SENSORS DESIGN AND MATHEMATICAL MODELING

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CHAOTIC DYNAMICS OF NON-LINEAR PROCESSES IN ATOMIC AND MOLECULAR SYSTEMS IN ELECTROMAGNETIC FIELD AND SEMICONDUCTOR AND FIBER LASER DEVICES: NEW APPROACHES, UNIFORMITY AND CHARM OF CHAOS

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Abstract. Work is devoted to the development of the theoretical foundations of the universal complex chaos-geometric and quantum-dynamic approach that consistently includes a number of new quantum models and a number of new or improved methods of analysis (correlation integral, fractal analysis, algorithms, average mutual information, false nearest neighbors, Lyapunov exponents, surrogate data, non-linear prediction, spectral methods, etc.) to solve problems quantitatively complete modeling and analysis of chaotic dynamics of nonlinear processes in atomic and molecular systems in a uniform and alternating electromagnetic field and quantum generator, laser systems and devices (including fibers, semiconductor lasers with feedback et al.). For considered class of systems and devices there are theoretically studied scenarios of generating chaos, obtained complete quantitative data on the chaos characteristics and different modes of operation.

Keywords: chaotic dynamics, atomic and molecular systems in electromagnetic field, semiconductor and fiber laser devices, chaos-geometric and quantum-dynamics approach

ХАОТИЧНА ДИНАМІКА НЕЛІНІЙНИХ ПРОЦЕСІВ В АТОМНИХ І МОЛЕКУЛЯРНИХ СИСТЕМАХ В ЕЛЕКТРОМАГНІТНОМУ ПОЛІ І НАПІВПРОВІДНИКОВИХ ТА ВОЛОКОННО-ЛАЗЕРНИХ ПРИСТРОЯХ: НОВІ ПІДХОДИ, ОДНОМАНІТНІСТЬ І КРАСА ХАОСУ

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Анотація. Робота присвячена викладенню універсального комплексного хаос-геометричного і квантово-динамічного підходу, що включає низку нових квантових моделей і нових або удосконалених методів аналізу (кореляційний інтеграл, фрактальний аналіз, алгоритми середньої взаємної інформації, хибних найближчих сусідів, показники Ляпунова, сурогатних даних, спектральні методи тощо), для вирішення задач кількісного моделювання і аналізу хаотичної динаміки нелінійних процесів в атомно-молекулярних системах в однорідному і змінному електромагнітному полі і квантово-генераторних, лазерних системах та приладах (у т.ч., волоконних, напівпровідникових лазерах із зворотним зв'язком і т.і.). Для розглянутого класу систем і пристроїв теоретично вивчені сценарії генерації хаосу, отримані кількісні дані по характеристикам хаотичної динаміки і різним режимам функціонування.

Ключові слова: хаотична динаміка, атомні і молекулярні системи в електромагнітному полі, напівпровідникові і волоконні лазери, хаос-геометричний і квантово-динамічний підходи

ХАОТИЧЕСКАЯ ДИНАМИКА НЕЛИНЕЙНЫХ ПРОЦЕССОВ В АТОМНЫХ И МОЛЕКУЛЯРНЫХ СИСТЕМАХ В ЭЛЕКТРОМАГНИТНОМ ПОЛЕ И ПОЛУПРОВОДНИКОВЫХ И ВОЛОКОННО-ЛАЗЕРНЫХ УСТРОЙСТВАХ: НОВЫЕ ПОДХОДЫ, ЕДИНООБРАЗИЕ И ОЧАРОВАНИЕ ХАОСА

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Аннотация. Работа посвящена изложению универсального комплексного хаос-геометрического и квантово-динамического подхода, который включает ряд новых квантовых моделей и ряд новых или усовершенствованных методов анализа (корреляционный интеграл, фрактальный анализ, алгоритмы средней взаимной информации, ложных ближайших соседей, показатели Ляпунова, суррогатных данных, нелинейный прогноз, спектральные методы и т.д.), для решения задач количественного моделирования и анализа хаотической динамики нелинейных процессов в атомно-молекулярных системах в однородном и переменном электромагнитном поле и квантово-генераторных, лазерных системах и приборах (в т.ч., волоконных, полупроводниковых лазерах с обратной связью и др.). Для рассмотренного класса с систем и устройств теоретически изучены сценарии генерации хаоса, получены количественные данные по характеристикам хаотической динамики и различным режимам функционирования.

Ключевые слова: хаотическая динамика, атомные и молекулярные системы в электромагнитном поле, полупроводниковые и волоконные лазеры, хаос-геометрический и квантово-динамический подходы

1. Introduction

At present time one of the extremely important and too complex areas of elements, systems and devices physics and sensor electronics is study of regular and chaotic dynamics dynamics of nonlinear processes in the different classes of quantum, quantum-generating systems and devices and quantum (atomic-molecular systems in an external electromagnetic field) [1-18]. Naturally, this is caused by a very rapid development in the last decade of so-called quantum instrument, including creation of new types of quantum systems and devices (laser diodes, chaotic quantum generators and lasers monohydric masers, atomic clocks, quantum Carnot machine with "radiation" working substance Bose Condensate systems in pairs of alkali atoms, etc.). For these systems and devices it is of a principle role an important manifestation of the effect of chaos, all the elements of chaotic dynamics. Chaotic fluctuations in the dynamics of laser diodes deserve much attention because of their potential for unprecedented application of the technologies, secure communication, the construction of the so-called chaotic lidar, optical reflectometer, true random number generators and so on. It is well known that the transition to chaos in dissipative regime of functioning of NMR-maser provides the construction based on a new type of detector signals with unprecedented sensitivity especially when approaching control parameter of the system to the point of so-called doubling bifurcation, and these detectors for weak signals unstable maser systems can operate in a range 1-10^6Hz.

It is worth to remind that dynamics of the cited systems in external electromagnetic field has features of the random, stochastic kind and its realization does not require the specific conditions. The importance of studying a phenomenon of stochasticity or quantum chaos in dynamical systems is provided by a whole number of technical applications, including a necessity of understanding chaotic features in a work of different electronic devices and systems. New field of investigations of the quantum and other systems has been provided by a great progress in a development of a chaos theory methods [1-12]. In previous our papers [3-5,13,14,19-21] we have given a review of new methods and algorythms to analysis of different systems of quantum physics, electronics and photonics. In this paper we have used the nonlinear method of chaos theory and the recurrence spectra formalism to study quantum stochastic futures and chaotic elements in dynamics of atomic systems in the external electroomagnetic fields. In this paper we present our new approaches to the universal quantum-dynamic and chaos-geometric modelling and analysis of the chaotic dynamics of nonlinear processes in atomic and molecular systems in intense electromagnetic fields and quantum-generator and laser systems and devices (including single-modal laser with an absorbing cell, a semiconductor laser coupled with feedback with delay, the system of semiconductor quantum generators, combined through a general cavity, fiber lasers). In order to make modelling chaotic dynamics it has been constructed improved complex system (with chaos-geometric, neural-network, forecasting, etc. blocks) that includes a set of new quantum-dynamic models and partially improved non-linear analysis methods including correlation (dimension D) integral, fractal analysis, average mutual information, false nearest neighbours, LE, KE power spectrum, surrogate data, nonlinear prediction, predicted trajectories, neural network methods etc. Very important feature of the work is establishing universal character of chaotic dynamics in different systems and devices.

