

переподготовке, достижении большей сбалансированности между спросом и предложением рабочей силы. В их задачи входит также установление контакта между работодателем и работниками, координация деятельности разных органов, в определенной мере связанных с реализацией политики занятости и деятельности на рынке труда.

Что касается государственного управления, то здесь предлагаются следующие разновидности организационных сетей: государственные сети, объединяющие разные государственные учреждения, их руководителей и специалистов по профессиональным и социальным принципам; общественные сети, которые включают тех, кто обслуживает систему государственного управления (ученых, экспертов); международные сети, которые быстро развиваются в условиях глобализации и создания мирового информационного пространства.

Практика формирования «рабочих сетей» (network) знаменует переход к новым организационным структурам и изменяет взаимоотношения между государством и населением.

#### Выводы.

Значительно важную перспективу в Украинских условиях могут иметь мероприятия, направленные на комплексное регулирование спроса и предложения труда. Центральное место среди них занимает рациональное использование гибких форм занятости. Появились новые категории работников. Которые традиционно работают в определенной фирме, но заняты неполный рабочий день, так как работают одновременно на разных работодателей. Этот процесс неоднозначный, поэтому необходимо проанализировать гибкие формы занятости в Украине и отработать адекватные мероприятия регулирования этой сферы.

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## **PHASE TRAJECTORIES MAPPINGS IN DISSIPATIVE SYSTEM IDENTIFICATION**

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### **INTRODUCTION**

Most of mechanical systems display non-linear characteristics provided certain parameters of outer excitation. Non-linearity is a common feature shared by mechanical systems, whereas their linear behaviour is an exception. One of the primary sources of non-linearity in mechanical system dynamics is non-linearity of dissipative characteristics. In fact, energy dissipation is the least understood aspect in mechanical systems. Models of viscous friction give but only a rough presentation of actual physics and their application is often motivated by convenience of subsequent calculations. Adequate description of energy correlations in dissipative systems has been made possible only by nonlinear models. Dry friction, sliding friction, hysteresis friction and aerodynamic resistance are the most prominent examples of non-linearity of dissipative characteristics.

The current methods of determination of dissipation characteristics in a material subjected to periodic deformation are based either on immediate measuring of energy dissipated in a sample – those are known as direct methods (e.g., the strain energy method, the thermal method, the hysteresis loop method), or on the determination of relative characteristics of energy dissipation – those are known as indirect methods (the method of damped vibrations, to the resonance curve method and the phase method). While analyzing the above mentioned methods and their variations, one should note that any one of them makes it possible to determine only average characteristics; none of these methods rules out the effects of dry friction on quantitative characteristics, and none of them possesses the versatile applicability for any mechanical systems being studied.

### **1. PHASE TRAJECTORIES OF OSCILLATIONS OF NONLINEAR SYSTEMS IN THE EXPANDED PHASE SPACE**

Dynamic behaviour of mechanical systems is usually presented as oscillating processes in various graphic forms such as time processes, the Lissajous patterns and hodograph. Such patterns of presentations enable to determine the type of a process and to perform numerical estimations of its characteristics, but do not disclose any properties of the governing system. Unlike them classic phase trajectories have the row of advantages.

A phase space in classic mechanics is represented as a multidimensional space. The number of measured values for a phase space is equal to the doubled number of degrees of freedom of the system being investigated [1]. The geometric presentation of a single phase trajectory or a set of trajectories allows coming to important conclusions about the oscillation characteristics. It is, foremost, true with the oscillations, which are described with nonlinear differential equations.

As it has been shown by the investigations of several authors [4, 5], the expansion of a phase space by taking into account the phase planes  $(y, \dot{y})$  and  $(\dot{y}, \ddot{y})$

substantially promotes the efficiency in analyzing a dynamic system behaviour. Hereby, we pass on to a three-dimensional phase space confined with three co-ordinate axes, i.e. displacement, velocity and acceleration. An interest taken into accelerations in dynamic systems is conditioned by the fact that these accelerations are more sensitive to high-frequency components in oscillating processes. It is precisely diagram  $\ddot{y}(y)$  that enables to define the type and the level of non-linearity in a system. The geometric presentation of an individual phase trajectory or of a family of trajectories allows coming to important conclusions about the properties of a model of the system being studied. Incorporation of phase trajectories on planes  $(y, \dot{y})$  and  $(\dot{y}, \ddot{y})$  enhances the capabilities of classic methods of the qualitative theory due to their extension onto the class of inverse problems of dynamics.

