

**UDC 624**

**FINITE ELEMENT MODELLING OF BRIDGE STRUCTURES BASED ON  
NON-DESTRUCTIVE AND NON-CONTACT ASSESSMENT METHODS**

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**Introduction**

Mechanical and physical properties and characteristics of structures (e.g., bridges) including strength of the materials used, details of connections and joints, rebar configurations and utility details are usually available through structural design engineers and/or bridge owners in the form of drawings and construction plans. Availability of information is vital when the performance and behaviour of the structures require scrutiny. No doubt, in the absence of such vital information, assessment of the behaviour of such structures will be extremely challenging if not impossible. Applications of non-destructive and non-contact measurement methods and assessment of structures in establishing the required design and construction information is of significant importance to structural engineers. Research by Gucunski et al. (2010), Parmar and Sharp (2009) and Lanbo and Tieshuan (2005) are just a few examples of the vast literature in existence on non-destructive health assessment methods for bridges, such as Ground Penetrating Radar (GPR), Acoustic Emission and Tomographic Imaging methods. Applications of Finite Element (FE) methods have also been proved to be viable in simulating and analysing the behaviour of bridges within the context of health assessment of structures. Roberts et al. (2003) and Meng et al. (2003) used GPR to provide reliable FE models to predict the behaviour of bridges. Teughels and De Roeck (2005) presented an FE model updating the approach for damage detection and parameter identification.

A combination of applications of non-destructive testing methods, non-contact measurement techniques and the FE method are reported in this paper. The main objective of this work was to establish the perceived structural defects, structural details and the dynamic response of a bridge structure under different loading conditions. This manuscript reports a specific part of a bigger project which aims to provide a method to assess the reliability of bridges for which the fundamental mechanical properties are missing. In the case considered, this paper reports the method for evaluation of the unknown mechanical properties. The health assessment of the bridge will then be presented in a separate manuscript.

The approach provided here is innovative for reinforced concrete bridges, although a similar approach was applied before to assess an old masonry arch bridge. (Lubowiecka et al., 2009) Conventional methods of bridge assessment within the context of health monitoring of infrastructures, such as a condition based survey using visualisation techniques, which is used frequently by industry, have their limitations in ascertaining the full extent of inflicted damage to structures, (Helmerich et al., 2012). Applications of conventional techniques have proved to be

more challenging in the absence of structural design details and drawings (this is often observed in the case of older structures), (Thakkar et al., 2006). It is also widely appreciated that the complexities of bridge structures demands a more rigorous and thorough approach when the long term integrity and functionality of these structures is required, (Lam et al., 2007).

Various methods for measuring the response of a structure have become widely used for the effective restoration and conservation of bridge structures, (Diamanti et al., 2008 and Lynch et al., 2004). Also a number of non-destructive evaluation (NDE) methods have been developed but few have been integrated for bridge health assessment purposes, (Scott et al., 2003, Rens et al. 1997 and White et al. 1992).

The Image by Interferometric Survey of Structures (IBIS-S) system is an advanced non-contact radar based sensor system with interferometric capabilities. When used appropriately, the data provided on displacements and natural frequencies due to the vibration caused by any static or dynamic loading will be of high sensitivity and accuracy, (Bernardini et al., 2007, Wilkins et al., 2003 and Pieraccini et al., 2008).

Accelerometers are sensors used in structural health monitoring that measure structural vibration at specific positions. Choosing the proper type of accelerometer sensors depends on the type of structure monitored, range of measurement and the resolution needed.

The methodology used in this investigation consists of three steps. Firstly, the response of the structure is monitored by non-contact and non-destructive methods. Second step is to generate the FE models. Lastly the FE models are properly correlated with the field data to find the unknown mechanical properties of the structure. As alluded to earlier, the application of non-contact measurement methods in assessing structures such as bridges within the context of the health monitoring of structures is not a new concept. Significant advancements have been made by design engineers and practitioners in this field in recent years. The most important advancements probably are: improvements in surveying and monitoring techniques as well as in equipment and data processing software. However, it is fair to say that there is a noticeable lack of an established integrated approach that utilises the results of non-contact measurement method applications to develop a reliable numerical model for simulating the structural behaviour of, for example, bridge structures under different loading conditions (static and dynamic). The development of such models is essential in assessing the reliability and integrity of bridge structures and their function in the future. The details of the structure studied in this research are presented here.

