistics costs per unit time.

The first-order necessary conditions (3)-(6):

$$\frac{\partial F_i}{\partial p_i} = b + \gamma \cdot p_j + w \cdot (k + \gamma) + \frac{d \cdot (k + \gamma)}{Q_i} - 2 \cdot p_i \cdot (k + \gamma) = 0, \qquad (3)$$

$$\frac{\partial F_i}{\partial Q_i} = \frac{d \cdot D_i}{Q_i^2} - \frac{l}{2} = 0, \qquad (4)$$

or

$$p_i = \frac{\mathrm{w}}{2} + \frac{b + \gamma \cdot p_j}{2 \cdot (k + \gamma)} + \frac{d}{2 \cdot Q_i}, \qquad (5)$$

$$Q_i^2 = \frac{2 \cdot d \cdot (b - k \cdot p_i + \gamma \cdot (p_j - p_i))}{l}. \qquad (6)$$

Substituting (6) into (5), we obtain the reaction curves of i-th retailer, $R_i(p_j)$ in implicit form (7)

$$p_i = \frac{\mathrm{w}}{2} + \frac{b + \gamma \cdot p_j}{2 \cdot (k + \gamma)} \pm \sqrt{\frac{d \cdot l}{8 \cdot (b - k \cdot p_i + \gamma \cdot (p_j - p_i))}}. \qquad (7)$$

Under the assumption of symmetric costs, it can be easily verified that there exists an equilibrium point when $p_i^e = p_j^e$, $Q_i^e = Q_j^e$. Thus, at the equilibrium point, the formulas (5), (6) will be in the form

$$p_i^e = \frac{b + (k + \gamma) \cdot (w + d/Q_i^e)}{2 \cdot k + \gamma}; \qquad (8)$$

Розвиток методів управління
та господарювання на транспорті
№ 2 (59), 2017

$$Q_i^e = \sqrt{\frac{2 \cdot d \cdot (b - k \cdot p_i^e)}{l}}. \qquad (9)$$

To find the stationary points, we substitute (8) into (9) and equate to zero (10)

$$Q_i^3 - \frac{2 \cdot d \cdot (k + \gamma) \cdot (b - k \cdot w)}{l \cdot (2 \cdot k + \gamma)} \cdot Q_i + \frac{2 \cdot d^2 \cdot k \cdot (k + \gamma)}{l \cdot (2 \cdot k + \gamma)} = 0. \qquad (10)$$

We have obtained a cubic equation in reduced form. Thus, depending on the parameter values, the number of real stationary points can be 1, 2 or 3.

It should be noted that the key parameter for retailers is a market potential. It is interesting to analyze the dependence of the number of real stationary points of the market potential. For this, the market potential can be presented in the formula (10) as (11)

$$b(Q_i) = k \cdot w + \frac{k \cdot d}{Q_i} + \frac{l \cdot (2 \cdot k + \gamma)}{2 \cdot d \cdot (k + \gamma)} \cdot Q_i^2. \qquad (11)$$

The graph of the function $b(Q_i)$ is shown in Figure 2. Figure 2 shows that the number of real roots of the cubic equation (10) depends on the level of market potential. When $b < b_i^{bif}$ the equation (10) has one real negative root q_3, when $b = b_i^{bif}$ — one real negative root q_3 and one real positive two-fold root q_i^{bif}, when $b > b_i^{bif}$ — three real roots $(q_1 > q_2 > q_3)$. Thus, Figure 2 shows an imperfect pitchfork bifurcation, where the level of market potential is a bifurcation point [20]. Figure 2 also shows that the positive order size exists only when $b \geq b_i^{bif}$, so the bifurcation point can be considered as a barrier to entry for retailers.

