UDC 621.31

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# PECULIARITIES OF THE UNCERTAINTY DETERMINATION FOR VECTOR QUANTITIES REPRODUCTION AND TRANSFER

*Peculiarities of the uncertainty determination in case of vector values measurement are analyzed. Analysis shows, that in case when both parameters of vector value are significant, appear correlation between these two measured parameters, caused by imperfection of the measuring instrument and uncertainty of both parameters of used standard. This correlation has to be determined and taken into account during the uncertainty determination. On the example of inductivity unit reproduction analysis show that neglecting of this fact could led to incorrect uncertainty determination.*

***Key words:*** *uncertainty, vector value, measurement, parameters, correlation, inductivity unit, reproduction, dissipation factor, standard.*

## Introduction

Uncertainty estimation of modern measuring equipment is the complicate process because of it need taking into account many uncertainty sources. Nevertheless, notwithstanding the fact that the uncertainty determination process is very complicate, all these processes are well described in fundamental works [1 – 4]. Usually, during the uncertainty determination process, we analyze the influence of the complex of destabilizing factors on concrete physical value. Generally, measured value is described as scalar one. Such approach is used even in the case then the vector physical value is measured. In this case two parameters of complex value are measured and uncertainty are usually determined separately for everyone parameter. It is evidently shown by structure of standards and transfer chains. Any one of these standards and transfer chains don't take into account correlation between two impedance parameters, caused by imperfection of the measurement instruments. This approximation is good enough when one of the impedance parameters is much greater than another: during the measurement of capacitance with low dissipation factor, resistance with low phase angle etc. Such approach permits to simplify as it is complicate process of uncertainty analysis.

## Problem

In practical measurements and sometimes during unit reproduction we have deal with measurement of the vector values, having both parameters being significant. In this case we have to take into account correlation between these two parameters, caused by imperfection of measuring instrument. This fact is well known. It complicates uncertainty analysis but not always it is taken into account.

## Decision

As the example, let we will analyze inductance unit reproduction, based on comparison of the inductive $Z_L$ and capacitive $Z_C$ standards on well known frequency ω using comparator, which measure ratio K of these impedances ($K = K_c + jK_k$). Here we will analyze the peculiarities of the uncertainty determination type B only. Formula (1) describes the comparison process:

$$\frac{Z_L}{Z_C} = K = K_c + jK_k . \qquad (1)$$

Here $K_c$ and $K_k$ are active and reactive components of impedance ratio, given by comparator [5].

Let we'll suppose that comparator determines coefficients $K_c$ and $K_k$ with additive $\Delta K_c$ and $\Delta K_k$, and multiplicative $\delta_c$ and $\delta_к$ uncertainties, so that these values are described by formulas:

$$K_c = K_{co}(1 + \delta_c) + \Delta K_c;$$
$$K_к = K_{ко}(1 + \delta_к) + \Delta K_k, \qquad (2)$$

where $\Delta K_c$ and $\Delta K_k$, $\delta_c$ and $\delta_к$ are the complex numbers, described by formulas:

$$\Delta K_c = \Delta K_{cc} + j\Delta K_{cк};$$
$$\Delta K_k = \Delta K_{кc} + j\Delta K_{кк}, \qquad (3)$$
$$\delta_c = \delta_{cc} + j\delta_{cк};$$
$$\delta_к = \delta_{kc} + j\delta_{kk}.$$

Let, as usually, suppose that impedance of capacitive and inductive standards are described by formulas:

$$Z_c = 1/j\omega C_o(1 - jtg\delta_c);$$
$$C_o = C_{он}(1 + \delta_0)(1 - j(tg\delta_{co} + \Delta tg\delta_c))$$
(parallel equivalent diagram);
$$Z_L = j\omega L + r; \quad tg\delta_L = r/\omega L \qquad (4)$$
(serial equivalent diagram),

where $C_{он}$ – measured value of standard capacitance; $\delta_0$ – uncertainty of standard capacitance measurement; $tg\delta_{co}$ – measured value of capacitive standard dissipation factor; $\Delta tg\delta_c$ – uncertainty of capacitive standard dissipation factor measurement; L and $tg\delta_L$ – inductance and dissipation factor of inductive standard to be determined.

