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Fractal dimensions of gypsum cave-mazes of Western Ukraine

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Резюме: *Горизонтально развитые гипсовые пещеры-лабиринты Западной Украины характеризуются сложной пространственной структурой, обладающей признаками фрактальной, которая может быть исследована с использованием соответствующих математических методов. Изучено (рассчитано) объемное и корреляционное измерения длиннейших гипсовых пещер региона. Предполагается, что результаты фрактального исследования могут быть использованы в целях прогноза еще не обнаруженных частей пещерных лабиринтов.*
<u>Ключевые слова</u>: лабиринтовые пещеры, фракталы, фрактальные измерения.

Андрейчук В.М., Бляхович Т., Доміно К. Фрактальні виміри гіпсових печер-лабіринтів Західної України // Спелеологія і Карстологія, - № 11, Сімферополь. – 2014. – С. 40-47.

Резюме: *Горизонтально розвинуті гіпсові печери-лабіринти Західної України характеризуються складною просторовою структурою, яка має ознаки фрактальної і може бути досліджувана з використанням відповідних математичних методів. Досліджено (розраховано) об´ємний та кореляційний виміри найдовших гіпсових печер-лабіринтів регіону. Висловлюється припущення, що результати фрактального дослідження можуть використовуватись з ціллю прогнозування ще невідкритих частин печер-лабіринтів.*
<u>Ключові слова:</u> лабіринтові печери, фрактали, фрактальні виміри.

Andreychouk V.N.., Błachowicz T., Domino K. Fractal dimensions of gypsum cave-mazes of Western Ukraine // Speleology and Karstology, - № 11, Simferopol. – 2014. – P. 40-47.

Abstract: *Gypsum maze caves of Western Ukraine are characterized by a complex spatial structure, which can be treated as fractals and can be studied using appropriate mathematical tools. Capacitance and correlation fractal dimensions of largest gypsum caves of the region were calculated. The results were used to predict findings of new, yet undiscovered parts of cave mazes.*
<u>Keywords:</u> maze caves, fractals, fractal dimentions.

INTRODUCTION

Natural objects possessing enough complicated spatial structure can be treated as fractals. Importantly, calculations of so-called fractal dimensions can be performed in most typical cases. There are many different types of fractal dimensions named as: capacity, correlation, informative, topological, boxed, Hausdorff, Lyapunov, to mention widely used terms or synonyms. However, a common feature for all types of these fractal quantitative measures is that the fractal dimension counts a self-similarity of an object at different spatial scales. In other words, a fractal dimension measures directly geometrical complexity of an object as a whole or additionally can be sensitive to uniformity of spatial distribution existing in a given object. Especially, a fractal dimension can keep information about surface roughness and edges complexity. Obviously, in order to perform proper analysis any information of interest should be collected in a form of an image for further numerical processing.

Regions and karst objects, including caves, usually have complicated spatial structure and possess a self-similarity property enabling treatment of them as fractals. Good examples of karst fractals are: the karst landscape densely dotted by craters and karst depressions, often overlapping each other, the corroded walls in caves covered by micro-forms, the rock massifs cut by nets of karsified fissures, and others structures. The problem is not discussed in details in scientific literature, - there are only a few works devoted to fractal problematics in karst (Curl, 1986; Laverty, 1987; Finnesand, Curl; 2009, Kusumayudha, Notosiswoyo, Gautama; 2000, Skoglund, Lauritzen, 2011, Piccini, 2011).

A specific example of a spatial, genetic, fractal-like organization are maze cave systems created by hypogenic

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speleogenesis. They often form enormous and dense nets of underground channels and corridors. Due to their spatial complexity such systems can be analyzed as fractal objects. Obviously, this fractal character of labyrinthine caves – as specific natural objects – is interesting and this is a novelty in the field. However, the important question is: if such analysis makes sense, which fractal dimension is optimally suitable for that purpose? Authors argue that this type of analysis does makes sense and try to specify one of a possible field of application, namely, for prediction of existence of not discovered yet (not explored) parts of cave networks. This aspect of research has both theoretical importance and practical meaning for speleologists trying to discover new unknown cave regions.

