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DESCRIPTION OF THE ACCURACY OF BROADLY UNBALANCED SENSOR RESISTANCE BRIDGES

After short introduction transfer coefficients of the unloaded four arms bridge of arbitrary variable arm resistances, supplied by current or voltage source, are given in Table 1. Their error propagation formulas are find and two rationalized forms of accuracy measures, i.e. related to the initial bridge sensitivities and of double component form as sum of zero error and increment error of the bridge transfer coefficients are introduced. Both forms of transfer coefficient measures of commonly used bridge - of similar initial arm resistances in balance and different variants of their jointed increments, are given in Table 3. As the example limited errors of some resistance bridges with platinum Pt100 industrial sensors of class A and B are calculated – Table 4 and analyzed. Presented approach is discussed and found as the universal solution for all bridges and also for any other circuits used for parametric sensors.

Keywords: *accuracy, transfer function, error, standard statistical measure, unbalanced sensor resistance bridges.*

Introduction

Accuracy of current and voltage supplied strain bridges has been analyzed by M. Kreuzer [1], [2], but only for very small sensor increments. This paper is based on earlier author proposals given in papers [1], [6] - [9]. As it has been pointed there, the generalized accuracy description of the 4R bridge of arbitrary variable arm resistances was not existing in the literature before above papers, but is urgently needed mainly for:

- initial conditioning circuits of analogue signals from broadly variable impedance sensors,
- identification of the changes of several internal parameters of the equivalent circuit of the object working as twoport X, when it is measured from its terminals for testing, monitoring and diagnostic purposes.

Near the bridge balance state, application of relative errors or uncertainties is useless, as they are rising to $\pm\infty$. In [3], [6- 9] this obstacle was bypassed by relating the absolute value of any bridge accuracy measure to the initial sensitivity of the current to voltage or voltage to voltage bridge transfer function. Initial sensitivities are valuable reference parameters as they do not change within the range of the bridge imbalance. In paper [9] the new double component approach to describing the bridge accuracy is described. It has form of sum of the initial stage and of the bridge imbalance accuracy measures. Such double component method of describing accuracy is commonly used for the broad range instruments, e.g. digital voltmeters. Relation of each components to accuracy measures of all variable and stationery bridge arm resistances have been developed. As the example formulas of accuracy measures of two bridges used for industrial Pt sensors will be presented and their limited errors calculated.

Basic formulas of bridge transfer functions

Four resistances (4R) connected in the closed loop can work as twoport type X with two pair of terminals AB and CD, shown on Fig 1. If some of its internal resistances R_i are variable the output voltage U'_{DC} may change sign for some set of them. This circuit is used in measurements for long time under the commonly known name - bridge. For work with sensors it is specially design. Preferred to use now is the ideal supply: by current $I_{AB} \to J = \text{const}$, $R_G \to \infty$ or by voltage $U_{AB} = \text{const}$, $R_G = 0$ and also the unloaded output, i.e.: $R_L \to \infty$, $U'_{DC} \to U^{\infty}_{DC}$.

For single variable measurements it is enough to know changes of one terminal parameter and the output circuit voltage U_{DC} is mostly used. With notations of fig. 1 formulas (1), (2) of U^{∞}_{DC} and bridge transfer functions (3), (4) together with their rationalized two-factor product forms are given in Table 1 [3, 4], where:

I_{AB}, U_{AB} - current or voltage on bridge supply terminals A B;

$R_i \equiv R_{i0} + \Delta R_i \equiv R_{i0}(1+\varepsilon_i)$ – arm resistance of initial value R_{i0} and absolute ΔR_i and relative ε_i increments;

r_{21}, k_{21} – current to voltage and voltage bridge transfer functions of the open-circuited output;

$$t_0 = \frac{R_{10}R_{30}}{\sum R_{i0}}, \quad k_0 = \frac{R_{10}R_{30}}{(R_{10}+R_{20})(R_{30}+R_{40})}$$ -

initial bridge open circuit sensitivities of r_{21} and of k_{21};

$\sum R_i \equiv \sum_{i=1}^{4} R_i = \left(1+\varepsilon_{\Sigma R}\right) \sum R_{i0}$ – sum of bridge arm resistances;

$\varepsilon_{\Sigma R}\left(\varepsilon_i\right)$, $\sum R_{i0}$ – its increment and initial value;

$f\left(\varepsilon_i\right)$, $f_E\left(\varepsilon_i\right)$ – normalized bridge imbalance function of r_{21} and of k_{21};

$\Delta L\left(\varepsilon_i\right); \varepsilon_{\Sigma R}\left(\varepsilon_i\right)$ – increment of the function $f\left(\varepsilon_i\right)$ numerator.

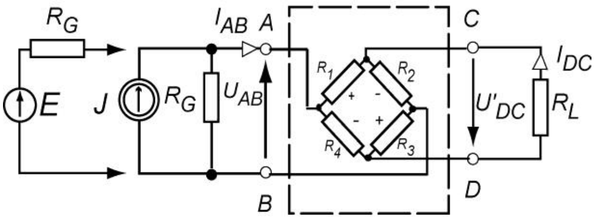


Fig. 1**.** Four arms circuit working as the twoport of type X with the voltage or current supply source branch

If transfer function $r_{21} = 0$ or $k_{12} = 0$, the bridge is in balance and from (3) and (4) its conditions of both supply cases and generally for any single source are the same: $R_1R_3 = R_2R_4$. The balance of the bridge can occurs for many different combinations of R_i, but the basic balance state is defined for all $\varepsilon_i = 0$, i.e. when:

$$R_{10}R_{30} = R_{20}R_{40} \qquad (5)$$

Bridge transfer functions (3), (4) =0can be simplified to products of their initial sensitivities t_0, k_0 in the balance and normalized unbalance functions $f\left(\varepsilon_i\right)$, $f_E\left(\varepsilon_i\right)$. Their formulas can be expressed by initial values R_{i0} and increments of all resistances, i.e. $R_i = R_{i0}(1+\varepsilon_i)$ and R_{i0} referencing to one of the first arm, i.e.: $R_{20} = mR_{10}$, $R_{40} = nR_{10}$ and from (3) $R_{30} = mnR_{10}$, as is shown in Table 1.

Table 1

Open circuit voltage of the resistance bridge and its transfer functions

a – **current supply**	b **– voltage supply**
$U'_{DC} \to U^{\infty}_{DC} = I_{AB} r_{21}$ (1)	$U'_{DC} \to U^{\infty}_{DC} = U_{AB} k_{21}$ (2)
$r_{21} \equiv \dfrac{U^{\infty}_{DC}}{I_{AB}} = \dfrac{R_1R_3 - R_2R_4}{\sum R_i} \equiv t_0 f\left(\varepsilon_i\right)$ (3)	$k_{21} \equiv \dfrac{U^{\infty}_{DC}}{U_{AB}} = \dfrac{R_1R_3 - R_2R_4}{\left(R_1+R_2\right)\left(R_3+R_4\right)} \equiv k_0 f_E(\varepsilon_i)$ (4)
where $t_0 \equiv \dfrac{m n R_{10}}{(1+m)(1+n)}$ $t_0 \equiv \dfrac{m n R_{10}}{(1+m)(1+n)}$, $\varepsilon_{\Sigma R} = \dfrac{\varepsilon_1 + m\varepsilon_2 + n\left(\varepsilon_4 + m\varepsilon_3\right)}{(1+m)(1+n)}$	$k_0 \equiv \dfrac{m}{(1+m)^2}$ $f_e\left(e_i\right) \equiv \dfrac{\Delta L\left(e_i\right)}{\left(1+e_{12}\right)\left(1+e_{34}\right)}$ $\varepsilon_{12} \equiv \dfrac{\varepsilon_1 + m\varepsilon_2}{1+m}$ $\varepsilon_{43} \equiv \dfrac{\varepsilon_4 + m\varepsilon_3}{1+m}$ *n* is arbitrary
$\boldsymbol{\varepsilon}_i = \left[\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4\right]^T$, $\Delta L\left(\varepsilon_i\right) = \varepsilon_1 - \varepsilon_2 + \varepsilon_3 - \varepsilon_4 + \varepsilon_1\varepsilon_3 - \varepsilon_2\varepsilon_4$	

Accuracy description of broadly variable resistances

The accuracy of measurements depends in complicated way on structure of the instrumentation circuit, values and accuracy of its elements including sensors and on various environmental influences of natural conditions and of the neighboring equipment. Two type of problems have been met in practice:

- description of circuits and measurement equipment by instantaneous and limited values of systematic and random errors, absolute or related ones, as well by statistical measures of that errors,
- estimation of the measurement result uncertainty, mainly by methods recommended by guide GUM.

Measures of accuracy (errors, uncertainty) of the single value of circuit parameter are expressed by numbers, of variable parameter - by functions of its values. In both cases they depend on equivalent scheme of the circuit, on environmental and parameters of instrumentation used or have to be use in the experiment.