### **The Pentagon Road Bridge**

The Pentagon Road Bridge (Figure 1) is located in Chatham Town Centre, Kent and carries an access road from Rope Walk to the Pentagon Shopping Centre, with a four span simply supported concrete deck of beam and slab construction. This concrete bridge was constructed in 1975. At the north end, support is via a leaf pier that is shared with the access road. At the south end, the bridge links the access road to the rooftop car park of the shopping centre.



Figure 1. General view of the Pentagon Road Bridge

Figure 2 illustrates the exact location of the bridge. Unfortunately, no data in terms of the mechanical behaviour of the bridge material is currently available. For example, the much needed structural parameters for analytical purposes, such as the compressive strength, tensile strength and the modulus of elasticity of the bridge material (concrete), are unavailable. Also, because of the lack of a proper maintenance scheme, there is reasonable doubt as to the integrity of the joints (whether the supports connecting the decks to the abutments and piers are performing as the original design anticipated).

There are a number of known defects on the bridge, including the visibility of rebar in some places due to concrete deterioration.



Figure 2. Location of the Pentagon Road Bridge (SE UK)

The Pentagon Road Bridge consists of 4 spans, standing on abutments at the extreme ends of the bridge, and 3 piers as mid supports, as illustrated in Figure 3.

Starting from the south end of the bridge, the bridge decks in the first and second span, with a depth of 110 cm, and span lengths equal to 26.6 and 16.2 m respectively, do not show any visible structural deterioration or problems. Deck thickness in the 10.5 m long third span is decreased to 50 cm. The slab depth and span length of the fourth span are 30 cm and 13.4 m, respectively. The fourth span is supported by a grid of beams. At the south end of this span, a cantilever structure is supported by the supporting beams. The third and fourth spans are in very poor condition and require an assessment of their adequacy prior to any remedial works being carried out. All of the bearings to the main spans that could be seen are in poor

condition. Apparent structural deterioration and cracks as well as hidden flaws and imperfections identified using the GPR device make investigation crucial. GPR is applied both to recognise the position and density of reinforcements and to locate zones with significant concrete degradation.

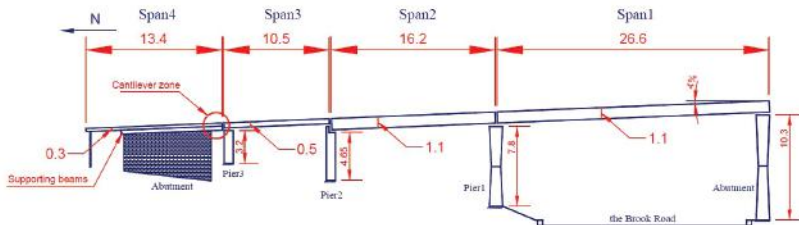


Figure 3. Model depiction of the Pentagon Road Bridge (Dimensions in metre)

Previously, a visual inspection had been carried out in 2009 by Jacobs Engineering U.K. Limited. Their report identified faults and made recommendations for the bridge, including relating to rebar visibility in some places due to concrete deterioration. Some of the points in the report are presented here to reflect the necessity of the health assessment studies provided. “Concrete repairs are required to the North End Leaf Pier Support, Piers 1 and 2 and all of the deck spans. The transverse beam at the South end of Span 4 is in very poor condition and required an assessment of its adequacy prior to any remedial works being carried out. The surfacing on the deck is nearing the end of its life with areas of reinstatement, some in poor condition, potholes and fretting. All of the bearings to the main spans that could be seen were in a poor condition. The fixed bearing at the east of the north deck of Span 1 has a broken bottom plate/shell and needs to be replaced. Report recommended that all of the bearings need replacements. The transverse joints and the longitudinal joint have faults that indicate that there is leakage at each. The joints should be replaced. The parapets are damaged with areas of corrosion and require repair and painting.”

### IBIS-S Monitoring Procedure and Results

IBIS-S is an advanced non-contact measurement technique that provides measurements of static or dynamic responses of a structure from several points, with a displacement sensitivity of 0.01mm. It was developed by the Department of Electronics and Telecommunication of Florence University and the Ingegneria Dei Sistemi Company, (Gentile and Bernardini, 2008).