In opinion of authors, the above mentioned goal is optimally fulfilled by the use of capacity and correlation fractal dimensions. These dimensions characterize fractal geometrical complexity of objects and may indicate internal regularities, or level of heterogeneity providing information about a genetic complexity (mono- or multi-factorial origin) manifested itself as a specific spatial realization of mazes. Thus, a capacity dimension enables estimation of the general level of structural complexity, a variety of an object as a whole. The lower is the value, the greater is spatial multiplicity of a given cave (or its part). On the other hand, a correlation dimension additionally senses variations in a cave structural distribution. Additionally, an important source of information can come from a comparison between both fractal dimensions what will be discussed below.

STUDIED OBJECTS – CAVE MAZES

Analytical studies were performed for four maze caves chosen from the set (fig. 1) of largest gypsum caves of the Western Ukraine region (in brackets a total length of passages and corridors in km is given): Optymistychna (188), Ozerna (111), Zoloushka (90), Kryshtaleva (22) (Klimchouk, Andreychouk, Turchinov, 2009). These caves are horizontal maze cave systems developed in hypogenic conditions and represent enormous and dense networks of underground passages and corridors. Area of cave fields (fig. 1) ranges from 0.3 to 2.5 km^2.

All the mentioned caves are located in Western Ukraine (Podilla and Bukovina regions) and developed in Miocenic gypsum layer of 20-25 m thickness. Underground waters penetrated the layer (from below) through the vertical and subvertical fractures causing formation of diverse internal morphological (speleomorphological) structures and their combinations. All the caves have similar (hypogenic) origin and were formed in confined phreatic conditions as a result of underground waters rising across the gypsum bed via dense networks of fissures in gypsum.

Fig. 1. Configurations and relative dimensions of some cave fields of the largest gypsum caves of Western Ukraine (including investigated caves). All contours are pictured at the same scale (after Klimchouk, Andreychouk, Turchinov, 2009).