The measures of broadly variable resistance R_i, e.g. of the stress or of the temperature sensors, could be expressed by two components: for its initial value R_{i0} and for its increment ε_i as is shown by formulas of Table 2.

Instantaneous absolute error Δ_{Ri} (6) and its two relative values δ_i and δ_{Ri} referenced to R_{i0} (see Fig. 2) or to R_i are given by formulas (7a,b), **relative limited errors** $\left|\delta_i\right|$, $\left|\delta_{Ri}\right|$ of the poorest case of values and signs of $\left|\delta_{i0}\right|$ and $\left|\Delta_{\varepsilon i}\right|$ or $\left|\delta_{\varepsilon i}\right|$ – by (8a,b), and **standard statistical measure** δ_{Ri} for random errors or uncertainties - by (9) and (9a,b). If random errors of increment and of initial value of resistance are statistically independent then correlation coefficient $k_i = 0$, but if they are strictly related each to the other then $k_i = \pm 1$. Exact k_i value can only be find experimentally. From (8a,b) follows that borders of the worse cases $\pm\left|\delta_i\right|$ of possible values of δ_i dependent linearly and $\pm\left|\delta_{Ri}\right|$ of δ_{Ri} dependent nonlinearly on ε_i when $\left|\delta_{i0}\right|$ and $\left|\Delta_{\varepsilon i}\right|$ or $\left|\delta_{\varepsilon i}\right|$ are constant [3], [6 – 9]. Distribution of the initial values and relative increments

e_i of the sensors set resistances depends on on their data obtained in the production process. Its actual values also depend on influences of the environmental conditions.

Table 2

Two-component formulas of the sensor resistance accuracy measures

Sensor resistance R_i		$R_i = R_{i0}(1+\varepsilon_i)$ if $\varepsilon_i > -1$ (5)
Absolute error $\Delta_i \equiv R_i - R_{i\,nominal}$		$\Delta_i = \Delta_{i0}(1+\varepsilon_i) + R_{i0}\,\Delta_{\varepsilon i} = \Delta_{i0} + \varepsilon_i(\Delta_{i0} + R_{i0}\delta_{\varepsilon i})$ (6)
Relative errors 1 2		$\delta_i = \delta_{i0}(1+\varepsilon_i) + \Delta_{\varepsilon i} = \delta_{i0} + \varepsilon_i(\delta_{i0} + \delta_{\varepsilon i})$ (7a) $\delta_{Ri} = \delta_{i0} + \frac{\Delta_{\varepsilon i}}{1+\varepsilon_i} = \delta_{i0} + \frac{\varepsilon_i}{1+\varepsilon_i}\delta_{\varepsilon i}$ (7b) where: $\delta_{i0} = \frac{\Delta_{i0}}{R_{i0}}$, $\delta_{\varepsilon i} = \frac{\Delta_{\varepsilon i}}{\varepsilon_i}$ - relative errors of: initial value R_{i0} and of resistance increment ε_i
Relative limited errors $\lvert\delta_i\rvert \equiv \frac{\lvert\Delta_i\rvert}{R_{i0}}$ $\lvert\delta_{Ri}\rvert \equiv \frac{\lvert\Delta_i\rvert}{R_i}$		$\lvert\delta_i\rvert = \lvert\delta_{i0}\rvert(1+\varepsilon_i) + \lvert\Delta_{\varepsilon i}\rvert = \lvert\delta_{i0}\rvert(1+\varepsilon_i) + \lvert\varepsilon_i\rvert\lvert\delta_{\varepsilon i}\rvert$ (8a) $\lvert\delta_{Ri}\rvert = \lvert\delta_{i0}\rvert + \frac{\lvert\Delta_{\varepsilon i}\rvert}{1+\varepsilon_i} = \lvert\delta_{i0}\rvert + \frac{\lvert\varepsilon_i\rvert}{1+\varepsilon_i}\lvert\delta_{\varepsilon i}\rvert$ (8b)
Statistical measure standard deviation of δ_{Ri} (for random error or uncertainty) $\bar{\delta}_{Ri} \equiv \frac{\bar{\Delta}_i}{R_i}$		$\bar{\delta}_{Ri} \equiv \frac{\bar{\Delta}_i}{R_i} = \sqrt{\bar{\delta}_{i0}^{\,2} + \left(\frac{1}{1+\varepsilon_i}\right)^2 \bar{\Delta}_{\varepsilon i}^{\,2} + 2k_i \frac{1}{1+\varepsilon_i}\bar{\delta}_{i0}\,\bar{\Delta}_{\varepsilon i}}$ (9) where: $\bar{\delta}_{i0}$, $\bar{\delta}_{\varepsilon i}$ – standard measures of initial value R_{i0} and of relative increment ε of resistance R_i, $k_i \subset (-1...0...+1)$ – correlation coefficient
Particular cases of correlation:	full: $k_i = \pm 1$	$\bar{\delta}_{Ri} = \left\lvert \bar{\delta}_{i0} \pm \left(\frac{1}{1+\varepsilon_i}\right)\bar{\Delta}_{\varepsilon i}\right\rvert$ (9a)
	no: $k_i = 0$	$\bar{\delta}_{Ri} = \sqrt{\bar{\delta}_{i0}^{\,2} + \left(\frac{1}{1+\varepsilon_i}\right)^2 \bar{\Delta}_{\varepsilon i}^{\,2}}$ (9b)

Description of the accuracy of bridge transfer functions

Instantaneous values of measurement errors of bridge transfer functions r_{21} and k_{21} result from the total differential of analytical equations (3) and (4) from Table 1. After ordering all components of δ_{Ri} their actual absolute errors $\Delta_{r_{21}}$ and $\Delta_{k_{21}}$ are given in the first line of the table 3.

Relative errors are preferable in measurement practice, but it is not possible to use them for transfer functions near the bridge balance as the ratio of absolute error $\Delta_{r21} \rightarrow \Delta_{r210} \neq 0$ and the nominal value $r_{21} \rightarrow r_{210} = 0$ (or for the voltage supplied bridge of Δ_{k21} and $k_{21} \rightarrow k_{210} = 0$) is rising to $\pm\infty$. Then other possibilities should be applied. Proposed are two possible ways to describe accuracy of the bridge transfer function r_{21} (or k_{21}):

- absolute error of the bridge transfer function may be referenced to initial sensitivity factor t_0 of r_{21} (or to k_0 of k_{21}) or to the range of transfer function r_{21max} - r_{21min} (or k_{21max} - k_{21min});
- initial error Δ_{r210} have to be subtracted from Δ_{r21} and then accuracy could be described by two separate terms: for zero and for transfer function increment, as it is common for digital instrumentation.

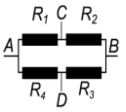
In **the first type method** the errors Δ_{r21} and Δ_{r21} are referenced to the initial sensitivities factors t_0 or k_0 as constant for each bridge, then to the full range of r_{21} or k_{21} as they could be change.

If resistances are expressed as $R_i = R_{i0}\ (1+\varepsilon_i)$, $R_j = R_{j0}\ (1+\varepsilon_j)$ and errors δ_{Ri} of resistances R_i are expressed as in (7b), by their initial errors δ_{i0} and incremental errors $\delta_{\varepsilon i}$, then both these formulas can be generalized to (10) and (11) for relative errors δ_{r21} and δ_{k21} of both transfer functions. In (10a) multiplier $(-1)^{i-1}$=+1 if i is 1, 3 or –1 if i is 2, 4. In (10) and (11) one could see that if errors δ_{Ri} of the neighboring bridge arms have the same sign they partly compensate each other.

From (10) and (11) are find limited relative errors (12) and (13) and relative random measures (14), (15).