Розвиток методів управління
та господарювання на транспорті
№ 2 (59), 2017

It is easy to notice that the function $b(Q_i)$ at $Q_i = q_i^{bif}$ has a local minimum. From the first order conditions (12)

$$\frac{db}{dQ_i} = \frac{2 \cdot l \cdot (2 \cdot k + \gamma) \cdot Q_i}{2 \cdot d \cdot (k + \gamma)} - \frac{k \cdot d}{Q_i^2} = 0, \qquad (12)$$

we find

$$Q_i^{bif} = \sqrt[3]{\frac{d^2 \cdot k \cdot (k + \gamma)}{l \cdot (2 \cdot k + \gamma)}}. \qquad (13)$$

Substituting (13) into (11), we find the bifurcation value of market potential of i-th retailer

$$b_i^{bif} = k \cdot w + \sqrt[3]{\frac{27 \cdot d \cdot l \cdot k^2 \cdot (2 \cdot k + \gamma)}{8 \cdot (k + \gamma)}}. \qquad (14)$$
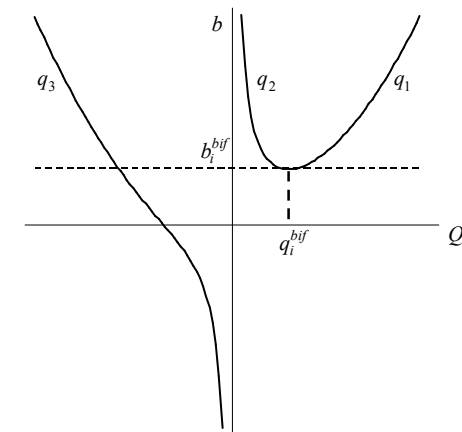

Fig. 2. Location of the real roots of equation (10) depending on the bifurcation parameter («imperfect pitchfork bifurcation» [20])

Розвиток методів управління
та господарювання на транспорті
№ 2 (59), 2017

The parametric solution. Using $\left(b_i^{bif}, Q_i^{bif}\right)$, we define stationary points of i-th retailer in parametric form. Consider the functions

$$b_i^h = k \cdot w + \sqrt[3]{h_i^b \cdot \frac{d \cdot l \cdot k^2 \cdot (2 \cdot k + \gamma)}{k + \gamma}}$$

and $Q_i^h = \sqrt[3]{h_i^Q \cdot \frac{d^2 \cdot k \cdot (k + \gamma)}{l \cdot (2 \cdot k + \gamma)}}$, where b_i^h, Q_i^h – parametric representation of the roots of the equation (10), h_i^b, h_i^Q – yet unknown parameters, $h_i^b = f\left(h_i^Q\right)$. Substituting b_i^h and Q_i^h into equation (10), determine the relationship between the parameters: $h_i^b = \left(h_i^Q + 2\right)^3 / \left(8 \cdot h_i^Q\right)$. Let $h_i^Q = h_i$, then $h_i^b = \left(h_i + 2\right)^3 / \left(8 \cdot h_i\right)$, $h_i \neq 0$.

Thus, the real roots of the equation (10) in the parametric form (15)

$$b_i^h = k \cdot w + \frac{h_i + 2}{2} \cdot \sqrt[3]{\frac{d \cdot l \cdot k^2 \cdot (2 \cdot k + \gamma)}{h_i \cdot (k + \gamma)}},$$

$$(15)$$

$$Q_i^h = \sqrt[3]{\frac{h_i \cdot d^2 \cdot k \cdot (k + \gamma)}{l \cdot (2 \cdot k + \gamma)}}.$$

When the $h_i < 0$ we obtain the root q_3, when $0 < h_i < 1$ – the root q_2, when the $h_i = 1$ – bifurcation point q_i^{bif}, when $h_i > 1$ – the root q_1 (Figure 2).

Substituting (15) into (5), we determine the price through the parameter (16)

$$p_i^h = w + \left(\frac{k \cdot (h_i + 2)}{2 \cdot (k + \gamma)} + 1\right) \cdot \sqrt[3]{\frac{d \cdot l \cdot (k + \gamma)^2}{k \cdot h_i \cdot (2 \cdot k + \gamma)^2}}.$$

$$(16)$$

We can give an economic interpretation to the h_i-parameter. The h_i-parameter is expressed through a relative indicator of economic efficiency – return on logistics costs, which is given by: $r_i = \left(GP_i - LC_i\right) / LC_i$.