Let, as well, suppose that operating frequency is described by formula:

$$\omega = \omega_0(1 + \delta_\omega), \qquad (5)$$

where $\omega_0$ – measured value of comparator operating frequency; $\delta_\omega$ – uncertainty of comparator operating frequency measurement.

 

Substituting (2), (3), (4) and (5) into (1) we can get:

$$L = \frac{K_{co}}{\omega^2 C_{он}}\left(1 - tg\delta_L \cdot tg\delta_{co}\right);$$

$$r = \frac{K_{ko}}{\omega C_{он}}\left(1 + \frac{tg\delta_{co}}{tg\delta_L}\right); \quad (6)$$

$$tg\delta_L = \frac{K_{ko}}{K_{co}}\left(1 + tg\delta_{co}(tg\delta_L + \frac{1}{tg\delta_L})\right).$$

Using named substitutions it is easy to show as well, that:

$$\delta L = \sqrt{X^2 + Y^2}\ ; \quad (7)$$

$$\delta(tg\delta_L) = \sqrt{(\delta_\omega)^2 + Z^2}\ , \quad (8)$$

where

$$X^2 = A^2 + (2\delta_\omega)^2 + (\delta_o)^2;$$

$$Y^2 = (tg\delta_L)^2 (B^2 + (\Delta tg\delta_C)^2);$$

$$Z^2 = (B^2 + (\Delta tg\delta_c)^2)\left((tg\delta_L)^2 + \frac{1}{(tg\delta_L)^2}\right);$$

$$A^2 = (\delta_{cc})^2 + (tg\delta_L)^2 (\delta_{kk})^2 + M^2;$$

$$B^2 = (\delta_{kc})^2 + (tg\delta_L)^2 (\delta_{ck})^2 + N^2;$$

$$M^2 = \left(\frac{\Delta K_{cc}}{K_{co}}\right)^2 + \left(\frac{\Delta K_{kk}}{K_{co}}\right)^2;$$

$$N^2 = \left(\frac{\Delta K_{kc}}{K_{ko}}\right)^2 + \left(\frac{\Delta K_{ck}}{K_{ko}}\right)^2.$$

Formulas shows that reproduced inductance value, calculated by formula (6), slightly differs on classical value ($L = 1/\omega^2 C_{он}$ ). This relative difference is equal to product of dissipation factors of inductive $tg\delta_L$ and capacitive $tg\delta_c$ standards.

Because of dissipation factor $tg\delta_L$ of inductive standard is rather great (usually 0,1 – 0,3), the uncertainty of capacitive standard dissipation factor uncertainty $\Delta tg\delta_c$ has to be low (last term in radicand, formula (7)). If, for example, we want to get this part of uncertainty less than 1 – 3 ppm, the uncertainty of capacitive standard dissipation factor has to be less than 3 – 10 ppm. Taking into account that for inductivity unit reproduction on point 100 mH and at 1 kHz we need usually capacitive standard having capacitance 25 – 250 nF or more, these requirements are very severe. If inductive standard on point of unite reproduction has lower impedance (1 mH, 10 mH), the dissipation factors of both inductive and capacitive standards increase and requirements to uncertainty of capacitive standard dissipation factor measurement become still more severe.

Formula (7) shows as well, that some part of the comparator uncertainty of measurement ($B^2$) enters into common uncertainty as product of $B^2$ and $tg\delta_L$.

Thus, analysis of the formula (7) shows that there are two parts of uncertainty δL sources:

– first part of sources enter into formula (7) with multiplier 1 (comparator uncertainty $A^2$, uncertainty of capacitive standard capacitance determination $\delta_o$, double uncertainty of frequency measurement);

– second part of uncertainty sources enter in formula (7) with multiplier $tg\delta_L$ (comparator uncertainty $B^2$, uncertainty of capacitive standard dissipation factor $\Delta tg\delta_c$ determination).