Table 3

Accuracy measures of the open-circuit 4R bridge in general case

Bridge type		**a) Measures** δ_{r21}, $\lvert\delta_{r21}\rvert$, $\bar{\delta}_{r21}$ **of function** r_{21} **accuracy**	b) **Measures** δ_{k21}, $\lvert\delta_{k21}\rvert$, $\bar{\delta}_{k21}$ **of function** k_{21} **accuracy**
General case  **$R_1 = R_{10}(1+\varepsilon_1)$** **$R_2 = R_{20}(1+\varepsilon_2)$** **$R_3 = R_{30}(1+\varepsilon_3)$** **$R_4 = R_{40}(1+\varepsilon_4)$** $\varepsilon_i \geq -1$	Actual errors: absolute and relative	$\Delta_{r21} = R_1 \frac{R_3 - r_{21}}{\Sigma R_i}\delta_{R1} - R_2 \frac{R_4 + r_{21}}{\Sigma R_i}\delta_{R2} + R_1 \frac{R_1 - r_{21}}{\Sigma R_i}\delta_{R3} - R_4 \frac{R_2 + r_{21}}{\Sigma R_i}\delta_{R4}$ $\boxed{\delta_{r21} \equiv \frac{\Delta_{r21}}{t_0} = \sum_{i=1}^{4} w'_{Ri}\delta_{Ri} = \sum_{i=1}^{4} w'_{Ri}\left(\delta_{i0} + \frac{\varepsilon_i}{1+\varepsilon_i}\delta_{\varepsilon i}\right)}$ (10) where: $w'_{Ri} = \frac{1+\varepsilon_i}{1+\varepsilon_{\Sigma R}}\left[(-1)^{i-1}(1+\varepsilon_j) - \frac{r_{21}}{R_{j0}}\right]$ (10a) subscript j=3, 4, 1, 2 when i =1, 2, 3, 4	$\Delta_{k21} = \frac{R_1 R_2}{(R_1+R_2)^2}(\delta_{R1} - \delta_{R2}) + \frac{R_3 R_4}{(R_3+R_4)^2}(\delta_{R3} - \delta_{R4})$ $\boxed{\delta_{k21} \equiv \frac{\Delta_{k21}}{k_0} = \sum_{i=1}^{4} w'_{ki}\,\delta_{Ri} = \sum_{i=1}^{4} w'_{ki}\left(\delta_{i0} + \frac{\varepsilon_i}{1+\varepsilon_i}\delta_{\varepsilon i}\right)}$ (11) where: *n* – arbitrary $w'_{k1} = -w'_{k2} = \frac{(1+\varepsilon_1)(1+\varepsilon_2)}{1+\varepsilon_{12}}$; $w'_{k3} = -w'_{k4} = \frac{(1+\varepsilon_3)(1+\varepsilon_4)}{1+\varepsilon_{34}}$ (11a)
	Limited errors	$\boxed{\lvert\delta_{r21}\rvert = \sum_{i=1}^{4}\lvert w'_{Ri}\rvert\,\lvert\delta_{Ri}\rvert = \sum_{i=1}^{4}\lvert w'_{Ri}\rvert\left(\lvert\delta_{i0}\rvert + \frac{\lvert\varepsilon_i\rvert}{1+\varepsilon_i}\lvert\delta_{\varepsilon i}\rvert\right)}$ (12)	$\boxed{\lvert\delta_{k21}\rvert = \sum_{i=1}^{4}\lvert w'_{ki}\rvert\,\lvert\delta_{Ri}\rvert = \sum_{i=1}^{4}\lvert w'_{ki}\rvert\left(\lvert\delta_{i0}\rvert + \frac{\lvert\varepsilon_i\rvert}{1+\varepsilon_i}\lvert\delta_{\varepsilon i}\rvert\right)}$ (13)
	Random measures	$\bar{\delta}_{r21} = \sqrt{\sum_{i=1}^{4} w'^2_{Ri}\,\bar{\delta}^2_{Ri}} = \sqrt{\sum_{i=1}^{4} w'^2_{Ri}\left(\bar{\delta}^2_{i0} + \frac{\varepsilon_i^2}{(1+\varepsilon_i)^2}\bar{\delta}^2_{\varepsilon i}\right)}$ (14) correlation coefficients $k_{ij} = 0$	$\bar{\delta}_{r21} = \sqrt{\sum_{i=1}^{4} w'^2_{ki}\,\bar{\delta}^2_{Ri}} = \sqrt{\sum_{i=1}^{4} w'^2_{ki}\left(\bar{\delta}^2_{i0} + \frac{\varepsilon_i^2}{(1+\varepsilon_i)^2}\bar{\delta}^2_{\varepsilon i}\right)}$ (15) correlation coefficients $k_{ij} = 0$
Measures of $r_{210} = 0$	**actual error** $\delta_{210} = \delta_{10} - \delta_{20} + \delta_{30} - \delta_{40}$ (16)	**limited error** $\lvert\delta_{210}\rvert_m = \sum\lvert\delta_{i0}\rvert$ (17)	**mean square measure,** $k_{ij} = 0$ $\bar{\delta}_{210} = \sqrt{\sum\bar{\delta}^2_{i0}}$ (18)

If some resistance R_i is constant, then ε_i=0, $\delta_{Ri}=\delta_{i0}$, but weight coefficient w'_{Ri} of its component in error $\Delta_{r_{21}}$ or $\Delta_{k_{21}}$ still depends on other arm increments $\varepsilon_{j\neq i}$.

In initial balance state, i.e. when all arm increments ε_i=0, the nominal transfer functions $r_{21}(0) \equiv r_{210}=0$ and $k_{21}(0) \equiv k_{210}=0$, but real resistances R_i have some initial errors δ_{i0} and usually $\Delta_{r210}=t_0\delta_{210}\neq 0$, $\Delta_{k210}=k_0\delta_{210}\neq 0$. All measures of balance state are given in the last line of Table 3 as formulas (16) – (18).

Relative error δ_{r21} could be presented as sum:

$$\delta_{r21} \equiv \frac{\Delta_{r21}}{t_0} = \delta_{210} + \delta_{r21\varepsilon}(\varepsilon_i) \qquad (19)$$

where: $\delta_{210} = \delta_{10} - \delta_{210} + \delta_{30} - \delta_{40}$ – initial (or zero) relative error of r_{21}=0;

$\delta_{r21\varepsilon}(\varepsilon_i)$ –relative error of normalized imbalance function f (ε_i) when $r_{21}\neq 0$, also referenced to t_0.

Similarly, an δ_{k21} error of the voltage transfer coefficient k_{21} is

$$\begin{aligned}\delta_{k21} &= \sum_{i=1}^{4} w'_{ki}\left(\delta_{i0} + \frac{\varepsilon_i}{1+\varepsilon_i}\delta_{\varepsilon i}\right) = \\ &= \delta_{210} + \delta_{k21\varepsilon}(\varepsilon_i)\end{aligned} \qquad (20)$$

Coefficients $w'_{ki} = \frac{1}{k_0}w_{ki}$ must be determined from (11) or (11a).

Initial relative error δ_{210} is similar for any mode of the supply source equivalent circuit of the bridge as twoport. Zero of the bridge may be corrected on different ways: by adjustment of the bridge resistances, by the opposite voltage on output or by the digital correction of converted output signal. In such cases from (19) it is

$$\delta_{r21\varepsilon} = \frac{1}{t_0}\sum_{i=1}^{4}\left[w_{Ri} - (-1)^{i-1}\right]\delta_{i0} + \frac{1}{t_0}\sum_{i=1}^{4} w_{Ri}\frac{\varepsilon_i}{1+\varepsilon_i}\delta_{\varepsilon i}\,. \qquad (21)$$

From (21) follows that related to t_0 error $\delta_{r21\varepsilon}$ of r_{21} increment depends not only on increment errors $\delta_{\varepsilon i}$ of resistances R_i but also on their initial errors $\delta_{i0}\neq 0$ even when initial error of the whole bridge δ_{210}=0, because after (12a) weight coefficients of δ_{i0} in (21) depends on ε_i. The component of particular error δ_{i0} despairs only when δ_{i0}=0. Functions of $\Delta_{\varepsilon i}$ or $\delta_{\varepsilon i}$ may be approximated for some ε_i intervals by constant values.

In the **second type method** absolute error of transfer function r_{21} after subtracting its initial value is

$$\Delta_{r21} - \Delta_{r210} = \sum_{i=1}^{4}\left[w_{Ri} - (-1)^{i-1}\right]\delta_{i0} + \sum_{i=1}^{4} w_{Ri}\frac{\varepsilon_i}{1+\varepsilon_i}\delta_{\varepsilon i}\,. \qquad (22)$$

And after referenced it to r_{21}, and substitution w_{Ri} from (12a)

$$\begin{aligned}\delta_{r21r} &\equiv \frac{\Delta_{r21} - \Delta_{r210}}{r_{21}} = \frac{\delta_{r21} - \delta_{210}}{f(\varepsilon_i)} = \\ &= \sum_{i=1}^{4} w'_{ri0}\,\delta_{i0} + \sum_{i=1}^{4} w'_{r\varepsilon i}\,\delta_{\varepsilon i}\,,\end{aligned} \qquad (23)$$

where

$$w'_{ri0} = (-1)^{i-1}\frac{\varepsilon_i + \varepsilon_j + \varepsilon_i\varepsilon_j - \varepsilon_{\Sigma R}}{\Delta L(\varepsilon_i)} - t_0\frac{1+\varepsilon_i}{R_{j0}(1+\varepsilon_{\Sigma R})}\,; \qquad (23a)$$

$$w'_{r\varepsilon i} = \left[\frac{(-1)^{i-1}(1+\varepsilon_j)}{\Delta L(\varepsilon_i)} - \frac{t_0}{R_{j0}(1+\varepsilon_{\Sigma R})}\right]\varepsilon_i\,; \qquad (23b)$$

Weight coefficients (23a, b) are finite for any value of r_{21} including r_{21}= 0 because if all $\varepsilon_i \to 0$ also $\Delta L \to 0$.