The IBIS-S radar system comprises of a sensor module, a control laptop and a power supply unit. The sensor module with two horn antennas (Figure 4 – top right) on a rotating head tripod (Figure 4 – left) sends and receives the electromagnetic waves. The sensors connect to a laptop (Figure 4 – middle right) and power supply unit (Figure 4 – bottom right). The laptop configures the acquisition parameters, processing the data and storing the acquired signals via dedicated software, (Gentile and Bernardini, 2008 and Bernardini et al., 2007).

IBIS-S produces a one dimensional image of the structure being surveyed and the separation between measurement points is achieved by range only. For this reason, IBIS-S is installed at one end of a structure looking along its length.



*Figure 4. View of IBIS-S system*

The principal advantage of using IBIS-S monitoring technology is the remote sensing of a large number of points without the need to access the structure. But conversely, IBIS-S is not mobile to monitor a structure. Moreover, the IBIS-S monitoring system is also suitable to provide practically permanent monitoring of static and dynamic responses.

IBIS-S is more effective with reflectors on the deck of bridges. It transmits and receives electromagnetic waves to evaluate the natural frequencies of the structure and the displacements at reflective points, (Gentile and Bernardini, 2008). Most of the structures have enough natural reflection to make it possible to perform a measurement without the installation of corner reflectors (Figure 5); however, corner reflectors do aid greatly in the acquisition of more precise data in the IBIS-S method.

IBIS-S is installed first at position A (beside the south end abutment) and then at position B (beneath the north end of the bridge), looking along the length of the structure. 15 corner reflectors were installed at approximately 4 metre intervals along one side as illustrated in Figure 6 as schematic view of the reflector alignment.



Figure 5. Corner reflectors

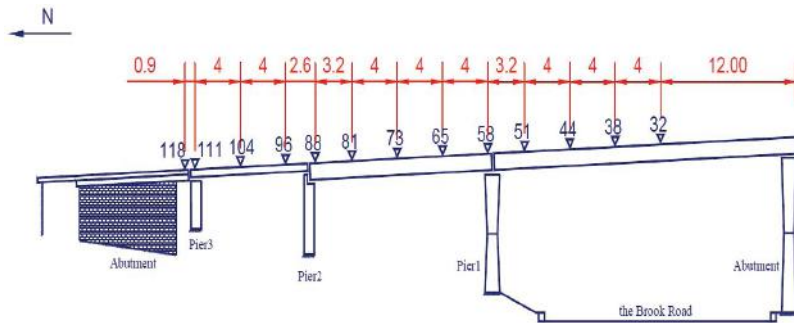


Figure 6. Position of IBIS-S Reflectors (Dimensions in metre)

The settings presented in Table 1 were used for data processing/

The IBIS-S test was carried out on 7th November 2010. The weather was dry and sunny and the temperature was 5°C. A 180kN vehicle was driven across the bridge four times during each acquisition to excite the structure as shown in Figure 7.



Settings for IBIS-S data processing

Item	Acquisition Frequency	Range Resolution	Structure Length	IBIS position height from bridge deck	Antenna Tilt	Interpolation Factor
Position A	200Hz	0.5m	60m	-11m	30 degrees	1
Position B	200Hz	0.5m	60m	-2m	20 degrees	1



Figure 7. The lorry passing over the bridge, with data acquisition at position A

In the graph below the four passes of the 180kN vehicle are clearly visible in the deflection time history at midpoint of span 1 while the IBIS-S is at position A as an example.

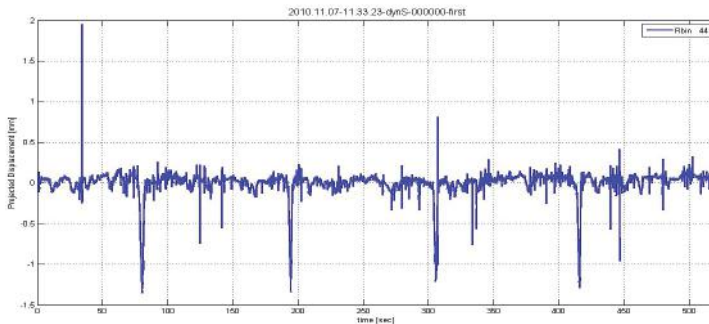


Figure 8. Deflection time history diagram at midpoint of span 1 while the IBIS-S is at position A