2. Universal chaos-geometric approach in analysis of chaotic dynamics of the nonlinear processes in systems and devices

As our approach has been presented earlier, here we are limited only by the key moments. Let us formally consider scalar measurements $s(n) = s(t_0 + n\mathrm{D}t) = s(n)$, where t_0 is the start time, Dt is the time step, and is n the number of the measurements. Further it is necessary to reconstruct phase space using as well as possible information contained in the $s(n)$. Such a reconstruction results in a certain set of d-dimensional vectors $\mathbf{y}(n)$ replacing the scalar measurements. Packard et al. [9] introduced the method of using time-delay coordinates to reconstruct the phase space of an observed dynamical system. The direct use of the lagged variables $s(n + \mathrm{t})$, where t is some integer to be determined, results in a coordinate system in which the structure of orbits in phase space can be captured. Then using a collection of time lags to create a vector in d dimensions,

$$\mathbf{y}(n) = [s(n), s(n+\tau), s(n+2\tau), \ldots, s(n+(d\text{-}1)\tau)], \quad (1)$$

the required coordinates are provided. In a nonlinear system, the $s(n + j\text{t})$ are some unknown nonlinear combination of the actual physical variables that comprise the source of the measurements. The dimension d is called the embedding dimension, d_E. Example of the Lorenz attractor given by Abarbanel et al. [8] is a good choice to illustrate the efficiency of the method.

According to Mañé and Takens [12], any time lag will be acceptable is not terribly useful for extracting physics from data. If t is chosen too small, then the coordinates s(n + jt) and s(n + (j + 1)t) are so close to each other in numerical value that they cannot be distinguished from each other. Similarly, if t is too large, then s(n + jt) and s(n + (j + 1) t) are completely independent of each other in a statistical sense. Also, if t is too small or too large, then the correlation dimension of attractor can be under- or overestimated respectively [3]. It is therefore necessary to choose some intermediate (and more appropriate) position between above cases. First approach is to compute the linear autocorrelation function

$$C_L(\delta) = \frac{\frac{1}{N}\sum_{m=1}^{N}[s(m+\delta)-\bar{s}][s(m)-\bar{s}]}{\frac{1}{N}\sum_{m=1}^{N}[s(m)-\bar{s}]^2}, \quad (2)$$

where $\bar{s} = \frac{1}{N}\sum_{m=1}^{N} s(m)$

and to look for that time lag where C_L(d) first passes through zero. This gives a good hint of choice for t at that $s(n + j\text{t})$ and $s(n + (j + 1)\text{t})$ are linearly independent. However, a linear independence of two variables does not mean that these variables are nonlinearly independent since a nonlinear relationship can differs from linear one. It is therefore preferably to utilize approach with a nonlinear concept of independence, e.g. the average mutual information. Briefly, the concept of mutual information can be described as follows. Let there are two systems, A and B, with measurements a_i and b_k. The amount one learns in bits about a measurement of a_i from measurement of b_k is given by arguments of information theory [3,7]

$$I_{AB}(a_i, b_k) = \log_2\left(\frac{P_{AB}(a_i, b_k)}{P_A(a_i)P_B(b_k)}\right), \quad (3)$$

where the probability of observing a out of the set of all A is $P_A(a_i)$, and the probability of finding b in a measurement B is $P_B(b_i)$, and the joint probability of the measurement of a and b is $P_{AB}(a_i, b_k)$. The mutual information I of two measurements a_i and b_k is symmetric and non-negative, and equals to zero if only the systems are independent. The average mutual information between any value a_i from system A and b_k from B is the average over all possible measurements of $I_{AB}(a_i, b_k)$,

$$I_{AB}(\tau) = \sum_{a_i, b_k} P_{AB}(a_i, b_k) I_{AB}(a_i, b_k)\,. \quad (4)$$

To place this definition to a context of observations from a certain physical system, let us think of the sets of measurements $s(n)$ as the A and of the measurements a time lag t later, $s(n + \text{t})$, as B set. The average mutual information between observations at n and n + t is then

$$I_{AB}(\tau) = \sum_{a_i, b_k} P_{AB}(a_i, b_k) I_{AB}(a_i, b_k)\,. \quad (5)$$

Now we have to decide what property of I(t) we should select, in order to establish which among the various values of t we should use in making the data vectors $\mathbf{y}(n)$. One could remind that the autocorrelation function and average mutual information can be considered as analogues of the linear redundancy and general redundancy, respectively, which was applied in the test for nonlinearity. The general redundancies detect all dependences in the time series, while the linear redundancies are sensitive only to linear structures. Further, a possible nonlinear nature of process resulting in the vibrations amplitude level variations can be concluded.

The goal of the embedding dimension determination is to reconstruct a Euclidean space R^d large enough so that the set of points d_A can be unfolded without ambiguity. In accordance with the embedding theorem, the embedding dimension, d_E, must be greater, or at least equal, than a dimension of attractor, d_A, i.e. $d_E > d_A$. However, two problems arise with working in dimensions larger than really required by the data and time-delay embedding [1,7,13,19]. First, many of computations for extracting interesting properties from the data require searches and other operations in R^d whose computational cost rises exponentially with d. Second, but more significant from the physical point of view, in the presence of noise or other high dimensional contamination of the observations, the extra dimensions are not populated by dynamics, already captured by a smaller dimension, but entirely by the contaminating signal. In too large an embedding space one is unnecessarily spending time working around aspects of a bad representation of the observations which are solely filled with noise. It is therefore necessary to determine the dimension d_A. There are several standard approaches to reconstruct the attractor dimension (see, e.g., [3,7-12]), but let us consider in this study two methods only. The correlation integral analysis is one of the widely used techniques to investigate the signatures of chaos in a time series. The analysis uses the correlation integral, $C(r)$, to distinguish between chaotic and stochastic systems. To compute the correlation integral, the algorithm of Grassberger and Procaccia [10] is the most commonly used approach. According to this algorithm, the correlation integral is

$$C(r) = \lim_{N\to\infty} \frac{2}{N(n-1)} \sum_{\substack{i,j \\ (1\le i<j\le N)}} H\left(r - \|\mathbf{y}_i - \mathbf{y}_j\|\right), \quad (6)$$

where H is the Heaviside step function with $H(u) = 1$ for $u > 0$ and $H(u) = 0$ for u £ 0, r is the radius of sphere centered on $\mathbf{y}_i$ or $\mathbf{y}_j$, and N is the number of data measurements. If the time series is characterized by an attractor, then the integral $C(r)$ is related to the radius r given by $d = \lim_{\substack{r\to 0 \\ N\to\infty}} \frac{\log C(r)}{\log r}$,

where d is correlation exponent that can be determined as the slop of line in the coordinates log $C(r)$ versus log r by a least-squares fit of a straight line over a certain range of r, called the scaling region.