Error δ_{r21r} is equivalent to error $\delta_{\varepsilon i}$ of the resistance R_i increment ε_i in formulas (7a,b). From (3) and (20) is:

for current to voltage transfer function r_{21}

$$\boxed{\Delta_{r21} = t_0\,\delta_{210} + r_{21}\,\delta_{r21r}} \qquad (24)$$

and from (4) similarly for voltage transfer function k_{21} is

$$\boxed{\Delta_{k21} = k_0\,\delta_{210} + k_{21}\,\delta_{k21k}} \qquad (25)$$

where $t_0\,\delta_{210}=\Delta_{r210}$, $k_0\,\delta_{210}=\Delta_{k210}$ – absolute errors of initial value r_{21} or k_{21} e.g. $r_{210} = 0$ or $k_{210}= 0$

δ_{r21r}, δ_{k21k} - related errors of transfer function increments r_{21} - r_{210} or k_{21} - k_{210} from the initial stage.

Two component accuracy equation (25) of k_{21} transfer function was funded by the same way as for r_{21}.

Actual values of instantaneous errors of r_{21} or k_{21} could be calculated only if signs and values of errors of all resistances are known. In reality it happens very rare. More frequently are used their limited systematic errors (of the worst case) for describing the permissible changes from nominal values of instrumentation parameters and statistical measures based on the standard deviation of the mean value of repeated measurement results. Formulas of this accuracy measures of r_{21} or k_{21} could be obtained after transformation of error formulas (10) – (25). All these accuracy measures is possible to find in one component or two component forms. One component formulas for arbitrary cases of 4R bridge are given in the table 3 and main particular cases - in [3], [6] - [9].

With (19) and (21) resulting limited error of transfer function r_{21} is

$$\left|\delta_{r21}\right| \equiv \frac{|\Delta_{r21}|}{t_0} = \sum_{i=1}^{4}\left|w'_{Ri}\right|\left(\left|\delta_{i0}\right| + \left|\frac{\varepsilon_i}{1+\varepsilon_i}\right|\left|\delta_{\varepsilon i}\right|\right) \le \left|\delta_{210}\right| + \left|\delta_{r21\varepsilon}(\varepsilon_i)\right|; \qquad (26)$$

where:

$$\left|\delta_{210}\right| \equiv \frac{|\Delta_{r210}|}{t_0} \le \left|\delta_{10}\right| + \left|\delta_{20}\right| + \left|\delta_{30}\right| + \left|\delta_{40}\right| = \sum_{i=1}^{4}\left|\delta_{i0}\right| -$$

limited (boundary) error of the zero initial state,

$$\left|\delta_{r21\varepsilon}(\varepsilon_i)\right| = \sum_{i=1}^{4}\left|w'_{Ri} - (-1)^{i-1}\right|\left|\delta_{i0}\right| + \sum_{i=1}^{4}\left|w'_{Ri}\right|\frac{|\varepsilon_i|}{1+\varepsilon_i}\left|\delta_{\varepsilon i}\right|.$$

From (19), (26) implies the inequality

$$\left|\delta_{r21}\right| \le \left|\delta_{210}\right| + \left|f(\varepsilon_i)\right|\left|\delta_{r21r}\right|; \qquad (27)$$

where

$$\left|\delta_{r21r}\right| = \frac{\left|\delta_{r21\varepsilon}\right|}{\left|f(\varepsilon_i)\right|} \equiv \sum_{i=1}^{4}\left|w'_{ri0}\right|\left|\delta_{i0}\right| + \sum_{i=1}^{4}\left|w'_{r\varepsilon i}\right|\left|\delta_{\varepsilon i}\right| -$$

relative error of increment $r_{21} - r_{210}$.

For random measures it is respectively

$$\bar{\delta}_{r21} < \sqrt{\bar{\delta}_{210}^2 + f^2(\varepsilon_i)\,\bar{\delta}_{r21r}^2} < \bar{\delta}_{210} + \left|f(\varepsilon_i)\right|\bar{\delta}_{r21r}; \qquad (28)$$

where $\bar{\delta}_{210} = \sqrt{\sum_{i=1}^{4}\bar{\delta}_{i0}}$, $\bar{\delta}_{r21r} \equiv \dfrac{\overline{\Delta_{r21} - \Delta_{r210}}}{|r_{21}|}$.

Despite the separation of the initial error $|\delta_{210}|$, for the unbalanced circuit the limited error $|\delta_{r21r}|$ of the r_{21} increment in (27) may still has components depended in part on the limited initial errors $|\delta_{i0}|$ of resistances R_i, and the random measure $\bar{\delta}_{r21r}$ of the r_{21} increment in (28) - respectively from $\bar{\delta}_{i0}$. Values of measures determined by the right-hand side of the inequalities (27) and (28) are slightly overstated, because their components are partially dependent.

It is also possible to refer the absolute measures of transfer coefficients of the 4R circuit to the range of their changes. For r_{21} at any values of increments ε_i such limited error $|\delta_{r21m}|$ is

$$\left|\delta_{r21m}\right| \equiv \frac{\left|\Delta_{r21}\right|}{\left|r_{21max} - r_{21minx}\right|} = \frac{1}{\left|\Delta f(\varepsilon_i)\right|_{Max}}\left|\delta_{210}\right| + \frac{\left|f(\varepsilon_i)\right|}{\left|\Delta f(\varepsilon_i)\right|_{Max}}\left|\delta_{r21r}\right|; \qquad (29)$$

where $|\Delta f(\varepsilon_i)|_{Max}=| f(\varepsilon_i)_{max}- f(\varepsilon_i)_{min}|$.

This description is similar to that as for the error of the digital voltmeter, and other instruments on a large range and resolution. The first component is constant and inversely proportional to the range of r_{21} changes and the second depends on the ratio of the relative imbalance function for the measured value and for the whole its range. Measures of the 4R bridge with the sensor of a certain tolerance are relatively larger for smaller ranges. If function $r_{21}(\varepsilon)$ is linear then formula (29) is simplified, for example, with coupled equal increments of $\pm\,\varepsilon$

$$\left|\delta_{r21m}\right| = \frac{|\delta_{210}|}{|\Delta f(\varepsilon)|_{Max}} + \frac{|\varepsilon|\,\left|\delta_{r21r}\right|}{\left|\varepsilon_{max} - \varepsilon_{min}\right|} \qquad (30)$$

For the finished product, such as IC measurement 4R circuit, the measures of individual resistance R_i are usually not known because the manufacturer provides permissible ranges of zero and of values of the transfer coefficient distribution, and very rare also distribution functions of these parameters. You will usually not be able to measure many times the parameters of sensor during its application, and used are the data received from the manufacturer or the first calibration. In the analysis of accuracy must therefore assume certain relations of the R_i resistance measures in the circuit, for example, the same of fixed arms and of the initial sensor resistances, i.e. $|\delta_{i0}|=|\delta_0|$ and for their increments $|\delta_{\varepsilon i}| \equiv$ $\equiv |\delta_\varepsilon|$.

Then $|\delta_{210}| = \Sigma|\delta_{i0}| \equiv 4|\delta_0|$. With less accurate sensors the measures of the more exact not variable resistances may also be negligibly small.

As an example of the application of the given formulas is to set the accuracy of the output voltage of 4R circuit working as twoport X. For supply source of current J with $R_G >> R_{AB}$ the output voltage is measured at the load $R_L >> R_{CD}$ with stabilized power supply or its ratio to the current J. From (1) and (24)

$$U_{DC}^{\infty} = J\left(r_{210} + \Delta r_{21}\right) \equiv \Delta U_{DC0}^{\infty} + J\, t_0\, f\left(\varepsilon_i\right). \quad (31)$$

The absolute error $\Delta_{U'DC}$ **of the output open-circuit voltage** referenced to the current J is

$$\frac{\Delta_{U'DC}}{J} = \Delta_{r21} + r_{21}\frac{\Delta_J}{J}. \quad (32)$$

where: Δ_J - the error with what is known the current J value or a certain its instability.

Regarding the error $\Delta_{U'DC}$ to the specified value of output voltage, e.g. corresponding to the range of

coefficient r_{21} changes, from (31), (32) the following **relative error** is obtained:

$$\delta'_{Um} \equiv \frac{\Delta_{U'DC}}{U'_{DCmax} - U'_{DCmin}} = \frac{t_0\,\delta_{210} + r_{21}\left(\delta_{r21r} + \delta_J\right)}{r_{21max} - r_{21min}} \quad (33)$$

where: $\delta_J \equiv \frac{\Delta_J}{J}$ – the relative error of current J.