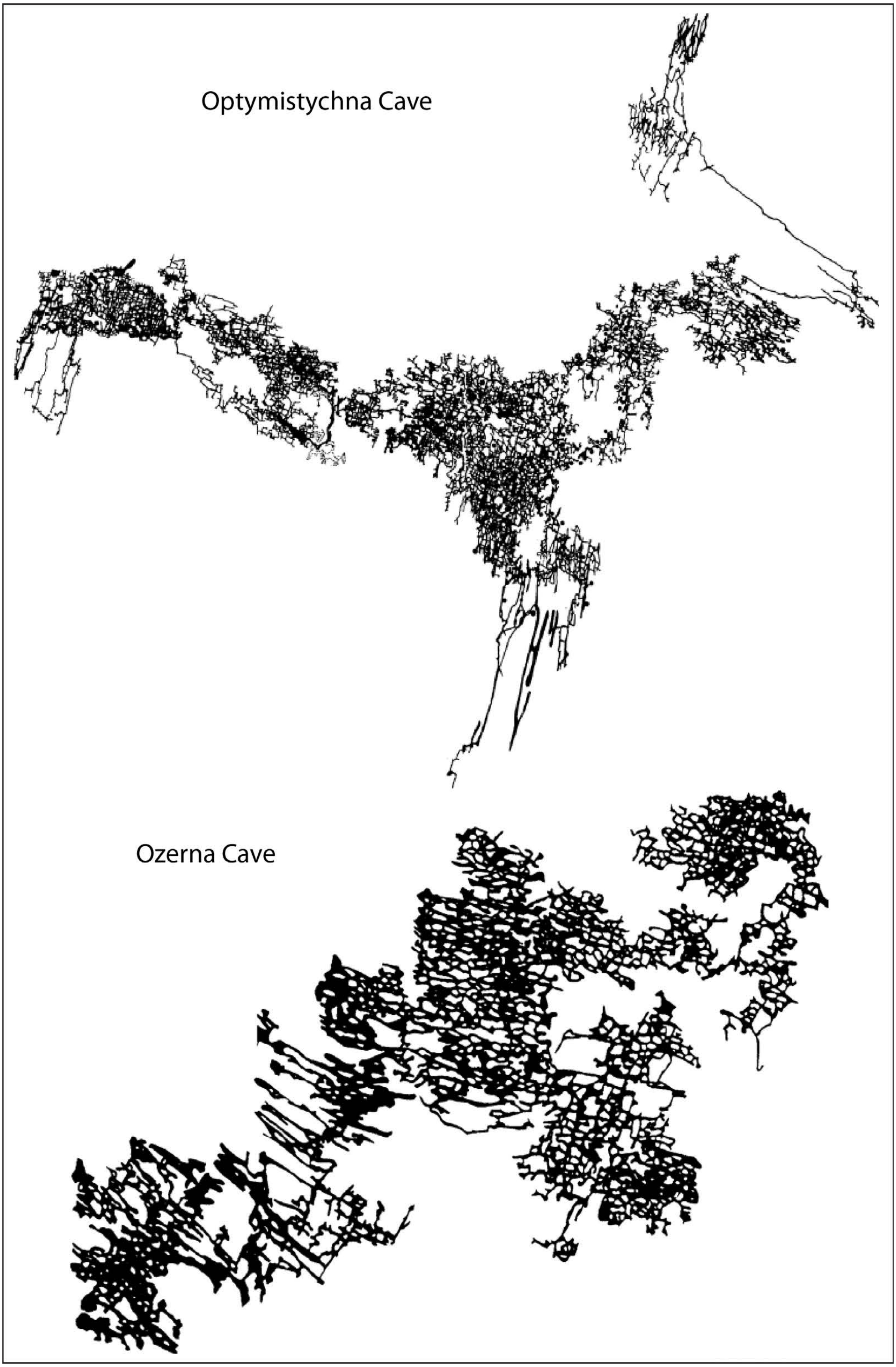


Fig. 2. Maps of caves: Optymistychna and Ozerna (spatial scales are different for the provided cases).