If the correlation exponent attains saturation with an increase in the embedding dimension, then the system is generally considered to exhibit chaotic dynamics. The saturation value of the correlation exponent is defined as the correlation dimension (d_2) of the attractor. The method of surrogate data [3,7-11] is an approach that makes use of the substitute data generated in accordance to the probabilistic structure underlying the original data. This means that the surrogate data possess some of the properties, such as the mean, the standard deviation, the cumulative distribution function, the power spectrum, etc., but are otherwise postulated as random, generated according to a specific null hypothesis. Here, the null hypothesis consists of a candidate linear process, and the goal is to reject the hypothesis that the original data have come from a linear stochastic process. One reasonable statistics is obtained as follows. If we denote Q_{orig} as the statistic computed for the original time series and Q_{si} for ith surrogate series generated under the null hypothesis and let m$_s$ and s$_s$ denote, respectively, the mean and standard deviation of the distribution of Q_s, then the measure of significance S is given by $S = \frac{|Q_{orig} - \mu_s|}{\sigma_s}$. An S value of ~2 cannot be considered very significant, whereas an S value of ~10 is highly significant. To detect nonlinearity in the amplitude level data, the one hundred realizations of surrogate data sets were generated according to a null hypothesis in accordance to the probabilistic structure underlying the original data. Often, a significant difference in the estimates of the correlation exponents, between the original and surrogate data sets, can be observed. In the case of the original data, a saturation of the correlation exponent is observed after a certain embedding dimension value (i.e., 6), whereas the correlation exponents computed for the surrogate data sets continue increasing with the increasing embedding dimension. The high significance values of the statistic indicate that the null hypothesis (the data arise from a linear stochastic process) can be rejected and hence the original data might have come from a nonlinear process. It is worth consider another method for determining d_E that comes from asking the basic question addressed in the embedding theorem: when has one eliminated false crossing of the orbit with itself which arose by virtue of having projected the attractor into a too low dimensional space? By examining this question in dimension one, then dimension two, etc. until there are no incorrect or false neighbours remaining, one should be able to establish, from geometrical consideration alone, a value for the necessary embedding dimension. Advanced version is presented in Ref. [3]

The LE are the dynamical invariants of the nonlinear system. In a general case, the orbits of chaotic attractors are unpredictable, but there is the limited predictability of chaotic physical system, which is defined by the global and local LE. A negative exponent indicates a local average rate of contraction while a positive value indicates a local average rate of expansion. In the chaos theory, the spectrum of LE is considered a measure of the effect of perturbing the initial conditions of a dynamical system. In fact, if one manages to derive the whole spectrum of the LE, other invariants of the system, i.e. KE and attractor's dimension can be found. The KE, *K*, measures the average rate at which information about the state is lost with time. An estimate of this measure is the sum of the positive LE. The inverse of the KE is equal to an average predictability. Estimate of dimension of the attractor is provided by the Kaplan and Yorke conjecture:

$$d_L = j + \frac{\sum_{\alpha=1}^{j} \lambda_\alpha}{|\lambda_{j+1}|}, \tag{7}$$

where j is such that $\sum_{\alpha=1}^{j} \lambda_\alpha > 0$ and $\sum_{\alpha=1}^{j+1} \lambda_\alpha < 0$, and the LE l_a are taken in descending order. There are a few approaches to computing the LE. One of them computes the whole spectrum and is based on the Jacobi matrix of system [3]. In the case where only observations are given and the system function is unknown, the matrix has to be estimated from the data. In this case, all the suggested methods approximate the matrix by fitting a local map to a sufficient number of nearby points. To calculate the spectrum of the LE from the amplitude level data, one could determine the time delay t and embed the data in the four-dimensional space. In this point it is very important to determine the Kaplan-Yorke dimension and compare it with the correlation dimension, defined by the Grassberger-Procaccia algorithm. The estimations of the KE and average predictability can further show a limit, up to which the amplitude level data can be on average predicted. Table 1 reflects the main blocks of a the universal complete complex chaos-geometric approach to chaotic dynamics in systems and devices. The basic idea of constructing model prediction of chaotic properties of complex systems is the use of the traditional concept of a compact geometric attractor, which evolve measurement data, plus implementation neural network algorithms. The meaning of the concept is in the doctrine of evolution attractor in the phase space of the system and in a sense the simulation ("guessing") temporal evolution.

It's about the fact that the phase space of a system orbit some continuously rolled on itself as a result of dissipative forces and the nonlinear part of the dynamics, so it is possible to find in the neighborhood of any point of the orbit **y** (n) other points of the orbit $\mathbf{y}^r(n)$, $r = 1, 2, \ldots, N_B$, arriving in the neighborhood of y (n) in different time moments which differ of n. Of course, then one can try to build different types of interpolation functions that take into account the whole neighborhood of the phase space, while explaining how the neighborhood evolve from y (n) around all points set near y(n + 1). In terms of the theory of neural networks, the simulation of the evolution of the system can be described by some generalized evolutionary neural dynamic equations. Simulating further the evolution of complex systems as appropriate neural network evolution with elements of self-learning, self-adaptability, etc., there is a significant opportunity to improve the quality of prediction of the evolutionary dynamics of modelling the attractor in a chaotic system. Modelling attractor by some record in memory, neural system evolutionary process, i.e. the transition from the initial state to the (next) final state, can be represented by a model of reconstruction of the full record on distorted information, that is a model of associative recognition. Domain of attraction of different attractors are separated by separatrises or by certain surfaces in the phase space, structure of which is quite complex. However, it imitates the properties of the chaotic object. The next step is to construct a parameterized nonlinear function F (x, a), which transform y (n) to $\mathbf{y}(n)$ в $\mathbf{y}(n+1) = \mathbf{F}(\mathbf{y}(n), \mathbf{a})$, and use different, including the neural network criteria for determining the parameters a. As the functional form of displaying, one may use, for example, polyno-