When all increments ε_i=0 but there is an zero error δ_{210} then error δ'_{Um0} is of the finite value. From (33) can be determinate an limited error (borders of error area) and also a random measures of output signal. Formulas for the accuracy measures of the voltage supplied 4R circuit can be obtained by the similar way.

The two-component method of the bridge transfer function r_{21} accuracy representation, separately for its initial value (eg. equal to zero) and for increment is similar like unified one used for digital instruments and of the broad range sensor transmitters. It is especially valuable if zero of the measurement track is set handily or automatically. Absolute measures could be transformed also by the linear or nonlinear function of the sensor set to the units of any particular measurand, e.g. in the case of temperature sensors - to ^{0}C [7 – 9].

Accuracy measures of 4R bridges of equal initial resistances

In the measurement practice the mostly used for sensors are four-arm bridge circuits of all resistances R_{i0} equal in the balance state (m=1, n=1). It is despite the fact that at the current supply for m>1, n>1 the value of initial sensitivity t_0 of this circuit is higher and for $m\to\infty$, $n\to\infty$ increases even up to four times. But at the same time increases also the power drawn by the system and non-standard sensors, such as two- or four-element differential sensor of different initial resistances R_{i0}, are need to use. The five basic variants A- E of the $4R_{10}$ bridge circuit used in practice together with their transfer coefficients r_{21}, k_{21}, instant and limited relative errors and random measures in terms of homogeneous and two-component form are given in Table 4. Formulas of accuracy measures for transfer functions r_{21} and k_{21} of these particular resistance bridges are much simpler then of general cases. Random measures are presented with assumption that all correlation coefficients k_{ij}=0. Formulas of k_{21} and its errors given are mainly for comparison as current supply is preferable one for resistance sensors.

A comparison of the formulas given in Table 4 for different variants of the $4R_{10}$ circuit shows a number of conclusions, including also some new, previously unknown, but important for the measuring technique.

• For the current supply the four variants A - D of the $4R_{10}$ bridge circuit have the linear transfer function $r_{21}(\varepsilon)$ as of any ε value. For the voltage supply the transfer function $k_{21}(\varepsilon)$ is linear for two of bridges A and B only, since circuit C and D are non-linear (formulas of their measures are not given in the Table 3).

• When circuit B - of opposite $\pm\varepsilon$ variable increments of R_1, R_2 and circuit E - of variable R_1 only is supplied by voltage, their initial sensitivity k_0 and $k_{21}(\varepsilon)$ does not depend on n and hence from the zero correction provided in the lower bridge branch R_3, R_4.

• Circuits B and D with opposite $\pm\varepsilon$ increments of one pair resistances R_1, R_2 or R_1, R_4 have a similar dependence of r_{21} on ε. First of them keeps a constant input resistance, and the second one - a constant output resistance. In the A circuit with four increments of $\pm\varepsilon$ both of these resistances are constant. If additional resistance is connected to the circuit input, or output respectively, the linearity is reminded, but the initial circuit sensitivity may be decreasing.

• Transfer functions $r_{21}(\varepsilon_1)$ and $k_{21}(\varepsilon_1)$ of the single sensor E circuit as two different hyperbolas intersect at ε_1=0 are nonlinear functions. The non-linearity of r_{21} is less as its denominator varies less on ε_1 than of k_{21}.

• Related measures are not dependent on the level of circuit resistances.

• For small ε the impact of measures of initial values R_{i0} is dominated.

• Errors of linear $4R_{10}$ circuits in a homogeneous form (column a) differently each other depend on ε,

• Dependency on ε of the r_{21} errors of the circuit B with variable R_1, R_2 or circuit D of variable R_1, R_4 is similar, but the resistances R_2 and R_4 turn into their roles.

• Despite the correction of zero, for the imbalance state of circuits the errors δ_{i0} still partially affect the errors δ_{r21r}, δ_{k21k} of the increments of transfer coefficients r_{21} and k_{21} (see column b).

• When limited errors $|\delta_{i0}|=|\delta_0|$ of the initial R_{i0} values of resistances R_i are equal and as well $|\delta_{\varepsilon i}|=|\delta_\varepsilon|$ of their $\pm\varepsilon$ increments, then for the all current supplied circuits A - D with linear transfer functions $r_{21}(\varepsilon)$ the errors

$|\delta_{r21r}|$ (column b) of increments of r_{21} are equal and independent from $|\varepsilon|$. Their absolute limited measures $|\Delta r_{21}|$ are linear function of ε and of the number z_v=(2, 4) of variable resistances, i.e:

$$|\Delta_{r21}| = t_0|\delta_{r21}| \le 0{,}25R_{10}\ [|\delta_{210}| + z_v\ |\varepsilon|(|\delta_0| + |\delta_\varepsilon|)]. \quad (34)$$

Dependences on ε of the limited errors $|\delta_{r21}|$ and $|\Delta r_{21}|$ of the transfer resistance r_{21} of these circuits have the similar form as for the limited errors $|\delta_i|$ and $|\Delta_i|$ of a single resistance R_i - Figure 2.

• Both voltage supplied $4R_{10}$ circuits A and B also have the same errors $|\delta_{k21k}|$ and $|\Delta_{k21}|$, but differently from the previous ones they dependent on ε:

$$|\Delta_{k21}| \le 0{,}25[|\delta_{210}| + z_v(\varepsilon^2|\delta_0| + |\varepsilon||\delta_\varepsilon|)]. \quad (35)$$

In imbalance the error $|\Delta_{k21}|$ depends on the initial error $|\delta_0|$ and on constant error $|\delta_\varepsilon|$ with coefficients ε^2 and $|\varepsilon|$.

Formulas (34) and (35) are presented for the first time and seems to be useful in practice.

• Error formulas of the linear $4R_{10}$ circuits A- D with sensors of coupled resistances are simpler than those of

the E circuit with single R_1 sensor.

Table 4 shows that all random measures of A-E bridges vary differently on ε and than their limited errors.

Errors of the circuit with zero setting

Initial signal is set equal to zero in two ways.

1st. Set to zero the bridge - via analogue or digitally-controlled regulation of one or two of its resistances R_i, such as a potentiometer or resistor ladder with switched contact on one of the bridge nodes. May arise small, usually negligible additional $\delta_{\varepsilon i}$ errors of regulated resistors. Then error of the bridge balance δ_{210}=0 and from (10a) and (21) follows

$$\delta'_{r21} \equiv \delta_{r21\varepsilon}(\varepsilon_i)\big|_{\delta_{210}=0} = \sum_{i=1}^{4}\left(w'^{/}_{Ri}\,\delta_{i0} + w'_{Ri}\frac{\varepsilon_i}{1+\varepsilon_i}\delta_{\varepsilon i}\right) \quad (36)$$

where:

$$w'^{/}_{Ri} = \frac{1}{1+\varepsilon_{\Sigma R}}\left[(-1)^{i-1}\left(\varepsilon_i + \varepsilon_j + \varepsilon_i\varepsilon_j\right) - \frac{r_{21}(1+\varepsilon_i)}{R_{j0}}\right] \quad (36a)$$

2nd. Setting to zero the signal of the system performs outside the bridge - for example on its output by voltage opposite to the initial one, or after processing the analog signal to digital.

Error of r_{21}function if $\varepsilon_i \ne 0$ is

$$\delta''_{r21\varepsilon} = \sum_{i=1}^{4}\left[w'_{Ri}(\varepsilon_i) - (-1)^{i-1}\right]\delta_{i0} + \sum_{i=1}^{4} w'_{Ri}\frac{\varepsilon_i}{1+\varepsilon_i}\delta_{\varepsilon i} \quad (37)$$

where:

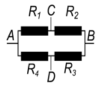
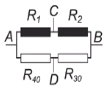
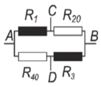
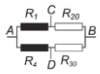
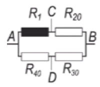
$$w'_{Ri}(\varepsilon_i) - (-1)^{i-1} = w'^{/}_{Ri}(\varepsilon_i) - \frac{\varepsilon_{\Sigma R}}{1+\varepsilon_{\Sigma R}}(-1)^{i-1} \quad (37a)$$

A comparison of (36a) and (37a) shows that, after correction of the initial circuit signal to zero, when $r_{21} \ne 0$, the first component in error $\delta''_{r21\varepsilon}$ is a bit larger than in $\delta'_{r21\varepsilon}$ on $\frac{\varepsilon_{\Sigma R}}{1+\varepsilon_{\Sigma R}}\delta_{210}$ and depends on the sign of the unbalanced bridge error. Limited errors of zero corrected circuits also slightly differ between themselves, i.e. at most about $\frac{|\varepsilon_{\Sigma R}|}{1+\varepsilon_{\Sigma R}}|\delta_{210}|$. Only for systems in which $\varepsilon_{\Sigma R}$=0 (i.e. bridges A, B, D from Table 2), the two above corrections of zero are equivalent but not so fully effective outside of the bridge balance. Even when errors $\delta_{\varepsilon i}$ of increments of the resistances R_i are negligible, or their proceeds to offset compensate each other, still error $\delta_{r21\varepsilon} \ne 0$. Errors $\delta_{r21\varepsilon}$ for both methods of correction are analyzed further in Example 1. A fully effective correction of zero and checking the accuracy of measurement channels with sensors require calibration for several values of the measured quantity. The measuring channel alone can be calibrated by replacing the sensor by other elements with its variable, dependent on mesurand parameters, e.g. by resistances corresponding to specified values of the mesurand.