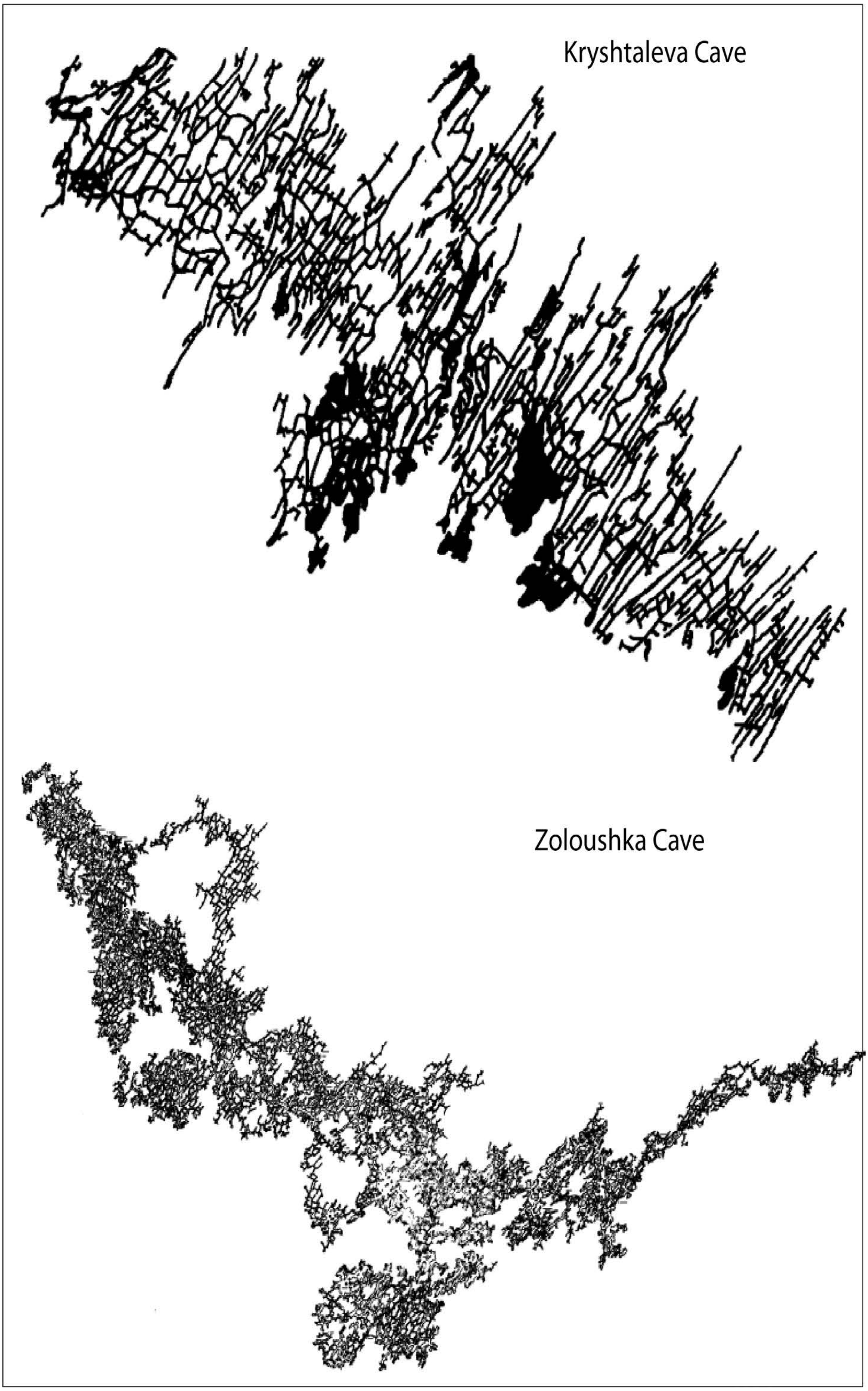


Fig. 3. Maps of caves: Kryshtaleva and Zoloushka (spatial scales are different for the provided cases).

Networks of fissures were evolutionary formed by subsequent overlapping of two main fracturing systems of different origin (lithogenetic and tectonic), resulting in formation of regular geometrical structures (polygons, crosses, etc.). For every cave considered here individual combinations (configurations) of overlapping polygonal (lithogenetic) or systematic (tectonic) networks are well distinguishable. Thus, fissures significantly extended by corrosion (up to dimensions of corridors) are seen at the cave maps (figs. 3 and 4). Also an extreme complexity of networks, as well as some regularities, are easily distinguishable on the maps.

BASIC FACTS AND RESEARCH METHODOLOGY

Fractals dimensions, including capacity and correlation ones, are quantities describing in some situations common figures, like lines, squares, cubes, providing normal integer values of these objects, that is: 1, 2, 3, respectively. A capacity dimension is based on counting of unit-boxes covering an object (Fig. 4 a, b). During this procedure boxes of down-scaled dimension are applied. The log-log dependence between number of boxes covering an object and a box size is linear within some range of variables. A capacity dimension is equal to a slope of that linear dependence. A capacity dimension of a normal figure, like a triangle, equals 2.

A correlation dimension methodology is similar to that of capacity dimension, as it is equal to a slope of linear log-log dependence between a correlation factor and unit-circles radii covering randomly chosen components of an object (Fig. 4 c). If points in a 2-dimensional object, for example in a triangle, are distributed completely randomly, then the correlation dimension equals 2. Importantly, a correlation dimension senses small-scale variations of an image, while a capacity dimension is not sensitive for local irregularities and represent uniquely an image as a whole (Baker, Gollub, 1998; Peitgen, Jürgens, Saupe, 2004).

Every dimension can be calculated from counting procedure of spatial unit objects of a length ε covering the measured object of the length L (Fig. 4 a). If the procedure provides $N(\varepsilon)$ counted squared objects (Fig. 4 b), the capacity dimension can be calculated from the following expression

$$L = N(\varepsilon)\cdot\varepsilon\,, \quad (1)$$

for a single dimensional object, or from the following formula

$$L^{d_{cap}} = N(\varepsilon)\cdot\varepsilon^{d_{cap}}\,, \quad (2)$$

if the capacity dimension d_{cap} is larger than 1. Taking logarithms of Eq. 2 one obtains

$$d_{cap} = \frac{\log(N(\varepsilon))}{\log(L)+\log((1/\varepsilon))}. \quad (3)$$

In practice, the capacity dimension can be derived from a linear log-log dependence between number of boxes $N(\varepsilon)$ and the square size ε, being the fractional part n of the analyzed size L. Thus, the slope of that dependence equals

$$d_{cap} \approx \frac{\log(N(\varepsilon))}{\log((1/\varepsilon))} = \frac{\log(N(\varepsilon))}{\log((n/L))}. \quad (4)$$