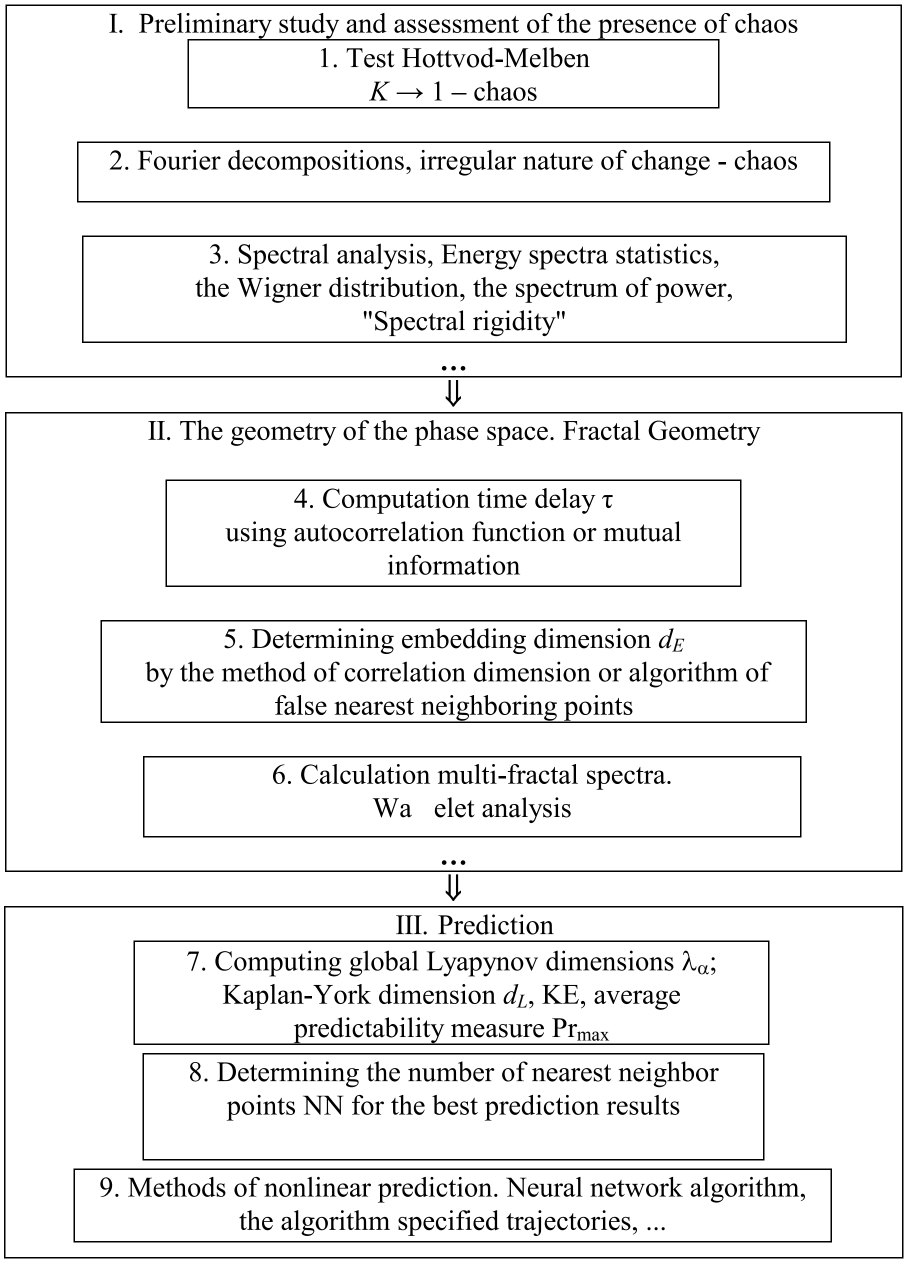

Figure 1. Chaos and neural network-geometric approach to nonlinear analysis and forecast chaotic dynamics processes in complex systems (devices).

mial basis functions. A measure of the quality of the curve fit to the data, which is determined from the condition, how exactly coincide **y**(k + 1) with **F**(**y**(k), **a**) is a local deterministic error: $e_D(k)$ = **y**(k + 1)-**F**(**y**(k), **a**).If the mapping F (y, a) is local, then for each neighbor to **y**(k) point, $\mathbf{y}^{(r)}(k)$ (r = 1, 2, …, N_B) can be written as $\varepsilon_D^{(r)}(k)$ = **y**(r, k + 1)-**F**($\mathbf{y}^{(r)}(k)$, **a**), where **y**(r, k + 1) is the point in phase space, which evolves y (r, k). To measure the quality of the curve fit to the data, the local cost function has the form (in fact, the function value for the error): $W(\varepsilon,k) = \sum_{r=1}^{N_B} \left|\varepsilon_D^{(r)}(k)\right|^2 / \sum_{r=1}^{N_B} \left[\mathbf{y}(k) - \langle \mathbf{y}(r,k) \rangle\right]^2$ and the parameters, determined by minimizing W(e, k), are dependent on parameter a. More formally, it is possible to start neural network algorithm, especially in terms of training an equivalent system of neural networks with the reconstruction and forecasting neural system state (correspondingly, correction of a). In Ref. [15] we have presented the results of modelling and recognition of complex patterns (trajectories). Taken as input noisy soliton-like pulse $f(t) = |A_0|^2\, ch^{-1}(bt)$, which imposed by an additive noise of intensity D = 0.000-0.0050, as well as a neural function $f(x)=1/[1+\exp(-\delta x)]$. The process of network training and playback signal was optimal at a certain level of noise (D = 0.0021); resulting in the PC experiments there has been revealed the unique possibility of stochastic resonance effect in the dynamical system with a noise