### Accelerometer Results and Validation of IBIS-S data

The accelerometer test was carried out on 11th March 2012 in parallel with the second round of IBIS-S monitoring survey. The weather was dry and sunny and the temperature was 16°C. Monitoring was carried out with the full support of the Medway Council's Highway health and safety division (Medway Council) and their subcontractors. It was necessary to introduce a traffic control system on the day as the bridge is heavily used by members of the public. Safe working practice was followed during the survey. Full access to the bridge was required, from time to time, during the survey. The aim of this study as discussed previously was to obtain results which could be compared and validated against the IBIS-S monitoring data. Figure 9 illustrates an IBIS-S reflector, a wireless sensor node and an accelerometer sensor.

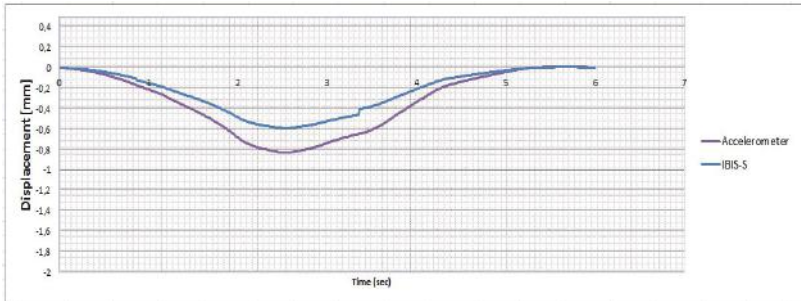


*Figure 9. An IBIS-S Reflector, a Wireless Sensor Node and an Accelerometer Sensor*

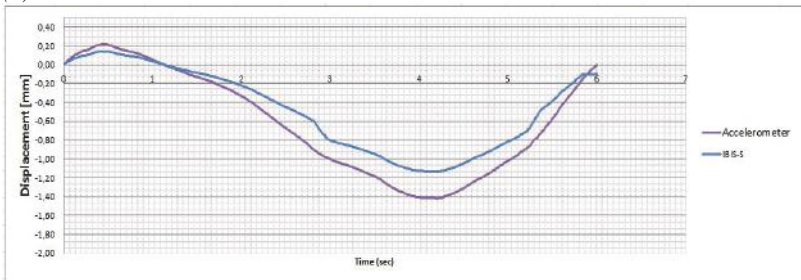
Before survey the accelerometers were calibrated. The results from the accelerometer consist of readings for the acceleration of the bridge on its vertical axis with a data acquisition frequency of 200Hz. These results were transformed into displacements. Initial displacements and velocities were set to zero, as no traffic load was applied to the bridge for a while before each trip of the test started.

To verify the accuracy of IBIS-S results, the time history of displacements at every reflector is compared with the data from the accelerometer monitoring procedure. Figure 10 illustrates a sample comparison between the results obtained from accelerometer nodes and IBIS-S reflectors at nodes 65 and 104 in Figure 6, respectively.





(a) Node 65



(b) Node 104

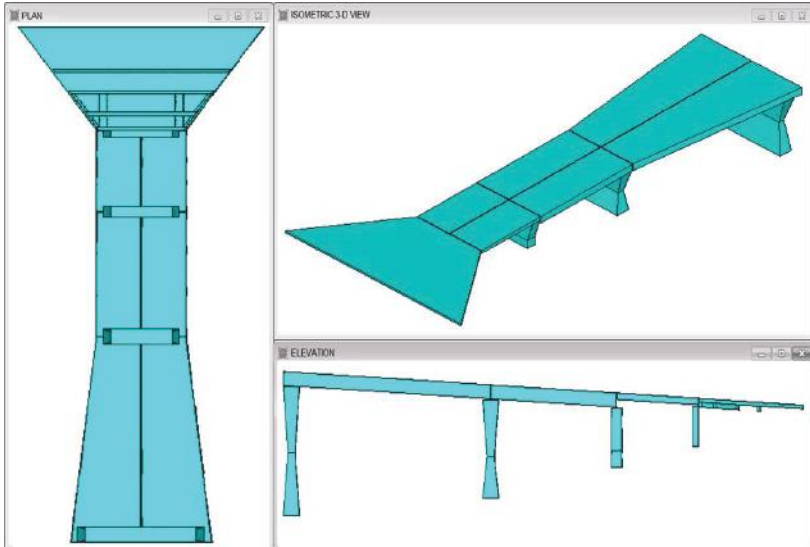
Figure 10. Comparison between Accelerometer and IBIS-S monitored data

The deflection time history pattern at every node monitored by the two methods is completely similar and the deflections monitored reveal a maximum relative difference of 23% between the IBIS-S and accelerometer results. Such an acceptable correlation between the monitored data from accelerometer network and IBIS-S survey validates the results for both.