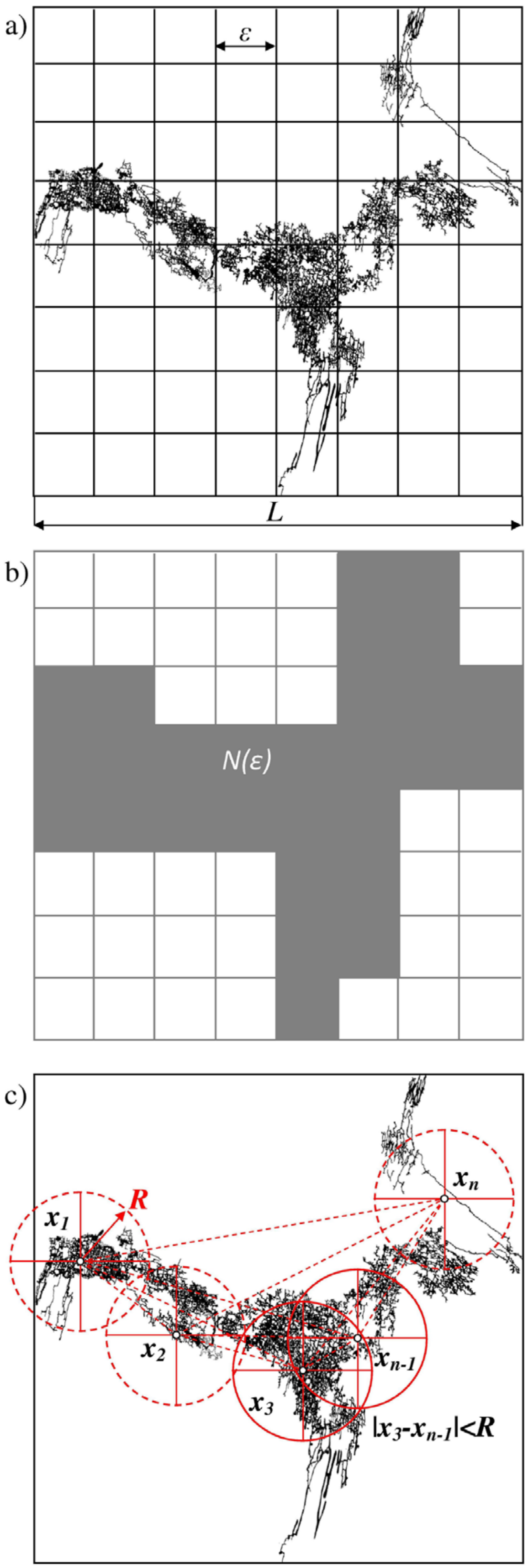


Fig. 4. An explanation of principles leading to capacity (a, b) and correlation (c) fractal dimensions.

Another type of fractal dimension is the correlation one. That type of dimension employs a correlation factor, which counts mutual distances of randomly distributed points, lying on an analyzed object. Every point lies in a center of a circle of radius R (Fig. 4 c). For increasing radius the $C(R)$ factor grows, however for enough large R-values the factor saturates since analyzed region can be completely included and covered by circles. The correlation factor is defined as follows

$$C(R) = \frac{1}{N^2} \sum_{i=1}^{i=N} \sum_{\substack{j=1 \\ j \neq i}}^{j=N} H\left[R - \left|x_i - x_j\right|\right], \quad (5)$$

where N is the number of points, and $H\left[R - \left|x_i - x_j\right|\right]$ is the Heaviside step function

$$H\left[R - \left|x_i - x_j\right|\right] = \begin{cases} 1 \text{ if } \left(R - \left|x_i - x_j\right|\right) > 0 \\ 0 \text{ if } \left(R - \left|x_i - x_j\right|\right) < 0 \end{cases}. \quad (6)$$