3. Chaos in dynamics of atomic systems in an intense electromagnetic field

In ref. [18] it has been developed a new non-perturbative quantum chaos-dynamic and geometric approach to modeling the chaotic dynamics of atomic systems in homogeneous magnetic field, which is based on the operator optimized perturbation theory and finite-difference solution of the Schrödinger equation for an atom in the field (in a cylindrical coordinate system z||B; $\Psi \sim e^{iM\varphi}$):

$$\left[\frac{\partial^2}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial}{\partial\rho}+\frac{\partial^2}{\partial z^2}-\frac{M^2}{\rho^2}-4\gamma^2\rho^2+ +\frac{4}{r}+V_C(r)+(\frac{E}{R_y}-\gamma M)\right]\Psi(\rho,z)=0\ , \quad (8)$$

where $g=B/Bo$, B_o=2.3505×10^{5}T, $V_c(r)$ – potential electron self-consistent field, including the Hartree potential plus the Kohn-Sham exchange-correlation potential (other notations are standard). The quantitative modeling of regular and chaotic dynamics, computation power and spectral parameters for the atoms of hydrogen, helium, neon in a uniform magnetic field (g = 0.01-10000) showed that the system generated quantum chaos, which is manifested in a very complex and irregular dependences of state energies upon the magnetic field amplitude, the presence of the level intersections (eg., for Ne quasi-intersections in dependence of the energy states $|0_N>$ and $|2p_0>$ upon the magnetic field amplitude at g = 158.7, $|2p_0>$ and $|1s^2>$ states- at g=40.2), in a photoionization cross sections, power spectra etc. We have calculated and carried our analysis of the photoionization spectrum, power spectrum, the energies and widths of resonances, the distribution of resonances in the H atom in the field of 5.96 T (the energy interval 20-80 cm-1). The experimental spectrum of hydrogen in the magnetic field 5.96 T is measured in [17]. According to our data, the density of states in the middle of each channel Landau resonance is 33 cm $^{-1}$ for the average resonance width - 0.004 cm^{-1}, which is consistent with experimental data Kleppner et al (1977): 0.004-0.006cm^{-1}.The same data have been also obtained for the Ba, namely, power spectrum, resonance structure of the barium photoionization spectrum. The speech is about a set of the low resonances вузьких (with widths 0.003-0.03cm^{-1}).

Further we present the results of modelling the chaotic dynamics of atomic systems in the crossed electric F_1 and magnetic g fields, based on the numerical solution of the Schrödinger equation:

$$H=1/2(p_\rho^2+l_z^2/\rho^2)+\gamma l_z/2+(1/8)\gamma^2\rho^2+ +(1/2)p_z^2+F_1 z\cdot\sin(\omega t)+V(r) \quad (9)$$

the operator perturbation theory and density functional method. Further we use the denotations:

$\tilde{f}=F_1\gamma^{-4/3}$, $e=E^{ion}g^{-2/3}$ (E^{ion} - ionization energy of the free atom).We have carried out modelling a chaotic dynamics for the Rydberg Li, Rb (n ~ 100 , m = 0) atoms in a static magnetic (B = 4.5T) and oscillating electric field with frequency w=10^{2}MHz (e=-0.03,v=0.32, $g^{-1/3}$ in the range 35-50; f = 0.000-0.070). Fig.2 shows the power spectrum of Rb: a- in a magnetic field (f = 0, the electric field is absent); (b) in a static magnetic field and oscillating electric field f = 0.0035 (our data).

Transition to chaos in system comprising inducing nonlinear resonances by magnetic field , its strong interaction and in excess of the critical field strength merger with the emergence of global chaos.

4. Chaotic features of atomic systems dynamics in a DC electric and electromagnetic fields

Here we present a new quantum-dynamic and chaos-geometric approach to modeling the chaotic dynamics of atomic systems in a dc electric and ac electromagnetic fields, based on the theory of quasi-stationary quasienergy states, optimized operator perturbation theory, method of model-potential, density functional formalism, a complex rotation coordinates algorithm method. The universal chaos-geometric approach has been used for modeling the chaos features. The new version of the operator perturbation theory generalizes the original method [19,20]. The master system of differential equations for the electronic wave function of an atomic system with N-electron core (described the Hellmann potential with parameters A, b) in a strong uniform electric field (of the intensity F) is as follows:

$$
\begin{aligned}
&f''_{E's} + \frac{|m|+1}{t} f'_{E's} + [E'/2 + (\beta_1' - (N + A\tilde{b}(k))/Z)/t - Ft/4] f_{E's} = 0 \\
&g_1'' + \frac{|m|+1}{t} g_1' + [E'/2 + \beta_2'/t + Ft/4]\, g_1 = 0 \\
&g_2'' + \frac{|m|+1}{t} g_2' + [E/2 + \beta_2'/t + Ft/4]\, g_2 = 2g_{Eb}\ ,
\end{aligned} \tag{10}
$$

The width of the resonance is defined as:

$$\mathrm{Im}\, E = \Gamma/2 = \pi |\langle \Psi_{Eb} | H | \Psi_{Es} \rangle|^2 . \tag{12}$$

In the case of the alternating electromagnetic field Hamiltonian atom is as follows:

$$H = \frac{1}{2}p^2 + V_{at}(r) + zF_0\cos(\omega t). \tag{13}$$

The field is periodic, of course one should use the Floquet theorem; then the eigen Floquet states $|\Psi_{E_j}(r,t)\rangle$ and quasienergies E_j are defined as the eigen functions and eigen values of the Floquet Hamiltonian $H_F = H - i\partial_t$. In the general form with using the method of complex coordinates the problem reduces to the solution of stationary Schrödinger equation, which is as follows in the model potential approximation:

$$(-1/2\cdot\nabla^2 + V_{at}(r) + \omega L_z + F_o z)\Psi_E(r) = E\Psi_E(r) \tag{14}$$

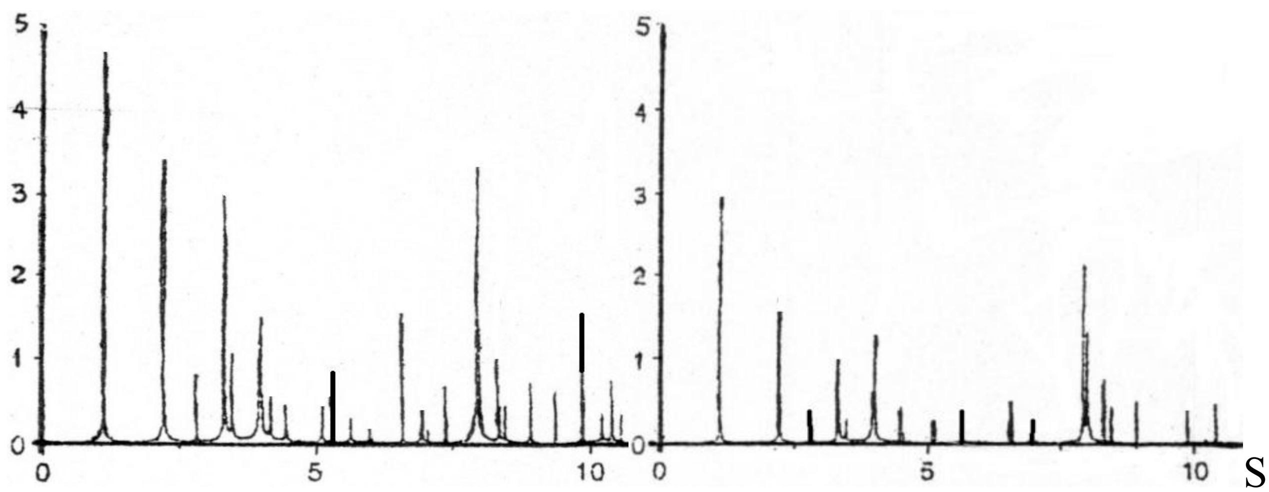

Fig.2. The power spectrum of Rb: : a- in a magnetic field (f = 0, the electric field is absent); (b) in a static magnetic field and oscillating electric field f = 0.0035 (our data).