Table 4

Accuracy measures of the open-circuit 4R bridges of equal all arm initial resistances $4R_{10}$

No	Bridge $4R_{10}$ parameters	Errors of R_i	**Related accuracy measures of bridge transfer functions r_{21} k_{21}:** a) related to initial sensitivities t_0 or k_0	b of increments r_{21}-r_{210} and k_{21}- k_{210}
1	<u>2</u>	3	4	5
1	**<u>Arbitrary</u> $\varepsilon_1, \ldots \varepsilon_4$** $R_i = R_{10}(1+\varepsilon_i)$ $\boxed{r_{21} = \frac{R_{10}}{4}\cdot\frac{\Delta L(\varepsilon_i)}{1+\frac{1}{4}\Sigma\varepsilon_i}}$	arbitrary	$\delta_{r21} \equiv \frac{\Delta_{r21}}{t_0} = \sum_{i=1}^{4} w'_{Ri}\delta_{Ri} = \sum_{i=1}^{4}(1+\varepsilon_i)\left[\frac{(-1)^{i-1}(1+\varepsilon_j)}{1+\frac{1}{4}\Sigma\varepsilon_i} - \frac{\Delta L(\varepsilon_i)}{4(1+\frac{1}{4}\Sigma\varepsilon_i)^2}\right]\delta_{Ri}$	$\delta_{r21r} \equiv \frac{\Delta_{r21} - \Delta_{r210}}{r_{21}} = \frac{1+\frac{1}{4}\Sigma\varepsilon_i}{\Delta L(\varepsilon_i)}\sum_{i=1}^{4}\left[\left(w'_{Ri} - (-1)^{i-1}\right)\delta_{i0} + w'_{Ri}\frac{\varepsilon_i}{1+\varepsilon_i}\delta_{\varepsilon i}\right]$
			where: $t_0 = \frac{R_{10}}{4}$, $w'^{/}_{Ri} = \frac{1}{t_0}w_{Ri}$ if i =1, 2, 3, 4 then j =3, 4, 1, 2	$\boxed{\delta_{r21} \equiv \delta_{210} + f(\varepsilon_i)\delta_{r21r}}$, $\boxed{\delta_{k21} \equiv \delta_{210} + f_E(\varepsilon_i)\,\delta_{k21k}}$
*2	$k_{21} = \frac{1}{4}\cdot\frac{\Delta L(\varepsilon_i)}{\left(1+\frac{\varepsilon_1+\varepsilon_2}{2}\right)\left(1+\frac{\varepsilon_3+\varepsilon_4}{2}\right)}$	* measures of k_{12} - on grey	$\delta_{k21} = \frac{(1+\varepsilon_1)(1+\varepsilon_2)}{\left(1+\frac{1}{2}\varepsilon_1+\frac{1}{2}\varepsilon_2\right)^2}(\delta_{R1}-\delta_{R2}) + \frac{(1+\varepsilon_3)(1+\varepsilon_4)}{\left(1+\frac{1}{2}\varepsilon_3+\frac{1}{2}\varepsilon_4\right)^2}(\delta_{R3}-\delta_{R4}) = \sum_{i=1}^{4} w'_{ki}\delta_{Ri}$	$\delta_{k21k} = \frac{1}{f_E(\varepsilon_i)}\sum_{i=1}^{4}\left[\left(w'_{ki} - (-1)^{i-1}\right)\delta_{i0} + w'_{ki}\frac{\varepsilon_i}{1+\varepsilon_i}\delta_{\varepsilon i}\right]$