Since the correlation factor is proportional to a radius $C(R) = const \cdot R^{d_{cor}}$, via the correlation dimension d_{cor}, then the latter can be calculated from the following expression

$$d_{cor} = \frac{\log(C(R))}{\log(R)} - \frac{\log(const)}{\log(R)} \quad (7)$$

and the dimension can be, in practice, calculated from the following expression

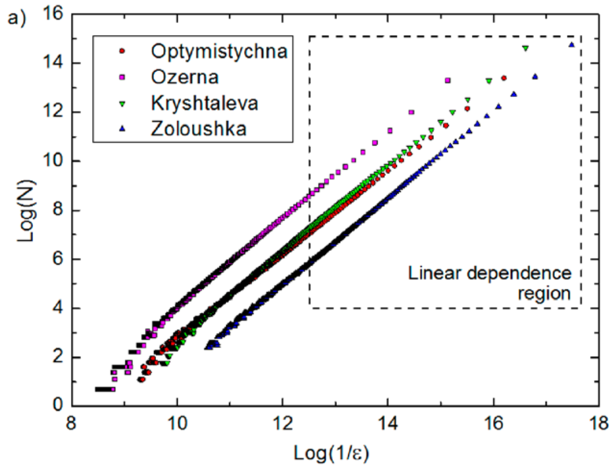
$$d_{cor} \approx \frac{\log(C(R))}{\log(R)}, \quad (8)$$

that is, can be derived from the linear log-log dependence between corresponding values.

INTERPRETATION OF RESULTS AND CONCLUSIONS

Performed image analysis of mono-colored maps enabled calculations of the capacity fractal dimension (Fig. 5) and the correlation fractal dimension (Fig. 6). Final results are given in tables 1 and 2.

The most fractal-like character has Optymistychna cave – the capacity and correlation dimensions are significantly different from other three cases – since calculated values of the capacity dimension, and the correlation dimension are equal to 1.71, and 1.76, respectively, and are the relatively smallest values for the considered caves. That fact shows onto a relatively more complex general geometrical structure. From a geomorphological point of view, it indicates also the significant participation in speleogenesis of both lithogenetic and tectonic factors associated with polygonal and crossing-like fissures systems. Significantly less complicated structure of Zoloushka and other caves indicates onto domination of one genetic factor (lithogenic or tectonic), which made a shape somehow more ordered. The regular features, represented by dominating number



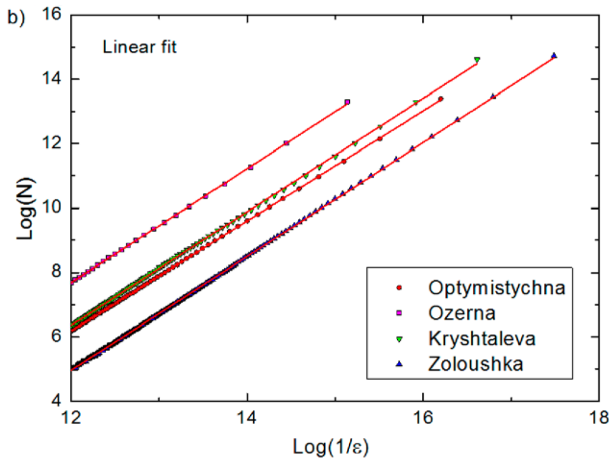


Fig. 5. The dependence between number of boxes covering analyzed pictures of caves and a box dimension (a). The capacity dimensions can be determined from linear fitting to linear dependence regions (b).

of passages, are clearly noticeable in Ozerna cave (in chosen parts) and in Kryshtaleva cave (as a whole).

This conclusion is confirmed by values of correlation dimension, which is sensitive for structure uniformity. Also, what is normal, it is slightly higher than that of a capacity one. From that perspective, the smallest correlation dimension of the Optymistychna cave (1.76) indicates onto larger spatial irregularities in a structure that in the Kryshtaleva cave (1.83), what is clearly visible in provided pictures.

Also, as it was mentioned, the important meaning for a quantitative description has a difference between capacity and correlation dimensions. In general, a larger value of a correlation dimension with respect to a capacity one, thus existence of a difference between these dimensions, is something normal, since it results from mathematical structure of calculations, is natural for most dynamical systems and possesses geometrical origin. However, comparable values, or even equal ones, might suggest that normal rules are somehow deviated, thus it can inform about aberrations from a fractal mechanism characterizing a building structure. In a spatio-structural language this can mean that some parts of cave are not yet discovered or, at