To calculate the energy state E, separation constant b_l one should solve the (6) with total H using the quantization terms:

$$
\begin{aligned}
&f(t) \to 0 \text{ at } t \Rightarrow \infty,\ \partial x(\beta, E)/\partial E = 0, \\
&x(\beta, E) = \lim_{t\to\infty} [g^2(t) + \{g'(t)/k\}^2]\, t^{|m|+1}
\end{aligned} \tag{11}
$$

i.e. to the stationary eigen value and eigen vectors task for some matrix A (with the consideration of several Floquet zones): $(A - E_j B)|E_j\rangle = 0$. As a decomposition basis, system of the Sturm functions of the operator perturbation theory basis is used.

As illustration, below we present some results of our calculations of ionization dynamics parameters for Rydberg atoms Li, Rb, Yb (Li: n_0=41-70;

Rb: n=51-70; Yb: n=60-80) in a microwave field (F=(1.2-3.2)×10^{-9} a.u.; w/2p=8.87, 36 HGz), in particular, a dependence of the ionization probability P upon the F, interaction time "atom-field", which are in physically reasonable agreement with experimental data by Munich group для Rb: n_0>60, F=(1.2-3.2)×10^{-9} a.u., w/2p = 8.87GHz) and Virginia group for Li: n_0<42, w/2p = 15GHz [22,25]. In Fig.3 we present our data on the dependence of ionization probability for atoms Li in the initially prepared states with l_0=0, m_0=0, n=63,67,69 on the field strength *F.*

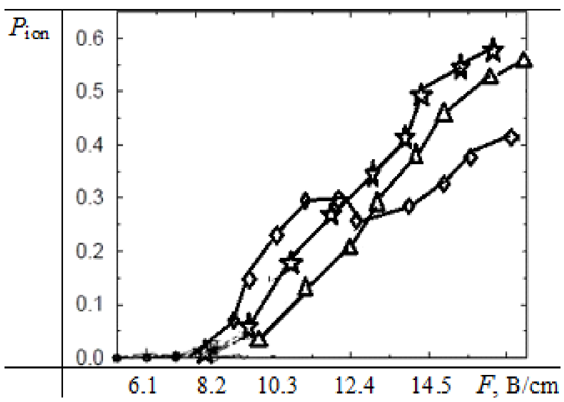

Fig.3. Our data on the dependence of ionization probability for atoms Li in the initially prepared states with l_0=0, m_0=0, n=63,67,69 on the field strength *F*: n=63 (◇), n=67 (△), n=69 (☆).

In table 1 we listed calculated dependence of the Rb ionization probability (l_0=0, m_0=0, n_0=60-66) upon the =2.8-3.1×10^{-9} a.u. [parameters: t = 327× 2p/w; frequency $w/2p$=36 GHz(I), 8.87GHz(II)]

Table 1.

Dependence of the ionization probability P for Rb (l_0=0, m_0=0, n_0=60-66) on F,n_0

n_0	F=2.8×10^{-9}; ω(I)	F=3.1×10^{-9}; ω(I)	F=2.8×10^{-9};ω(II)	F=3.1×10^{-9}; ω (II)
60	0,236	0,252	0,18	0,21
63	0,347	0,358	0,27	0,31
65	0,339	0,347	0,25	0,29
66	0,359	0,371	0,31	0,35

Our calculations have shown that in dynamics of ionization Li, Rb, Yb Rydberg states in the microwave field for main quantum numbers n_0 (n_0 ~ 63) there are the local violations of probability smooth growth associated with the complex Floquet spectrum, link between the quasi-stationary states and a continuum, the growing influence of multiphoton resonances. The picture becomes by more complicated due to the single-photon near-resonance transitions with quasi-random detuning from resonance and quantum phase shift due to scattering Rydberg electron on the atomic core. It is in agreement with alternative comments in [22,25]. Then we discovered a huge effect broadening of resonances, their intensive interaction in the spectrum of ytterbium heavy atoms in an external field, and quantitatively detected spectral quantum chaos in the distribution of the highly-lying stationary states, Rydberg, autoionization, Stark resonances in Yb spectrum (including resonances $4f^{13}6s^2np$, $4f^{13}6s^2nf$ з n>20, J=2^-); Two potions sets of levels (> 30 conf. , 80 conf.) are studied. The average distance S_n between the levels is ~0.03 eV, with accounting for the Rydberg series states which converged to have a mean value of 190 eV^{-1}, and taking into account the levels of Ry series converging to excited states of Yb^+ this number can reach 10^3 eV^{-1}. Our analysis shows that the distribution of S_n corresponds to a chaotic Wigner distribution. In Table. 2 there are listed our data and the available exp. data [25] on energies and widths width (cm^{-1}) for resonances $4f^{13}$ $[^2F_{7/2}]6s^2np[5/2]_2$, $nf[5/2]_2$ Yb in the electric field of 80V/cm.

Table 2.

Resonances $4f^{13}$ $[^2F_{7/2}]6s^2np[5/2]_2$, $nf[5/2]_2$ Yb in the electric field 80V/cm: E,Γ (cm^{-1}).

n	E_{exp}	E	Γ	E_{exp}	E	Γ
20	71428.1	71429	0.98	71559.1	71561	1.27
30	71698.8	71697	3.62	71732.4	71734	2.6
34	-	71741	2.65	-	71763	2.15
35	-	71748	1.82	-	71770	1.83
46	-	71797	1.79	-	71813	1.65

Further we have used the chaos-geometric approach to estimate parameters of chaotic dynamics for the Rydberg atoms Li, Rb, Yb in microwave field: correlation dimension, LE l_a(+~0.2,+~0.07), KE (~ 0.3). We have constructed the quantitative diagram of effects of the quantum fluctuations, stabilization, destabilization, delocalization and performance of the Kolmogorov-Arnold-Mozer theorem in atomic dynamics. We have found that the regime of the chaotic ionization for the Li, Rb in a microwave field at $\omega_o = \omega n_o^3$>0.88 (Li), w_0>0.31 (Rb) switches to dynamic stabilization one.