### Finite Element Models and the Inverse Approach

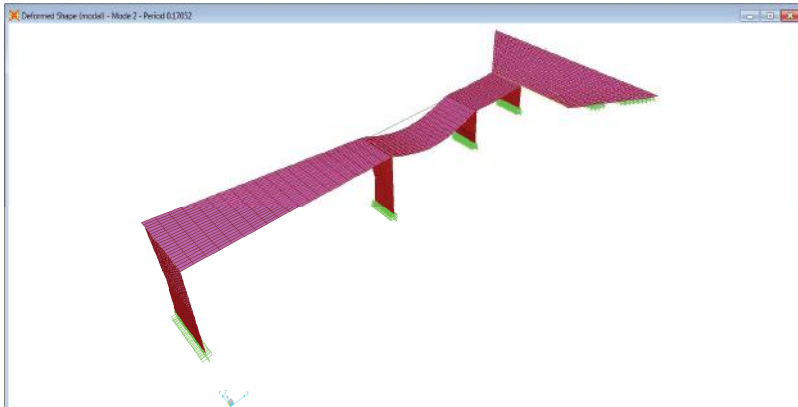
SAP2000, structural analysis program, was used to model the structure which is capable of modelling 3-D structures under moving loads (dynamic loading). The elements used in the model consisted of thick shell and beam elements. The shell elements were used to model the bridge decks, piers and abutment and the beam elements were utilised to represent the supporting beams in the last span of the bridge.

Figure 11 demonstrates elevation, plan and isometric 3-D views of the model. The geometry of the bridge is introduced to SAP2000 software as precisely as possible. However, at this stage, the material properties were input as accurate estimates (what is perceived in normal design procedures).

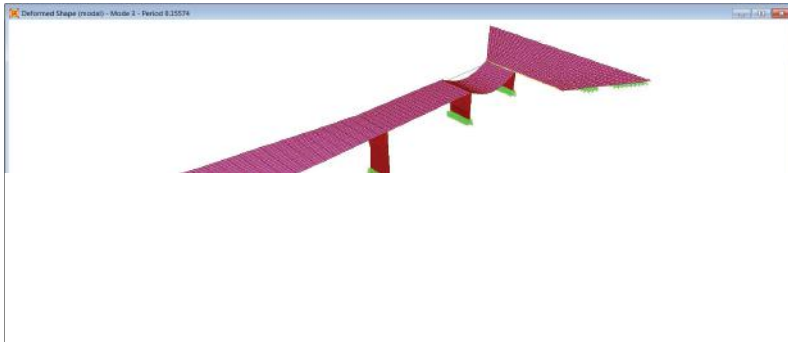


*Figure 11. Plan, elevation and 3-D view of the SAP model*

To verify the validation of the developed FE model, the natural frequencies of spans 2 and 3 obtained from SAP2000 were compared with those calculated from the structural dynamics equation, (Chopra, 1995). Figure 12 illustrates the periods of the two above mentioned spans.



(a) *Span 2*



(b) Span 3

Figure 12. Periods of two spans calculated by SAP 2000

The natural frequencies of a simply supported beam can be calculated using formula (1).

$$f = \frac{K}{2\pi L^2} \sqrt{\frac{EI}{m}} \tag{1}$$

Where:

$f$ : Natural frequency

$K$ : A constant dependant on the mode shape (equal to 9.87 for the first mode)

$E$ : Modulus of Elasticity

$I$ : Second moment of inertia of beam cross section

$m$ : Mass per unit length of beam

Table 2 presents the comparison which validates the SAP models.

Table 2.