## 2. IDENTIFICATION OF DYNAMIC MODELS OF MECHANICAL SYSTEMS

In the past two decades, the issues of construction of mathematical models and prediction of dynamic behaviour of structural elements proceeding from recorded experimental data have attracted considerable interest.

In spite of intensive investigations into the above mentioned matter, which have been undertaken in the scientific centers in different countries (supported by numerous publications on theoretical research and experiments, a number of specialized conferences [8, 9, 11]) as well as the important results obtained, there is no, so far, the only universal effective approach, which would allow for correct determination, prediction and analysis of dynamic properties in construction elements. Most of the methods of structural identification are based on the use of special types of outer excitation for a wide range of frequencies, such as symmetric monoharmonic excitation and rectangular impulse. These types of excitation are often unrealizable in mechanical systems. The methods based on the Fourie transformation do not allow classifying and localizing non-linearity [6, 7] and are inapplicable to investigating stochastic processes [3, 8, 9, 11]. It should be also noted that the application of Winer series and Hilbert transformation for identification of non-smooth non-linear dynamic characteristics is unjustified. [9, 11].

## 3. DIFFERENTIAL EQUATION OF FORCED VIBRATIONS OF A SYSTEM WITH STRUCTURAL FRICTION

Let's consider forced vibrations of a mechanical system with structural friction. We studied system are described by a differential equation of a view

$$m \ddot{y} + h(y, \dot{y}) + r(y) = 0. \quad (1)$$

where  $m$  is mass;  $y, \dot{y}, \ddot{y}$  are generalized co-ordinates, velocity and acceleration;  $r(y)$  is elastic force;  $h(y, \dot{y})$  - force of structural friction.

One of the most widespread models of structural friction forces having piecewise linear relation to displacement were analyzed. The graphs of a time processes of forces of structural are adduced for the most widespread models of structural are given in [5].

Let's note, that the phase space of dynamic systems is multi-dimensional. Diverse selection of parameters of phase planes is also possible. For the first time attempt to apply phase trajectories on planes  $(y, \dot{y})$  and  $(\dot{y}, \ddot{y})$  to investigation of dynamic systems was made in the monograph [4]. As follows from the obtained results, the phase trajectories on a plane  $(y, \dot{y})$  can be rather effectively utilized for identification of dynamic systems. In the monograph [4] the results of qualitative investigation of oscillations of conservative systems having the non-linear dissipative and elastic characteristics of different types are shown.

The most simplest model of structural friction is Coulomb model  $h(y, \dot{y}) = H_1 \text{sign} \dot{y}$ . Most prime model of dry friction. It was taken up in the analysis of dynamic behavior of structures under seismic loading. The relation  $h(y, \dot{y}) = H_1(1 + y/y_c) \text{sign} \dot{y}$  is used for the mechanical systems with initial static displacement. So the given model describes behavior of sheet spring plates and ground basis.

## 4. ANALYSIS OF THE OBTAINED RESULTS

The correlation of three models of the dissipative characteristics was performed: 1) viscous friction model  $h(y, \dot{y}) = H_1 \dot{y}$ , 2) Coulomb model  $h(y, \dot{y}) = H_1 \text{sign} \dot{y}$  and 3) structural friction  $h(y, \dot{y}) = H_1(1 + y/y_c) \text{sign} \dot{y}$ .

The outer asymmetrical periodic excitation of the following form was applied to a system  $P(t) = P_1 \cos \omega t$ .

The numerical modeling had been executed for the following parameters of a system (1):  $m = 1$ ;  $H_1 = 1$ ;  $y_c = 0.5$ . Amplitudes of harmonic excitement had taken the values  $P_1 = 0.1; 0.5; 1$ . The results of numerical modelling are also shown in Figs. 1.

As follows, from given relations the presence of structural friction not only results in amplitudes of resonance oscillations decreasing, but also to a considerable decrease of resonance frequencies. So, system with viscous friction has the resonance frequency  $\omega_{r,1} = 0.69 \text{ rad/s}$ , and system with Coulomb friction -  $\omega_{r,2} = 0.12 \text{ rad/s}$ ; for a system with structural friction  $\omega_{r,3} = 0.16 \text{ rad/s}$ .

In contrast to systems with viscous friction, the time processes  $\ddot{y}(t)$  of both systems it is possible to note instantaneous decreasing of acceleration in points of their maximum values. However, value of jumps for positive and negative values of accelerations equal and constants for systems with Coulomb friction. While for systems with structural friction, values of jumps a different. It depends, both on the sign of acceleration  $\ddot{y}(t)$ , and value of displacement  $y(t)$ .