4. Chaos in dynamics of molecular systems in an intense electromagnetic field

Here we present the chaotic dynamics analysis for diatomic molecules in an intense electromagnetic field), which, firstly, based on the numerical solution of the time-dependent Schrödinger equation and realistic Simons-Parr-Finlan model for the potential of diatomic molecule *U (x)* (the quantum unit) and, secondly, the universal approach to analysis of nonlinear chaotic dynamics (chaos-geometric unit). The problem is reduced to solving the Schrödinger equation:

$$i\partial\Psi/\partial t = [H_0 + U(x) - d(x)E_M\varepsilon(t)\cos(\omega_L t)]\ \Psi \tag{15}$$

where E_M - the maximum field strength, $e(t)=E_0 cos(nt)$ corresponds the pulse envelope. Molecule in the field gets induced polarization and its high-frequency component can be defined as:

$$p_c^{(x,y)}(t) = \left(\frac{1}{T}\right)\oint\langle\psi(t)|\hat{d}_{x,y}|\psi(t)\rangle cos\omega t dt, \tag{16}$$

where *T* — period of the external field, d –dipole moment. Then there is simply determined the power spectrum. As usually, to avoid the numerical noise during the Fourier transformation, the attenuation technique used, i.e. at t> t_p p (t) is replaced by

$$p(t)cos^2\{\pi(t-t_p)/[2(T-t_p)]\},\ (t_p < t < T\ )$$

with T=1.6t_p.

Numerical calculations of the dynamics of the diatomic molecule GeO in the linearly polarized field (molecule and field parameters are as : $\hbar\Omega$=985.8 cm^{-1}, $y\hbar\Omega$=4.2cm^{-1}, B = 0.48 cm^{-1}, d_0 = 3.28 D, M=13.1 а.е.м.; the radiation intensity 2.5-25 GW/cm^2, respectively: W = 3.39-10.72cm^{-1}) have been carried out. According to classical-dynamical treating [26], these parameters correspond to chaotic regime. The analysis shows that more than 200 vibrational-rotational molecular levels are involved into a chaotic dynamics. Fig. 4 shows the theoretical time dependence of polarization for GeO molecule in an intense field in a chaotic regime.

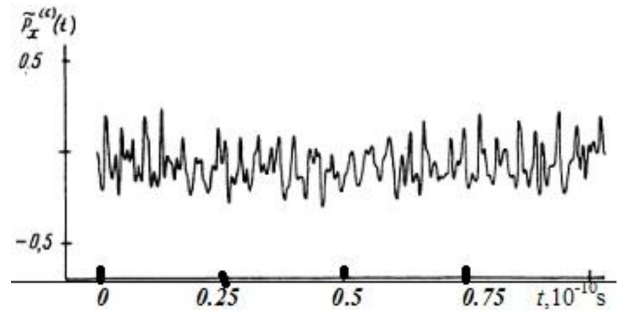

Fig.4. Time dependence of polarization for GeO molecule in intense field in a chaotic regime.

We at first have calculated the quantitative parameters for the GeO molecule chaotic dynamics in a linearly polarized field of intensity 25 GW / cm^2, namely: correlation dimension (2.73), the embedding dimension (3), Kaplan-York dimension (2.51), LE (first two LE are positive: + 0.146 + 0.0179), KE, etc.

5. Chaotic dynamics of non-linear processes in semiconductor and erbium fiber laser devices

Here we present the results of the first complete quantitative study of low- and high-dimensional dynamics of the generation of chaos in semiconductor GaAs / GaAlAs laser device with feedback with a delay or disruption in which non-stability was provided by changing the force feedback (current injection). Fischer et al [27] have performed an experimental study of the dynamics of the generation of chaos in semiconductor GaAs / GaAlAs Hitachi HLP1400 laser; instability in the system was caused by feedback from the delay at the change of control parameters such as force feedback m (current injection). Of course, depending on m in the system there are arisen the multi-stability of different classes; its modulation period is approximately T_n=2t/(2n+1), n=0, 1,2,… State of the n = 0 is called as basic. With respect to frequency modulation, other states are called as the third harmonic, fifth harmonic and so on. Fig. 5 shows the measured data for the time dependences of the intensity for a semiconductor laser device with feedback: a) Up figure - time series, which shows the chaotic wandering between the ground state and the state of the third harmonic; b) Down figure - the time series for the system in a state of global chaotic attractor.

The chaotization scenario in the system is from the beginning in converting of regular classes into individual chaotic states with the increasing the parameter m by means of the period doubling bifurcation sequence. Then there is arisen a global chaotic attractor after the merger of individual chaotic attractors according to the more complicated scenario. In Table. 3 there are listed the numerical data on the correlation dimension d_2, embedding dimension, based on the algorithm of false nearest neighboring points (d_N) with percentage of the false neighbors (%) calculated for different values of the delay time t according to the analysis of two rows (two regimes: (I) -Chaos and (II)- Hyperchaos) .

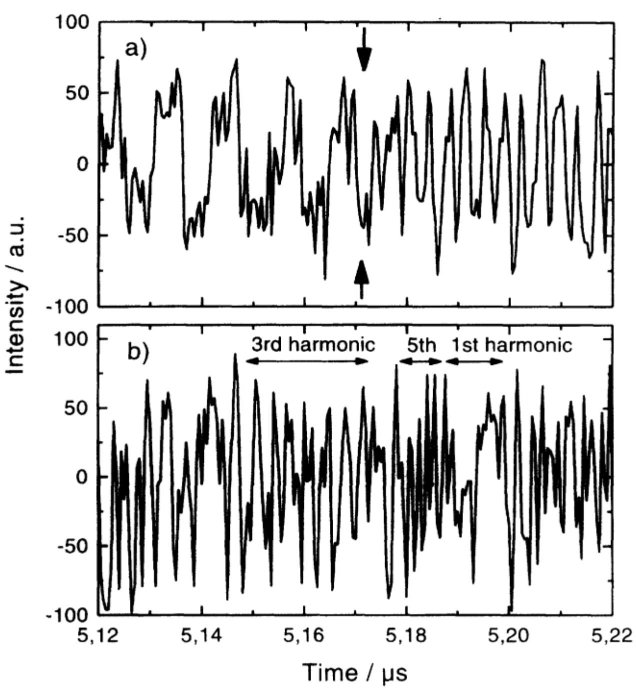

Fig.5. Time series of intensity in GaAs / GaAlAs Hitachi HLP1400 laser (experimental data by Fischer et al (Marburg, Germany, 1994)/

Table 3.