1	2	3	4	5
3	**Jointed R₁,... R₄** $\varepsilon_i = \pm\varepsilon$ $\vert\varepsilon\vert\le 1$ 	arbitrary	$\delta_{r21} = (1+\varepsilon)(\delta_{10}+\delta_{30}) - (1-\varepsilon)(\delta_{20}+\delta_{40}) + \varepsilon(\delta_{\varepsilon1}+\delta_{\varepsilon3}-\delta_{\varepsilon2}-\delta_{\varepsilon4})$	$\delta_{r21r} = \frac14(\delta_{10}+\delta_{30}+\delta_{20}+\delta_{40}+\delta_{\varepsilon1}+\delta_{\varepsilon3}-\delta_{\varepsilon2}-\delta_{\varepsilon4})$
4			$\vert\delta_{r21}\vert = (1+\varepsilon)(\vert\delta_{10}\vert+\vert\delta_{30}\vert) + (1-\varepsilon)(\vert\delta_{20}\vert+\vert\delta_{40}\vert) + \vert\varepsilon\vert\sum_1^4\vert\delta_{\varepsilon i}\vert$	$\vert\delta_{r21r}\vert \equiv \frac{\vert\Delta_{r21}-\Delta_{r210}\vert}{\vert r_{21}\vert} = \frac14\left(\vert\delta_{210}\vert+\sum_1^4\vert\delta_{\varepsilon i}\vert\right)$
5	A	$\vert\delta_{i0}\vert=\vert\delta_0\vert$ $\vert\delta_{\varepsilon i}\vert=\vert\delta_\varepsilon\vert$	$\vert\delta_{r21}\vert = 4(\vert\delta_0\vert+\vert\varepsilon\vert\,\vert\delta_\varepsilon\vert)$	$\vert\delta_{r21r}\vert = \vert\delta_0\vert+\vert\delta_\varepsilon\vert$
6	**R₁=R₃=R₁₀(1+ε) R₂=R₄=R₁₀(1-ε)** $\boxed{r_{21}=0{,}25R_{10}\cdot4\varepsilon}$	$\bar\delta_{i0}\equiv\bar\delta_0$ $\bar\delta_{\varepsilon i}\equiv\bar\delta_\varepsilon$	$\bar\delta_{r21} = 2\sqrt{(1+\varepsilon^2)\bar\delta_0^2+\varepsilon^2\bar\delta_\varepsilon^2}$ correlation coefficient: $k_{ij}=0$	$\bar\delta_{r21r} \equiv \frac{\overline{\Delta_{r21}-\Delta_{r210}}}{\vert r_{21}\vert} = 0{,}5\sqrt{\bar\delta_0^2+\bar\delta_\varepsilon^2}$ $k_{ij}=0$
7	$k_{21}=0{,}25\cdot4\varepsilon$	arbitrary	$\delta_{k21} = (1-\varepsilon^2)(\delta_{10}-\delta_{20}+\delta_{30}-\delta_{40}) + \varepsilon(1-\varepsilon)(\delta_{\varepsilon1}+\delta_{\varepsilon3}) - \varepsilon(1+\varepsilon)(\delta_{\varepsilon2}+\delta_{\varepsilon4})$	$\delta_{k21k} = -\frac14[\varepsilon(\delta_{10}-\delta_{20}+\delta_{30}-\delta_{40}) + (1-\varepsilon)(\delta_{\varepsilon1}+\delta_{\varepsilon3}) - (1+\varepsilon)(\delta_{\varepsilon2}+\delta_{\varepsilon4})]$
8		$\vert\delta_{i0}\vert=\vert\delta_0\vert$ $\vert\delta_{\varepsilon i}\vert=\vert\delta_\varepsilon\vert$	$\vert\delta_{k21}\vert = 4[(1-\varepsilon^2)\vert\delta_0\vert+\vert\varepsilon\vert\,\vert\delta_\varepsilon\vert]$	$\vert\delta_{k21k}\vert = \vert\varepsilon\vert\,\vert\delta_0\vert+\vert\delta_\varepsilon\vert$
9		$\bar\delta_{i0}\equiv\bar\delta_0$ $\bar\delta_{\varepsilon i}\equiv\bar\delta_\varepsilon$	$\bar\delta_{k21} = 2\sqrt{(1-\varepsilon^2)^2\bar\delta_0^2+\varepsilon^2(1+\varepsilon^2)\bar\delta_\varepsilon^2}$ $k_{ij}=0$	$\bar\delta_{k21k} = 0{,}5\sqrt{\varepsilon^2\bar\delta_0^2+\bar\delta_\varepsilon^2}$ $k_{ij}=0$
10	**Jointed R₁, R₂** $\varepsilon_1=\varepsilon=-\varepsilon_2$ $\vert\varepsilon\vert\le1$	arbitrary	$\delta_{r21} = (1-0{,}5\varepsilon)[(1+\varepsilon)\delta_{10}+\varepsilon\delta_{\varepsilon1}-\delta_{40}] - (1+0{,}5\varepsilon)[(1-\varepsilon)\delta_{20}+\varepsilon\delta_{\varepsilon2}-\delta_{30}]$	$\delta_{r21r} = \frac14[(1-\varepsilon)\delta_{10}+(1+\varepsilon)\delta_{20}+\delta_{30}+\delta_{40}]+\delta_{\varepsilon1}(0{,}5-\varepsilon)-\delta_{\varepsilon2}(0{,}5+\varepsilon)$
11		$\vert\delta_{i0}\vert=\vert\delta_0\vert$ $\vert\delta_{\varepsilon1}\vert=\vert\delta_{\varepsilon2}\vert=\vert\delta_\varepsilon\vert$	$\vert\delta_{r21}\vert = 4(1-0{,}25\varepsilon^2)\vert\delta_0\vert+2\vert\varepsilon\vert\,\vert\delta_\varepsilon\vert$	$\vert\delta_{r21r}\vert = \vert\delta_0\vert+\vert\delta_\varepsilon\vert$ if $\vert\varepsilon\vert\le0{,}5$; $\vert\delta_{r21r}\vert = \vert\delta_0\vert+2\vert\varepsilon\vert\,\vert\delta_\varepsilon\vert$ if $\vert\varepsilon\vert\ge0{,}5$
12	B	$\bar\delta_{i0}\equiv\bar\delta_0$ $\bar\delta_{\varepsilon1}=\bar\delta_{\varepsilon2}\equiv\bar\delta_\varepsilon$	$\bar\delta_{r21} = 2\sqrt{[1+(1-\varepsilon^2)^2]\bar\delta_0^2+\varepsilon^2(1+\varepsilon^2)\bar\delta_\varepsilon^2}$ $k_{ij}=0$	$\bar\delta_{r21r} = \frac12\sqrt{(1+0{,}5\varepsilon^2)\bar\delta_0^2+0{,}125(1+4\varepsilon^2)\bar\delta_\varepsilon^2}$ $k_{ij}=0$
13	**R₁=R₁₀(1+ε) R₂=R₁₀ (1-ε)** R₃=R₄ = R₁₀	arbitrary	$\delta_{k21} = (1-\varepsilon^2)(\delta_{10}-\delta_{20})+\varepsilon(1-\varepsilon)\delta_{\varepsilon1}+\varepsilon(1+\varepsilon)\delta_{\varepsilon2}+\delta_{30}-\delta_{40}$	$\delta_{k21k} = -0{,}5\varepsilon(\delta_{10}-\delta_{20})+0{,}5(1-\varepsilon)\delta_{\varepsilon1}+0{,}5(1+\varepsilon)\delta_{\varepsilon2}$
14		$\vert\delta_{i0}\vert=\vert\delta_0\vert$ $\vert\delta_{\varepsilon1}\vert=\vert\delta_{\varepsilon2}\vert=\vert\delta_\varepsilon\vert$	$\vert\delta_{k21}\vert = 4(1-0{,}5\varepsilon^2)\vert\delta_0\vert+2\vert\varepsilon\vert\,\vert\delta_\varepsilon\vert$	$\vert\delta_{k21k}\vert = 2\vert\varepsilon\vert\,\vert\delta_0\vert+\vert\delta_\varepsilon\vert$
15	$\boxed{r_{21}=\frac{R_{10}}{4}2\varepsilon}$ $k_{21}=\frac14\cdot2\varepsilon$	$\bar\delta_{i0}\equiv\bar\delta_0$ $\bar\delta_{\varepsilon1}=\bar\delta_{\varepsilon2}\equiv\bar\delta_\varepsilon$	$\bar\delta_{k21} = \sqrt2\sqrt{[1+(1-\varepsilon^2)^2]\bar\delta_0^2+\varepsilon^2(1+\varepsilon^2)\bar\delta_\varepsilon^2}$ $k_{ij}=0$	$\bar\delta_{r21r} = \frac1{\sqrt2}\sqrt{\varepsilon^2\bar\delta_0^2+(1+\varepsilon^2)\bar\delta_\varepsilon^2}$ $k_{ij}=0$
16	**Jointed R₁, R₃** $\varepsilon_1=\varepsilon=\varepsilon_3$ $\varepsilon\ge-1$	arbitrary	$\delta_{r21} = (1+\varepsilon)(\delta_{10}+\delta_{30})-\delta_{20}-\delta_{40}+\varepsilon(\delta_{\varepsilon1}+\delta_{\varepsilon3})$	$\delta_{r21r} = 0{,}5(\delta_{10}+\delta_{30}+\delta_{\varepsilon1}+\delta_{\varepsilon3})$
17			$\vert\delta_{r21}\vert = (1+\varepsilon)(\vert\delta_{10}\vert+\vert\delta_{30}\vert)+\vert\delta_{20}\vert+\vert\delta_{40}\vert+\vert\varepsilon\vert(\vert\delta_{\varepsilon1}\vert+\vert\delta_{\varepsilon3}\vert)$	$\vert\delta_{r21r}\vert = 0{,}5(\vert\delta_{10}\vert+\vert\delta_{30}\vert+\vert\delta_{\varepsilon1}\vert+\vert\delta_{\varepsilon3}\vert)$
18			$\bar\delta_{r21} = \sqrt{(1+\varepsilon)^2(\bar\delta_{10}^2+\bar\delta_{30}^2)+\bar\delta_{20}^2+\bar\delta_{40}^2+\varepsilon^2(\bar\delta_{\varepsilon1}^2+\bar\delta_{\varepsilon3}^2)}$ $k_{ij}=0$	$\bar\delta_{r21r} = \frac1{\sqrt2}\sqrt{\bar\delta_{10}^2+\bar\delta_{30}^2+\bar\delta_{\varepsilon1}^2+\bar\delta_{\varepsilon3}^2}$ $k_{ij}=0$
19	C R₂=R₄=R₁₀	$\vert\delta_{i0}\vert=\vert\delta_0\vert$ $\vert\delta_{\varepsilon1}\vert=\vert\delta_{\varepsilon3}\vert=\vert\delta_\varepsilon\vert$	$\vert\delta_{r21}\vert = 4(1+0{,}5\varepsilon)\vert\delta_0\vert+2\vert\varepsilon\vert\,\vert\delta_\varepsilon\vert$	$\vert\delta_{r21r}\vert = \vert\delta_0\vert+\vert\delta_\varepsilon\vert$
20	$\boxed{r_{21}=\frac{R_{10}}{4}\cdot2\varepsilon}$ $k_{21}=\frac14\frac{\varepsilon}{2+\varepsilon}$	$\bar\delta_{i0}\equiv\bar\delta_0$, $\bar\delta_{\varepsilon i}\equiv\bar\delta_\varepsilon$	$\bar\delta_{r21} = 2\sqrt{(1+\varepsilon+0{,}5\varepsilon^2)\bar\delta_0^2+0{,}5\varepsilon^2\bar\delta_\varepsilon^2}$ $k_{ij}=0$	$\bar\delta_{r21r} = \frac1{\sqrt2}\sqrt{\bar\delta_0^2+\bar\delta_\varepsilon^2}$ $k_{ij}=0$
21	**Jointed R₁, R₄** $\varepsilon_1=\varepsilon=-\varepsilon_4$ $\varepsilon\le$	arbitrary	$\delta_{r21} = (1-0{,}5\varepsilon)[(1+\varepsilon)\delta_{10}+\varepsilon\delta_{\varepsilon1}-\delta_{20}] - (1+0{,}5\varepsilon)[(1-\varepsilon)\delta_{40}+\varepsilon\delta_{\varepsilon4}-\delta_{30}]$	$\delta_{r21r} = \frac14[(1-\varepsilon)\delta_{10}+(1+\varepsilon)\delta_{40}+\delta_{30}+\delta_{20}+(2-\varepsilon)\delta_{\varepsilon1}-(2+\varepsilon)\delta_{\varepsilon4}]$
22		$\vert\delta_{i0}\vert=\vert\delta_0\vert$ $\vert\delta_{\varepsilon1}\vert=\vert\delta_{\varepsilon3}\vert=\vert\delta_\varepsilon\vert$	$\vert\delta_{r21}\vert = 4(1-0{,}25\varepsilon^2)\vert\delta_0\vert+2\vert\varepsilon\vert\,\vert\delta_\varepsilon\vert$	$\vert\delta_{r21r}\vert = \vert\delta_0\vert+\vert\delta_\varepsilon\vert$
23	D $r_{21}=\frac{R_{10}}{4}\cdot2\varepsilon$, $k_{21}=\frac14\frac{2\varepsilon}{1-0{,}25\varepsilon}$	$\bar\delta_{i0}\equiv\bar\delta_0$, $\bar\delta_{\varepsilon i}\equiv\bar\delta_\varepsilon$	$\bar\delta_{r21} = \sqrt2\sqrt{[1+0{,}25\varepsilon^2+(1-0{,}5\varepsilon^2)^2]\bar\delta_0^2+(1+0{,}25\varepsilon^2)\varepsilon^2\,\bar\delta}$ $k_{ij}=0$	$\bar\delta_{r21r} = \sqrt{(1+0{,}5\varepsilon^2)\bar\delta_0^2+(2+0{,}5\varepsilon^2)\bar\delta_\varepsilon^2}$ $k_{ij}=0$
24	**Variable R₁** only $\varepsilon_1\ge-1$	arbitrary	$\delta_{r21} = \frac{(1+\varepsilon_1)\delta_{10}+\varepsilon\delta_{\varepsilon1}+(1+0{,}5\varepsilon_1)^2\delta_{30}-(1+0{,}5\varepsilon_1)(\delta_{20}+\delta_{40})}{(1+0{,}25\varepsilon_1)^2}$	$\delta_{r21r} = \frac{(\frac12-\frac1{16}\varepsilon_1)\delta_{10}+(\frac12+\frac3{16}\varepsilon_1)\delta_{30}+\frac1{16}\varepsilon_1(\delta_{20}+\delta_{40})+\delta_{\varepsilon1}}{1+0{,}25\varepsilon_1}$
25	 E	$\vert\delta_{10}\vert\ne\vert\delta_{20}\vert=\vert\delta_{30}\vert=\vert\delta_{40}\vert=\vert\delta_0\vert$	$\vert\delta_{r21}\vert = \frac{(1+\varepsilon_1)\vert\delta_{10}\vert+\vert\varepsilon_1\vert\,\vert\delta_{\varepsilon1}\vert+(3+2\varepsilon_1+\frac14\varepsilon_1^2)\vert\delta_0\vert}{(1+0{,}25\varepsilon_1)^2}$	$\vert\delta_{r21r}\vert = \frac{\frac12\vert1-\frac18\varepsilon_1\vert\,\vert\delta_{10}\vert+\frac12(1+\frac58\varepsilon_1)\vert\delta_0\vert+\vert\delta_{\varepsilon1}\vert}{1+0{,}25\varepsilon_1}$
26	**R₁=R₁₀(1+ε₁)** R₂= R₃= R₄=R₁₀	$\vert\delta_{i0}\vert=\vert\delta_0\vert$ $\vert\delta_{\varepsilon1}\vert\ne0$	$\vert\delta_{r21}\vert = 4\vert\delta_0\vert+\frac{\vert\varepsilon_1\vert}{(1+0{,}25\varepsilon_1)^2}(\vert\delta_0\vert+\vert\delta_{\varepsilon1}\vert)$	$\vert\delta_{r21r}\vert = \vert\delta_0\vert+\frac{\vert\delta_{\varepsilon1}\vert}{1+0{,}25\varepsilon_1}$ if $\varepsilon_1\le8$; $\vert\delta_{r21r}\vert = \frac{0{,}375\,\varepsilon_1\vert\delta_0\vert+\vert\delta_{\varepsilon1}\vert}{1+0{,}25\varepsilon_1}$ if $\varepsilon_1\ge8$
27	$\boxed{r_{21}=\frac{R_{10}}{4}\frac{\varepsilon_1}{1+0{,}25\varepsilon_1}}$	$\bar\delta_{i0}\equiv\bar\delta_0$, $\bar\delta_{\varepsilon i}\equiv\bar\delta_\varepsilon$	$\bar\delta_{r21} = \frac{\sqrt{[2\varepsilon_1(1+0{,}5\varepsilon_1)+(1+(1+0{,}5\varepsilon_1)^2)^2]\bar\delta_0^2+\varepsilon^2\bar\delta_{\varepsilon1}^2}}{(1+0{,}25\varepsilon_1)^2}$ $k_{ij}=0$	$\bar\delta_{r21r} = \frac{\sqrt{(1+\frac12\varepsilon_1+\frac3{16}\varepsilon_1^2)\bar\delta_0^2+4\bar\delta_{\varepsilon1}^2}}{2(1+0{,}25\varepsilon_1)}$ $k_{ij}=0$
28		arbitrary	$\delta_{k21} \equiv \frac{\Delta_{k21}}{k_0} = \frac{(1+\varepsilon_1)(\delta_{10}-\delta_{20})+\varepsilon_1\delta_{\varepsilon1}}{(1+0{,}5\varepsilon_1)^2}+\delta_{30}-\delta_{40}$	$\delta_{k21k} = \frac{-0.25\varepsilon_1(\delta_{10}-\delta_{20})+\delta_{\varepsilon1}}{1+0{,}5\varepsilon_1}$
29	$k_{21}=\frac14\frac{\varepsilon_1}{1+0{,}5\varepsilon_1}$	$\vert\delta_{i0}\vert=\vert\delta_0\vert$ $\vert\delta_{\varepsilon1}\vert=\vert\delta_{\varepsilon3}\vert=\vert\delta_\varepsilon\vert$	$\vert\delta_{k21}\vert = \frac{(1+\varepsilon_1)\vert\delta_{210}\vert+\vert\varepsilon_1\vert\,\vert\delta_{\varepsilon1}\vert+0{,}25\varepsilon_1^2(\vert\delta_{30}\vert+\vert\delta_{40}\vert)}{(1+0{,}5\varepsilon_1)^2}$	$\vert\delta_{k21k}\vert = \frac{0{,}25}{1+0{,}5\varepsilon_1}\vert\varepsilon_1\vert(\vert\delta_{10}\vert+\vert\delta_{20}\vert)+\vert\delta_{\varepsilon1}\vert$
30		$\bar\delta_{i0}\equiv\bar\delta_0$, $\bar\delta_{\varepsilon i}\equiv\bar\delta_\varepsilon$	$\bar\delta_{r21} = \frac{\sqrt{(1+\varepsilon_1)^2(\bar\delta_{10}^2+\bar\delta_{20}^2)+(1+0{,}5\varepsilon_1)^4(\bar\delta_{30}^2+\bar\delta_{40}^2)+\varepsilon_1^2\bar\delta_{\varepsilon1}^2}}{(1+0{,}5\varepsilon_1)^2}$ $k_{ij}=0$	$\bar\delta_{k21k} = \frac{\sqrt{0{,}25^2\varepsilon_1^2(\bar\delta_{10}+\bar\delta_{20})+\bar\delta_{\varepsilon1}^2}}{1+0{,}5\varepsilon_1}$ $k_{ij}=0$
0	**Measures of balance accuracy**	**actual error** $\delta_{210}=\delta_{10}-\delta_{20}+\delta_{30}-\delta_{40}$	**limited error** $\vert\delta_{210}\vert_m = \sum\vert\delta_{i0}\vert$	**mean square measure,** $k_{ij}=0$ $\bar\delta_{210}=\sqrt{\sum\bar\delta_{i0}^2}$