Table 1

Summary of results

Cave	Capacity dimension d_{cap}	Uncertainty of d_{cap}	Correlation dimension d_{corr}	Uncertainty of d_{corr}
Optymistychna	1.71	0.02	1.76	0.03
Ozerna	1.78	0.03	1.79	0.03
Kryshtaleva	1.76	0.03	1.83	0.03
Zoloushka	1.76	0.02	1.80	0.03

Table 2

Spatial scales for pixels in analyzed images

Cave	Pixel size (m)	Picture dimension (pixels)
Optymistychna	2.22	3295 x 2952
Ozerna	2.22	1936 x 1437
Kryshtaleva	0.39	4048 x 2983
Zoloushka	2.22	6263 x 3749

least, not included in graphical charts. Just from this hypothesis results a predictive importance of a comparative analysis of both dimensions. How much it is correct, that will be revealed by future speleological investigations of caves.

Thus, from the presented point of view, Ozerna cave really stands out. Thus, looking onto cave picture, and taking into an account the fact that the correlation and the capacity dimensions are comparable (the difference equals 0.01), it might indicate onto existence of parts not yet discovered, which should complete structural morphology and increase correlation dimension to the higher value of about 0.05-0.07, under the assumption that a difference between both dimensions is a solid rule for caves. A larger difference and smaller «reservoir» for undiscovered parts has Zoloushka (the difference equals 0.02-0.04), next is Optymistychna cave (0.03-0.05), and finally the smallest possibility for undiscovered part might reveal Kryshtaleva cave (0.04-0.07).

As a curiosity of described caves we would like to present a hypothetical cave, with no internal structure, possessing a single compact volume, derived graphically from Ozerna cave (Fig. 7.). For this case, both the capacity and correlation dimensions are now grater, more closer to the numerical value of 2, and both the dimension are equal within the obtained accuracy of calculations.

All analyzed caves can be treated like fractals and their capacity and correlation fractal dimension were calculated. It is a hope of authors that presented calculations of fractal dimensions provided a lot of information, which interpreted from this methodology perspective, would support future speleomorphologic and speleogenetic investigations.

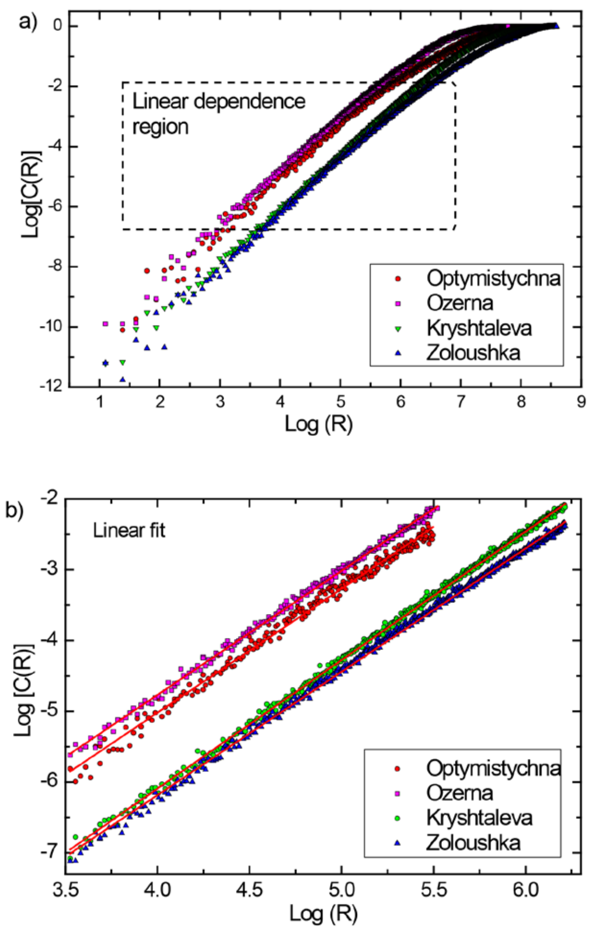


Fig. 6. The dependence between correlation factor and radii of circles associated with randomly distributed points representing caves (a). The correlation dimensions can be determined from fitting using linear regression (b).

Fig. 7. Hypothetical cave derived from Ozerna Cave. Its fractal dimensions are equal to 1.84+/-0.02, and 1.84+/-0.02, for the capacity dimension and correlation dimension, respectively.

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