Substitute the parametric solution (15)-(16) into the original profit function (17)

$$F_i^h = GP_i - LC_i = \frac{k \cdot (h_i + 2) + 2 \cdot (k + \gamma)}{4 \cdot (2 \cdot k + \gamma)} \cdot \sqrt[3]{\frac{h_i \cdot d^2 \cdot l^2 \cdot k \cdot (k + \gamma)}{2 \cdot k + \gamma}} - \sqrt[3]{\frac{h_i \cdot d^2 \cdot l^2 \cdot k \cdot (k + \gamma)}{2 \cdot k + \gamma}}. \quad (17)$$

From (17) we determine the return on logistics costs:

$$r_i = \frac{k \cdot (h_i - 4) - 2 \cdot \gamma}{4 \cdot (2 \cdot k + \gamma)}, \text{ where the } h_i = \frac{(2 \cdot k + \gamma) \cdot (4 \cdot r_i + 2)}{k}.$$

Thus, the solution (15)-(16), where the parameter is return on logistics costs, is expressed as follows (18):

$$b_i^r = k \cdot w + \left(k + (2 \cdot k + \gamma) \cdot (2 \cdot r_i + 1)\right) \cdot \sqrt[3]{\frac{d \cdot l}{(4 \cdot r_i + 2) \cdot (k + \gamma)}},$$

$$Q_i^r = \sqrt[3]{\frac{d^2 \cdot (4 \cdot r_i + 2) \cdot (k + \gamma)}{l}}, \quad (18)$$

$$p_i^r = w + 2 \cdot (r_i + 1) \cdot \sqrt[3]{\frac{d \cdot l}{(4 \cdot r_i + 2) \cdot (k + \gamma)}}.$$

At the bifurcation point $(h_i = 1)$ the return on logistics costs is equal: $r_i^{bif} = -\frac{3 \cdot k + 2 \cdot \gamma}{4 \cdot (2 \cdot k + \gamma)}$. Let us define the boundaries of changes in profitability, depending on the coefficients of price sensitivity of demand. Since $\gamma \in (0; k)$, then $r_i^{bif} \in (-0,417; -0,375)$. When $h_i > 1$, return on logistics costs will increase, so the retailers are interested only in the root q_1.

Розвиток методів управління
та господарювання на транспорті
№ 2 (59), 2017

Now it is necessary to analyze the extrema of the function (2). For this, we define the Hessian matrix (19)

$$H_i = \begin{bmatrix} \dfrac{\partial^2 F_i}{\partial p_i^2} & \dfrac{\partial^2 F_i}{\partial p_i \partial Q_i} \\ \dfrac{\partial^2 F_i}{\partial Q_i \partial p_i} & \dfrac{\partial^2 F_i}{\partial Q_i^2} \end{bmatrix} = \begin{bmatrix} -2\cdot(k+\gamma) & -\dfrac{d_i\cdot(k+\gamma)}{Q_i^2} \\ -\dfrac{d_i\cdot(k+\gamma)}{Q_i^2} & -\dfrac{2\cdot d_i\cdot(b-k\cdot p_i+\gamma\cdot(p_j-p_i))}{Q_i^3} \end{bmatrix}. \quad (19)$$

As it is known, the type of extrema of the function depends on the character of definiteness of the Hessian matrix evaluated at the stationary points. The character of definiteness of the Hessian matrix (19) depends on stationary points (18) and may be different. We are interested in the dependence of the indicator r_i. Since we need the maximum, we will find conditions under which the Hessian matrix is negative definite.

According to Sylvester's criterion the matrix (19) is negative definite when $|H_i| > 0$ or

$$4\cdot Q_i\cdot(b-k\cdot p_i+\gamma\cdot(p_j-p_i))-d\cdot(k+\gamma) > 0. \quad (20)$$

Substituting the parametric solution (18) into the condition (20), we obtain a sufficient condition for a maximum: $r_i > -37{,}5\%$. Thus, only root q_1 include the point of maximum of the function (2), $Q_i^e \in q_1$, $Q_i^e > Q_i^{bif}$.

Thus, the condition for entering the market for retailers

$$b > b(r=-0{,}375)=k\cdot w+(6\cdot k+\gamma)\cdot\sqrt[3]{\dfrac{d\cdot l}{32\cdot(k+\gamma)}}.$$

The Bertrand-Nash equilibrium between retailers will be achieved at r_i^e and r_j^e ensuring equality of the potential:

$b_i^r(r_i^e)=b_j^r(r_j^e)=b$ and at the point of intersection of the reaction curves: $R_i(p_j^r(r_j^e))=R_j(p_i^r(r_i^e))$, $r_i^e=r_j^e$.