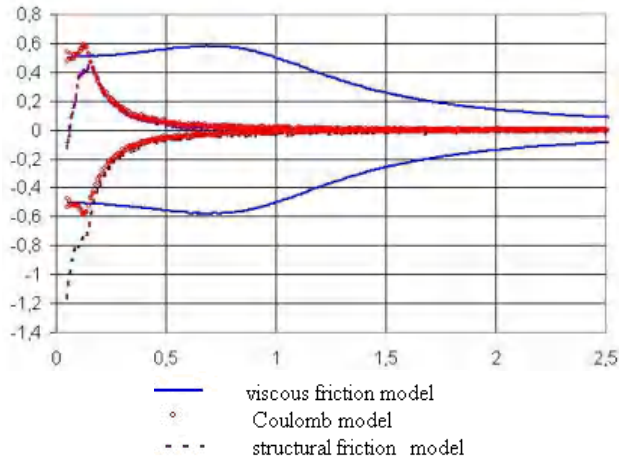


Figure 1. Amplitude-frequencies characteristics of dissipative systems

The hysteretic loops on a phase plane  $(y, \dot{y})$  for systems with Coulomb friction back are symmetric concerning origin of the coordinate. It changes length with a constant step, distance between their orbits are constant. The phase trajectories on a plane  $(\dot{y}, \ddot{y})$  represent members of elliptical arcs. It displaced from each other in the left and right half-planes. The phase trajectories on a plane  $(y, \ddot{y})$  for systems with structural friction are nonsymmetrical concerning an axis  $y$ .

### 5. DISCRETE MAPPINGS OF PHASE TRAJECTORIES IN THE EXPANDED PHASE SPACE

Let us consider a mechanical system described by the following differential equation:

$$m\ddot{y} + H(y, \dot{y}) + R(y) = 0, \quad (2)$$

where  $m$  is mass; functions  $H(y, \dot{y})$  and  $R(y)$  describe dissipative and elastic force, respectively.

In order to obtain information about the structure of forces  $H(y, \dot{y})$  and  $R(y)$ , let us apply outer periodic excitation to the system (1). Thus, we investigate the system

$$m\ddot{y} + H(y, \dot{y}) + R(y) = P(t), P(t+T) = P(t). \quad (3)$$

The qualitative analysis of  $T$ -periodic oscillations of the system (1) is based on studying Poincare trajectories on phase plane  $(y, \dot{y})$ . It consists in studying distinctive trajectories (equilibrium conditions, limit cycles, separatrix) and their

stability on plane  $(y, \dot{y})$  or so-called the phase plane. Acceleration of system  $\ddot{y}$  at every moment of time  $t$  is uniquely determined by displacement  $y$  and velocity  $\dot{y}$  according to equation (3).

#### a. Testing for non-linearity

We will assume that functions describing dissipative and elastic forces are unknown to us. The first question consists of that, to set, the system is linear or not.

Let us denote a sequence of points by  $\{\Pi_k\} = \{y_k, \dot{y}_k, \ddot{y}_k\}$ ,  $k = 1, \dots, n$ , describing the measured displacement, velocity and acceleration in the system (4) at the discrete moments of time  $t = t_k = t_0 + kT$ , where  $T$  is a period of the outer excitation. When we represent these points in the extended phase space  $(y, \dot{y}, \ddot{y})$  we obtain a set of points parameterized by time  $t_k$ . Ideally, a measurement error is nill, then

$$m\ddot{\bar{y}}_k + H(\bar{y}_k, \dot{\bar{y}}_k) + R(\bar{y}_k) = c \text{ for } k = 1, \dots, n, \quad (4)$$

where  $c = F(t_0) = F(t_k)$  is a constant value for all values of  $k$ . That means that all points are located on the surface, which can be described by equation  $m w + h(u, v) + r(u) = 0$  in  $(u, v, w)$ -space. If functions  $H(y, \dot{y})$  and  $R(y)$ , which describe dissipative and elastic force characteristics in the mechanical system, are linear, then the surface in the expanded phase space transforms into a plane, i.e. all points of sequence  $\Pi_k$  are to lie on plane  $E$ . Then, there are two real numbers,  $a_1$  and  $a_2$ , and with these numbers all points of sequence  $\Pi_k$  have to satisfy the condition

$$m\ddot{\bar{y}}_k + a_1\dot{\bar{y}}_k + a_2\bar{y}_k = c \text{ for } k = 1, \dots, n, \quad (5)$$