Comparison of natural frequencies from FE models with analytical calculations

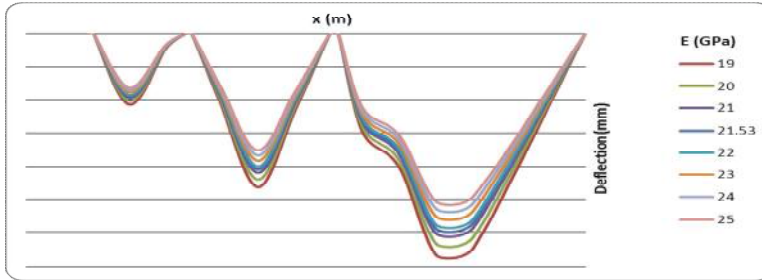
Span	FE Period(s)	FE frequency (rad/s)	Analytical frequency (rad/s)
2	0.170	5.87	5.94
3	0.156	6.43	6.59

This proves that the FE model is predicting the dynamic response of the structure correctly.

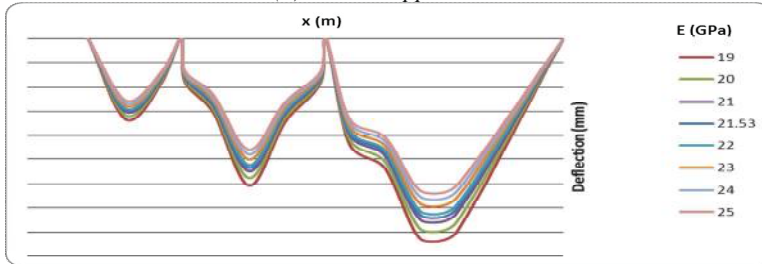
As stated earlier, data relating to the material properties and support configurations of the bridge deck were not readily available for analytical purposes. It is necessary to emphasise that the main focus of this part of the study was investigation of the linear elastic behaviour of the bridge. The two main parameters under consideration here, the support configurations of slabs at joints and the modulus of elasticity of concrete, have been considered as two main variables to be studied. It is essential to study the dependence of these parameters first. To investigate this parametric dependence a series of FE models with simple deck to

peer connections and another series with rigid connections were created. For each series, ten different models each with a specific value assigned for the Modulus of Elasticity of concrete were generated.

To find the proper deck support condition and Modulus of Elasticity, the maximum deflection of the bridge deck is demonstrated for every model in Figure 13.



(a) Fixed support condition



(b) Simple support condition

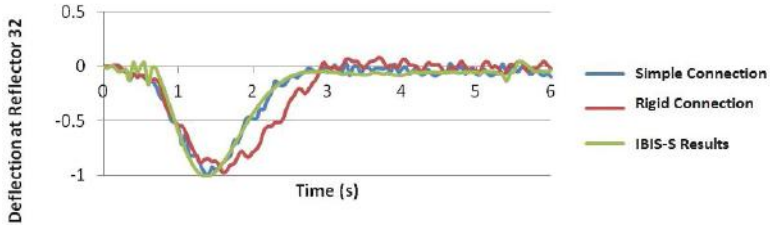
Figure 13. Deflection of the bridge deck with respect to variation of Young's Modulus of Concrete

Based on Figure 13, the variation of the connection condition does not affect the variation of the structural response with respect to the Modulus of Elasticity. The maximum deflection of the bridge decreases smoothly at every point on the bridge as the Modulus of Elasticity is increased regardless of the connection condition. Thus, the independence of the two variables under consideration is obvious. That is to say, although the behaviour of the structure is a function of these two parameters, change in the value of one of these parameters will not affect the value of the other (they are independent variables).

The mentioned connections were primarily assumed to behave as simple connections prior to any modifications being applied. Then these connections were considered as rigid connections prior to running the developed model again. In the modelling process, the speed of the lorry was set at 11.18 m/s in order to simulate the actual speed of the lorry while the monitoring survey with IBIS-S was materialised. The resulting deflections from both the finite element and the IBIS-S monitoring survey were normalised to a value of unity (-1) in order to exclude the effect of the

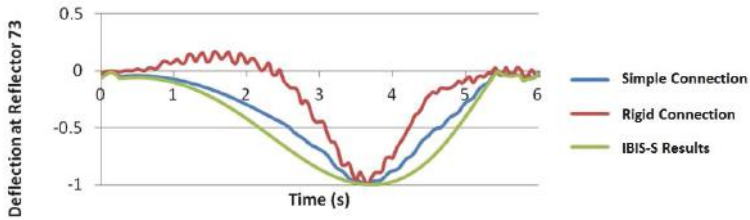
Modulus of Elasticity of the bridge material (concrete). As examples of the resulting diagrams, the illustrations presented in Figure 14 demonstrate the normalised deflection time history of the structure at Reflectors 32 and 73 near the midpoint positions of spans 1 and 2, respectively.