The trigonometric solution. Let us find the roots of the cubic equation (10) explicitly. Since the discriminant of the cubic equation (10) for $b > b_i^{bif}$ is negative, we will seek the trigonometric solution. Introduce the function (21)

$$\varphi_i = \arccos\left(-\sqrt{\left(\dfrac{b_i^{bif}-\mathrm{k}\cdot w}{b-\mathrm{k}\cdot w}\right)^3}\right). \quad (21)$$

Then the roots of the cubic equation (10) are equal

$$z_{i1} = 2\cdot\sqrt{\dfrac{2\cdot d\cdot(b-k\cdot w)\cdot(k+\gamma)}{3\cdot l\cdot(2\cdot k+\gamma)}}\cdot\cos\dfrac{\varphi_i}{3};$$

$$z_{i2} = 2\cdot\sqrt{\dfrac{2\cdot d\cdot(b-k\cdot w)\cdot(k+\gamma)}{3\cdot l\cdot(2\cdot k+\gamma)}}\cdot\cos\dfrac{\varphi_i-2\cdot\pi}{3}; \quad (22)$$

$$z_{i3} = 2\cdot\sqrt{\dfrac{2\cdot d\cdot(b-k\cdot w)\cdot(k+\gamma)}{3\cdot l\cdot(2\cdot k+\gamma)}}\cdot\cos\dfrac{\varphi_i+2\cdot\pi}{3}.$$

In the analysis of the sufficient conditions it has been found that the equilibrium order size belongs to the right-hand branch of the function $b(Q_i)$ (Figure 2): $Q_i^e \in q_1 = \max\{z_{i1}, z_{i2}, z_{i3}\}$. Compare the roots of (22) with each other. For the $b > b_i^{bif}$ the range of function (21) is equal: $\varphi_i \in (\pi/2; \pi)$. For $\varphi_i \in (\pi/2; \pi)$ the inequality $\cos\dfrac{\varphi_i}{3} > \cos\dfrac{\varphi_i-2\cdot\pi}{3} > \cos\dfrac{\varphi_i+2\cdot\pi}{3}$, therefore $q_1 = z_{i1}$, $q_2 = z_{i2}$, $q_3 = z_{i3}$. The equilibrium order size: $Q_i^e \in z_{i1}$.

Розвиток методів управління
та господарювання на транспорті
№ 2 (59), 2017

Numerical example and sensitivity analisis. We illustrate the obtained results on the numerical example, using data from [4]. Data are presented in Table 1.

Table 1

Initial numerical example data

b	k	γ	w	l	d
100	1,0	0,5	20	4,0	500

The equilibrium variables are presented in Table 2.

Table 2

Equilibrium variables

p_i^e	p_j^e	Q_i^e	Q_j^e	F_i^e	F_j^e	r_i^e	r_j^e
54,8	54,8	106	106	1148	1148	2,7	2,7

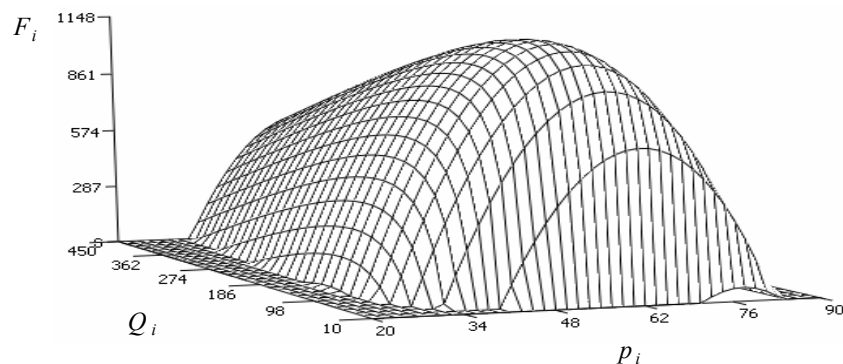Profit function of the i-th retailer at $p_j = p_j^e$ is shown in Figure 3.

Fig. 3. Profit function of the i-th retailer

Now we study the impact of changes in the values of the key parameters b, k, γ, w, d, l on the equilibrium profit of the i-th retailer. We change one parameter at a time, keeping the other parameters unchanged. The results are summarized in Table 3 (Figure 4).