This analysis shows that we can estimate uncertainty of inductance unit reproduction, considering $tg\delta_L$ as the sensitivity of the δL to the appropriate sources $B^2$ and $\Delta tg\delta_c$. In this case sensitivity should be considered as the constant.

Let we'll analyze uncertainty of $tg\delta_L$ reproduction, given by formula (8).

In this case dependence of the uncertainty $\delta(tg\delta_L)$ on different sources is much more complicate. Only one source of uncertainty ($\delta_\omega$) enters in formula (8) with sensitivity 1. Uncertainty $\Delta tg\delta_c$ of the capacitive standard dissipation factor and uncertainty $B^2$ of the comparator measurement enters in formula (8) with complicate coefficient, being function of the $tg\delta_L$. It easy to that $\delta(tg\delta_L)$ sharply increase when $tg\delta_L$ decrease.

Components of comparator measurement uncertainty used in values $A^2$ and $B^2$ could be found using replacing method (to find multiplicative component $\delta_c = \delta_{cc} + j\delta_{ск}$ and $\delta_к = \delta_{кс} + j\delta_{kk}$) and by measurements of comparator initial parameters ($\Delta K_c = \Delta K_{cc} + j\Delta K_{ск}$ and $\Delta K_k = \Delta K_{кс} + j\Delta K_{кк}$).

## Conclusion

To determine the uncertainty of the vector value measurement it is need to find in either way correlation between two components of the vector value caused by measuring instrument and used standards. Uncertainty of anyone component of vector value has to be calculated taking into account this correlation. Neglecting of this correlation could led to the incorrect uncertainty estimation.

## Literature List


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*Поступила в редколлегию 20.07.2009*

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### ОСОБЛИВОСТІ ОЦІНКИ НЕВИЗНАЧЕНОСТІ ВІДТВОРЕННЯ ТА ПЕРЕДАВАННЯ ОДИНИЦЬ ВЕКТОРНИХ ВЕЛИЧИН

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*Аналізуються особливості оцінки невизначеності вимірювання векторних величин. Аналіз показує, що в ситуаціях, коли два параметра векторної величини мають суттєве значення, треба визначати кореляційні коефіцієнти, які обумовлені неідеальністю вимірювальної апаратури та невизначеністю обох парамерів еталона, який використовується. Ці кореляційні коефіцієнти треба брати до уваги, коли визначається невизначеність вимірювання векторної величини. Для ситуації, коли відтворюється одиниця індуктивності, показано, що не виконання цих умов веде до росту невизначеності вимірювань.*

***Ключові слова:*** *невизначеність, векторна величина,вимірювання, параметри, кореляція, одиниця індуктивності, відтворення, фактор втрат, еталон.*

### ОСОБЕННОСТИ ОЦЕНКИ НЕОПРЕДЕЛЕННОСТИ ВОСПРОИЗВЕДЕНИЯ И ПЕРЕДАЧИ ЕДИНИЦ ПАРАМЕТРОВ ВЕКТОРНЫХ ВЕЛИЧИН

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*Исследуются особенности расчета неопределенности измерений векторных величин. Показано, что в тех случаях, когда векторная величина описывается двумя параметрами, каждый из которых имеет существенное значение, нельзя пренебрегать корреляцией результатов измерений этих параметров, вызванной неидеальностью измерительной аппаратуры или неопределенностью измерений обоих параметров используемых образцовых мер. На примере анализа неопределенности воспроизведения единицы индуктивности показано, что пренебрежение этим обстоятельством приводит к росту неопределенности измерений.*

***Ключевые слова:*** *неопределенность, векторная величина, измерение, параметры, корреляция, единица индуктивности, воспроизведение, фактор потерь, эталон.*