which is the evidence of the system linearity. Let us change the amplitude of governing force  $F(t)$  for  $a_3 F(t)$ , where  $a_3 > 0$  is the real positive number. Then, if the system under investigation is a linear one, the relevant multitude of results for measured value  $\Pi_k^{(a_3)}$  meets the condition  $\Pi_k^{(a_3)} = a_3 \Pi_k$ , that is the second evidence of linear of the system. If there are constants  $a_1$  and  $a_2$ , such that all measured points lie on the plane determined by the values of  $a_1$ ,  $a_2$  and  $c$  or in the vicinity of that plane, we can make a conclusion that the system (4) is linear or weak non-linear. In the case of linearity of functions  $H(y, \dot{y})$  and  $R(y)$ , which describe dissipative and elastic force characteristics in the mechanical system, all points of projections of sequence  $\{\Pi_k\}$  along plane  $E$  on planes  $(\bar{y}, \dot{\bar{y}})$ ,  $(\ddot{\bar{y}}, \bar{y})$  and  $(\ddot{\bar{y}}, \dot{\bar{y}})$  are located on a straight line.

#### b. Phase Trajectories Mappings of Dissipative System

A dynamic system described by the following non-linear 2-nd order differential equation was considered as an example:

$$m\ddot{y} + H_1 \left( 1 + \frac{y}{y_c} \right) \text{sign } \dot{y} + r(y) = 0. \quad (6)$$

In order to check the theoretical assumptions suggested we shall analyse a normalized dimensionless equation. The numerical modelling had been executed for the following parameters of a system (6):  $m=1$ ;  $H_1=1$ ;  $y_s=0.5$ . The outer asymmetrical periodic excitation of the following form was applied to a system:  $P(t) = P_1 \cos \omega t + P_0$ . The frequency of an outer excitation had taken the following values  $\omega = 0.5; 1; 2$ . Amplitudes of harmonic and constant component had taken the values  $P_1 = 0.1; 0.5; 1$  and  $P_0 = -0.1; 0.1; 1$  respectively. The elastic characteristic was assumed to be linear  $r(y) = y$ . The results of numerical modelling are also shown in Fig. 2. On a Fig.2 point were obtained for fixed values of amplitude of an external disturbance and three miscellaneous values of frequencies.

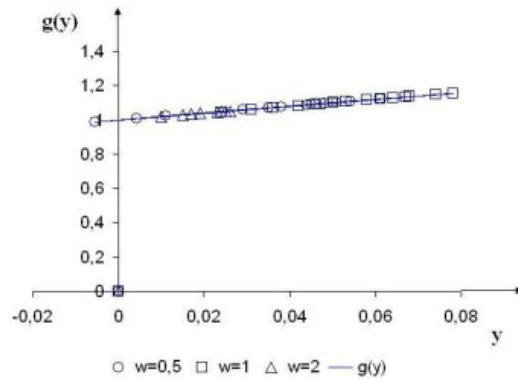


Figure 4. Influence of frequency of an outer excitation on accuracy of an estimation of parametric relation  $g(y)$

Analysing the presented relations, one may mark a good coincidence of numerical and analytical estimations of the function  $g(y)$ . Estimations of the function  $g(y)$  for negative displacement values are preferred to vary by values of constant component of an outer excitation. Let's note, that unlike conventional methods based upon the usage of method of least squares and Wiener series, the change of parameters of an outer excitation does not affect the accuracy of an estimation of dissipation parameters.

## 6. CONCLUSION

The application of phase trajectories in plane  $(y, \ddot{y})$  is suggested by the author. In contrast to trajectories in plane  $(y, \dot{y})$ , they don't require a lot of number of geometrical construction in identification of dynamic systems. It leads to the improvement of accuracy. The graphical criteria of non-linearity of dynamic system are developed. The preferable regimes of outer excitation to define separately structural damping force are obtained.

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### АНАЛІЗ НАПРУЖЕНО-ДЕФОРМОВАНОГО СТАНУ ЦЕГЛЯНОЇ КЛАДКИ З УРАХУВАННЯМ ПОШКОДЖЕНЬ ТА СУМІСНОЇ РОБОТИ НЕСУЧИХ СТІН БУДІВЛІ З ОСНОВОЮ

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**Постановка проблеми.** Контроль технічного стану несучих будівельних конструкцій при обстеженні будівель та споруд набуває все більшого значення зі збільшенням зношення основних житлових та виробничих фондів, що були збудовані у 60-70 роки ХХ сторіччя – у період масового будівництва.