Table 3

The impact of changes in the values of the key parameters on the equilibrium profit of the i-th retailer

$\%$ changes	b	k	γ	w	d	l
-25	409	1859	1170	1334	1199	1199
-20	531	1683	1166	1295	1188	1188
-15	667	1527	1162	1258	1178	1178
-10	815	1387	1157	1221	1168	1168
-5	975	1261	1153	1184	1158	1158
$+5$	1334	1046	1143	1112	1139	1139
$+10$	1531	952	1139	1077	1130	1130
$+15$	1742	868	1134	1043	1121	1121
$+20$	1964	790	1129	1009	1112	1112
$+25$	2199	719	1124	975	1103	1103

Based on the results of Table 3, the following observation can be made.

1. A higher value of market potential b results in higher values of equilibrium profit of the i-th retailer. Additionally, we find that equilibrium profit of the i-th retailer are highly sensitive to changes in b.

2. A higher value of other parameters k, γ, w, d, l results in lower values of equilibrium profit of the i-th retailer. Additionally, we find that equilibrium profit of the i-th retailer are highly sensitive to changes in k.

Розвиток методів управління
та господарювання на транспорті
№ 2 (59), 2017

Розвиток методів управління
та господарювання на транспорті
№ 2 (59), 2017

Розвиток методів управління
та господарювання на транспорті
№ 2 (59), 2017

Also we have got that ordering and holding costs equally influence on the equilibrium profit of the i-th retailer.
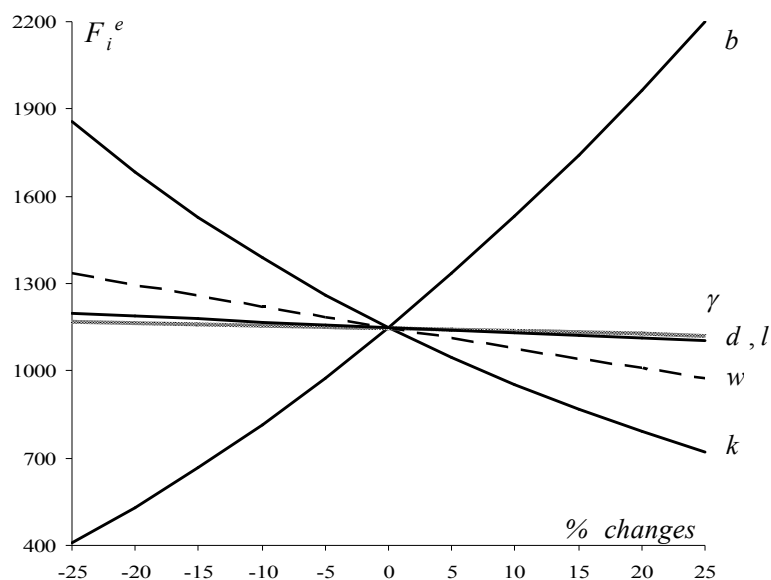

Fig. 4. Sensitivity analysis of equilibrium profit of the i-th retailer

Conclusions. In this article we have analyzed the price competition between retailers in duopoly model under symmetric costs. The market potential is considered to be the bifurcation parameter of retailers. The necessary and sufficient conditions for the existence of extrema are analyzed. The optimal solution in parametric form, where the parameter is the return on logistics costs, is found. The solution is determined in explicit form, taking into account the bifurcation point. The theoretical results are illustrated by a numerical example. Based on the results obtained, retailers can plan the level of profitability of logistics costs and assess the level of the entry barrier to the market.

REFERENCES

1. *Chan L.M.A. Coordination of Pricing and Inventory Decisions: A Survey and Classifications / L.M.A. Chan, Z.J.M. Shen, D. Simchi-Levi, J.L. Swann // Handbook of Quantitative Supply Chain Analysis: Modelling in E-Business Era. – Springer-Verlag US. – 2004. – № 5. – P. 335-392.*
2. *Abad P.L. Determining Optimal Selling Price and Lot Size when the Supplier offers All-Unit Quantity Discounts / P.L. Abad // Decision Sciences. – 1988. – Vol. 19. – № 3. – P. 622-634.*
3. *Chen C.K. An Analysis of Optimal Inventory and Pricing Policies under Linear Demand / C.K. Chen, K.J. Min // Asia-Pacific Journal of Operational Research. – 1994. – № 11. – P. 117-129.*
4. *Otake T. Inventory and Pricing Policies for a Duopoly of Substitute Products / T. Otake, K.J. Min // Proceedings of the Fifth Industrial Engineering Research Conference, Minneapolis, Minnesota. – 1996. – P. 293-298.*
5. *Whitin T.M. Inventory Control and Price Theory / T.M. Whitin // Management Science. – 1955. – № 2. – P. 61-68.*
6. *Thomas J. Price-Production Decisions with Deterministic Demand / J. Thomas // Management Science. – 1970. – № 16. – P. 747-750.*
7. *Kunreuther H. Optimal Pricing and Inventory Decisions for Non-Seasonal Items / H. Kunreuther, J.F. Richard // Econometrica. – 1971. – Vol. 39. – № 1. – P. 173-175.*
8. *Smith N.R. Optimal Pricing and Production Master Planning in a Multiperiod Horizon Considering Capacity and Inventory Constraints / N.R. Smith, J.R. Limon, L.E. Cardenas-Barron // Mathematical Problems in Engineering. – 2009.*