The correlation dimension d_2, embedding dimension, based on the algorithm of false nearest neighboring points (d_N) with percentage of the false neighbors (%) calculated for different values of the delay time t

Chaos (I)			Hyperchaos (II)		
τ	d_2	(d_N)	τ	d_2	(d_N)
58	3.4	5 (8.1)	67	8.4	11 (15)
6	2.2	4 (1.05)	10	7.4	8 (3.4)
8	2.2	4 (1.05)	12	7.4	8 (3.4)

Table. 4 shows the results of a calculation of the LE, the Kaplan-York attractor dimension, KE K_{entr}. For the studied series there are positive and negative values of LE The resulting Kaplan-York dimension in both cases is very close to the correlation dimension, which is determined by the algorithm by Grassberger and Procaccia; Moreover, the Kaplan-York dimension is smaller than the dimension of attachment, which confirms the correctness of the choice of the latter.

Table 4.

Data on LE: l_1-l_4 in the в порядку убування, d_L – Kaplan- York dimension, K_{entr} – KE

Regime	λ_1	λ_2	λ_3	λ_4	d_L	K_{entr}
Chaos (I)	0.151	0.00001	-0.188	-0.067	1.8	0.15
Hyperchaos (II)	0.517	0.192	-0.139	-0.042	7.1	0.71

Further we present the results of analysis and prediction of the chaotic dynamics for temporal dependence of the GaAs / GaAlAs (Hitachi HLP1400) mode laser intensities. All results are obtained on the basos of the universal chaos-geometric approach. Calculation of assessing the reliability (success) and efficiency of the forecasting model for the system showed that chaos mode correlation coefficient (r) between the actual and prognostic rows, ranked among the neighbors (NN), is: r = 0.96 (NN = 90), r = 0.97 (NN = 220), r = 0.97 (NN = 250); as a result we can talk about physically reasonable, quite successful prediction of temporal evolution of intensity especially for system in the low--dimensional (D ~ 2), chaos, some worse for hyper chaos regime (D ~ 7). Nonetheless, the implementation of the model indicates the possibility of a radically new direction of research in physics of dynamical systems and devices in terms sufficiently reliable quantitative prediction of their evolution in the future, at least in the short-term version.

Further we present the original results of a complete quantitative study of the chaos generation dynamics in the one-ring erbium fiber laser (EDFL) using control parameters: the modulation frequency *f* and dc bias voltage *V* of the additional electro-optical modulator (EOM). Feng et al. [28] experimentally observed generation of chaos in the EDFL (laser parameters: the output power 20.9 mV, wavelength 1550.190 nm) with the EOM which is made from crystal $LiNbO_3$. In the first series of measurements (Exp.1) the DC bias voltage is maintained at 10V, frequency modulation control parameters was *f*= 64-75MHz. Fig. 6a shows the measured time dependence of the output voltage V_{out} of the frequency modulation: A. *f*=75 MHz (one-period state); B. *f*=68MHz (double-period state); C. *f*=64MHz (chaotic state). In a second series of measurements the modulation frequency is kept at 60 MHz, and its dc bias voltage V was changed from 4 to 10V (fig.6b). Theoretical examination shows that depending on the values of *f, V* laser device is in turn in the one-period (f = 75 MHz, V = 10V or f = 60MHts, V = 4V), double-period (f = 68MHz, V = 10V or f = 60MHz , V = 6V), chaotic (f = 64MHz, V = 10 V and f = 60MHz, V = 10V) states. Using our universal chaos-geometric approach we have calculated values of LE, correlation dimension, embedding dimension, the Kaplan-York dimension, the KE K_{entr} for two time series. The relevant data are listed in the Table. 5

Table 5.

Results of calculation of the LE amplitude level: l_1-l_4 in descending order, d_L - Kaplan-York dimension, K_{entr} - KE

Row	λ_1	λ_2	λ_3	λ_4	d_L	K_{entr}
I	0.168	0.0212	-0.223	-0.323	2.85	0.19
II	0.172	0.0215	-0.220	-0.318	2.88	0.19

In general, our theoretical analysis shows that the chaos in the EDFL device is generated via scenario of intermittency by increasing the DC bias voltage and period-doubling bifurcation sequence by reducing the EOM modulation frequency.

7. Conclusions

So, we have carried out modelling chaotic dynamics of nonlinear processes in different classes of systems and devices using the same new uniform chaos-geometric and quantum dynamical approach and confirmed the universality and charm of chaotic phenomena. It is carried out computing energy and spectral parameters for hydrogen, helium, neon, ytterbium in a uniform magnetic field (g~ 0.01-10000) and found anti-crossings, complex power spectra with chaotic elements (inducing nonlinear resonances, then, their strong interaction, creating stochastic layers and global stochasticity). It is carried out modelling of chaotic dynamics of the Li, Rb Rydberg states in (n = 115,125; m = 0) in a static magnetic field B = 4.5T and oscillating electric field with frequency f= 102MHz) and shown that stochastic changing, fragmentation, extinction and again appearing of the peaks in power spectrum is occurred. We have presented a new approach to modelling the chaotic dynamics of diatomic molecules in intense electromagnetic field, which is , firstly, based on the numerical solution of the time-dependent Schrödinger equation and realistic model Simons-Parr-Finlan potential for diatomic molecules (quantum unit) and, secondly, the universal chaos-geometric nonlinear analysis unit, which includes the application of methods of correlational integral, LE and spectrum strength etc to analysing time series of populations, induced polarization. There are determined quantitative parameters of the GeO molecule chaotic dynamics in linear polarization filed (intensity of 25 GW / cm^2), including, correlation D (2.73), embedding D, Kaplan-York D (2.51), LE (the first two are positive, +, +), KE etc. We have carried out quantitative low- and high-D chaos dynamics generation studying in semiconductor GaAs/GaAlAs laser device with delayed feedback with governing (feedback strength, current injection). It is shown that the firstly arising periodic states turns into individual chaotic states and then global chaotic attractor with scenario through period-doubling bifurcation, which then significantly modified. It is numerically investigated chaos dynamics generation in the erbium one-ring fibre laser (EDFL, 20.9mV strength, l= 1550.190nm) with the control parameters: the modulation frequency *f* and dc bias voltage of the electro-optical modulator. It is shown that in depending upon *f, V* values there are realized 1-period (*f* = 75MHz, V = 10V and *f* = 60MHz, V = 4V), 2-period (*f* = 68 MHz, V = 10V or *f* = 60MHz, V = 6V), chaotic (*f* = 64MHz, V = 10 V and *f* = 60MHz, V = 10V) regimes; there are calculated LE, correlation, embedding, Kaplan-York dimensions, KE and theoretically shown that chaos in the erbium fiber laser device is generated via intermittency by increasing the DC bias voltage and period-doubling bifurcation by reducing the frequency modulation computers.

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