### Inverse Solution for Connections



(a) For midpoint of span 1

### Inverse Solution for Connections



(b) For midpoint of span 2

Figure 14. Normalised deflections from rigidly and simply connected bridge decks and IBIS-S monitoring

Figure 14 also clearly illustrates an appropriate representation of the connection (simple connection) behaviour of slab 1, as also confirmed for every other reflector position. To that effect, simple connections were used for the rest of the reported parametric studies in this paper.

The IBIS-S results (data) were also used to determine the actual value of the Modulus of Elasticity of concrete materials of the bridge. It is known that deflection of any structural element is inversely proportional to the Young's Modulus. The Modulus of Elasticity of concrete introduced to the Finite Element models was 21.53GPa, corresponding to a concrete with specific compressive strength value of 20.68 MPa (equal to 3000psi).

Therefore, the ratio of the actual deflection (in this case IBIS data) to the analytical deflection should be equal to the ratio of the assumed Modulus of Elasticity to the actual Modulus of Elasticity. Table 3 defines this comparison in detail. From this it can be deduced that the actual Modulus of Elasticity value is equal to 19.72GPa with the value of 1.092 as average ratio between the Finite Element model and the maximum deflections obtained by IBIS-S.



Table 3.

Comparison of deflections from FE models with IBIS-S results for Young's Modulus of Elasticity

Reflector No.	Max. FE Deflection (mm)	Max. IBIS-S Deflection (mm)	FE/IBISS
32	-1.41	-1.18	1.19
38	-1.462	-1.4	1.04
44	-0.947	-0.92	1.03
51	-0.768	-0.75	1.03
58	-0.323	-0.23	1.40
65	-0.612	-0.54	1.13
73	-1.07	-0.98	1.09
81	-0.54	-0.47	1.15
88	-0.314	-0.28	1.12
96	-0.322	-0.35	0.92
104	-0.592	-0.59	1.00
111	-0.03	-0.031	0.98

### Conclusions

This paper presents a methodology for the assessment of the structural response of bridge structures, in particular structures which lack information regarding their design and material properties and characteristics. This paper also presents a novel approach in modelling such structures using available specialist FE based software in combination with the applications of non-destructive and non-contact methods and systems.

Data compiled as a result of the applications of non-destructive and non-contact methods were successfully used for developing a more realistic and enhanced FE model of a bridge. The Pentagon Road Bridge (a 40 year old bridge) was used as the main focus of this study as there are no design drawings or details of the bridge available.

The Modulus of the Elasticity of concrete and the deck support condition were the two main unknown structural properties to be investigated. First the independence of these variables was proved. Therefore, the two parameters could be studied separately. To that effect, while studying the support conditions, it was not necessary to include the Modulus of Elasticity within the study. A maximum deflection of unity (-1) was assumed for any of the bridge spans in order to facilitate the calculations. The effect of the support configurations for two extreme cases, i.e., simple connection and rigid connection, were studied. This approach resulted in realising that the simple connection offers a more reliable support mechanism for the slabs.

In studying the influence of the Modulus of Elasticity of the concrete on the structural response of the bridge slabs (deflection) independently, an assumed value was introduced into the model starting with 21.53GPa (perceived/suggested value for structural concrete with compressive strength equal to 20.68 MPa (3000 psi)).

As a result of this approach, a set of theoretical values for deflections was compiled. Then these values were compared with the deflection data obtained during the fieldwork (actual monitoring of the bridge) using the IBIS-S sensor system. As a result of this comparison, necessary modifications and calibrations to the developed FE model were applied. Finally, the best estimate for Young's Modulus was equal to 19.72GPa.

The authors believe that the presented work and approach could be successfully applied to any other bridge within the context of the health monitoring of structures.

### **Acknowledgements**

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