Розвиток методів управління
та господарювання на транспорті
№ 2 (59), 2017

Розвиток методів управління
та господарювання на транспорті
№ 2 (59), 2017

9. *Tripathi R.P. Economic Order Quantity for Deteriorating Items with Non Decreasing Demand and Shortages Under Inflation and Time Discounting / R.P. Tripathi // International Journal of Engineering, Ttransactions C: Aspects. – 2015. – Vol. 28. – № 9. – P. 1295-1302.*

10. *Melnikov S.V. Optimal pricing and inventory decisions of retailer in EOQ model with linear demand / S.V. Melnikov // Development of administration and economic management methods on transport. – Odessa: ONMU, 2016. – Vol. 1 (54), – P. 21-43.*

11. *Sinha S. Coordination and price competition in a duopoly common retailer supply chain / S. Sinha, S.P. Sarmah // Computers & Industrial Engineering. – 2010. – Vol. 59. – P. 280-295.*

12. *Huang Y. Coordinating Pricing and Inventory Decisions in a Multi-Level Supply Chain: A Game-Theoretic Approach / Y. Huang, G.Q. Huang, S.T. Newman // Transportation Research Part E: Logistics and Transportation Review. – 2011. – Vol. 47. – № 2. – P. 115-129.*

13. *Feng Q. Supply chain contracting under competition: Bilateral bargaining vs. Stackelberg / Q. Feng, L.X. Lu // Production and Operations Management. – 2013. – Vol. 22. – № 3. – P. 661-675.*

14. *Alaei S. Analysis of Production - Inventory Decisions in a Decentralized Supply Chain / S. Alaei, M. Behravesh, N. Karegar // Prague Economics Papers. – 2014. – № 2. – P. 198-216.*

15. *Modak N.M. Pricing policy and coordination for a two-layer supply chain of duopolistic retailers and socially responsible manufacturer / N.M. Modak, S. Panda, S.S. Sana // International Journal of Logistics Research and Applications. – Vol. 19. – № 6.*

16. *Мельников С.В. Синергетика логистической системы / С.В. Мельников // Методи та засоби управління розвитком транспортних систем: Зб. наук. праць. – Одеса: ОНМУ, 2008. – Вип.14. – С.149-162.*

17. *Melnikov S.V. Modeling the effects of horizontal firm mergers under conditions of vertical market / S.V. Melnikov // Actual Problems of Economics. – Vol. 4. – № 118. – P. 250-255.*

18. *Min K. Inventory and pricing policies under competition / K. Min // Operations Research Letters. – 1992. – Vol. 12. – № 4. – P. 253-261.*

19. *Sadigh A.N. A Game Theoretic Approach to Coordination of Pricing, Advertising, and Inventory Decisions in a Competitive Supply Chain / A.N. Sadigh, S.K. Chaharsooghi, M. Sheikhmohammady // Journal of Industrial and Management Optimization. – 2016. – Vol. 12. – № 1. – P. 337-355.*

20. *Zhang W.-B. Synergetic Economics: Time and Change in Nonlinear Economics / W.-B. Zhang. – Berlin: Springer-Verlag, 1991.*

Рецензенти:

доктор економічних наук, старший науковий співробітник відділу ринку транспортних послуг Інституту проблем ринку та економіко-екологічних досліджень НАН України **С.В. Ільченко**

доктор економічних наук, професор, завідувач кафедри «Менеджмент та маркетинг на морському транспорті» Одеського національного морського університету **М.Я. Постан**