Examples

The following are some examples using the formulas of Table 4 to estimate the accuracy of $4R_{10}$ circuits.

Example 1. Comparison of the instantaneous and limited errors of the $4R_{10}$ circuit type A as the bridge with sensor of four increments of $\pm\varepsilon$, ($f(\varepsilon)=4\varepsilon$, $t_0=0{,}25R_{10}$), for two ways of correcting zero: in the bridge and beyond.

If zero is corrected by initial resistances R_{i0} then the initial error $\delta_{210}=\Sigma\delta_{i0}=0$, hence: $\delta_{10}+\delta_{30}=\delta_{20}+\delta_{40}$. In the instantaneous error δr_{21} of bridge A – Table 4, line 3 of column a), the sum of first two components become equal to zero and the error $\delta_{r21}=\varepsilon\ (\delta_{\varepsilon1}-\delta_{\varepsilon2}+\delta_{\varepsilon3}-\delta_{\varepsilon4})$. If actual values of the R_i increment errors $\delta_{\varepsilon i}$ are not known, but their limited errors $|\delta_{\varepsilon i}|$ are given, then the limited relative error $|\delta_{r21}|=|\varepsilon|\Sigma|\delta_{\varepsilon i}|$, while the absolute error $|\Delta_{r21}| \le 0.25R_{10}\,|\varepsilon|\ \Sigma|\delta_{\varepsilon i}|$.

When the circuit 4R is slightly imbalanced for $\varepsilon=0$, then the initial zero error $\delta_{210} \ne 0$. If zero-offset signal $Jt_0\delta_{210}$ is corrected in the measurement path after the bridge, then result obtain from (27) and row 4 of column b) is greater than limited error estimated above, i.e.: $|\delta_{r21}| \le 4|\varepsilon|\cdot|\delta_{r21r}| = |\varepsilon|\ (\Sigma|\delta_{i0}|+\Sigma|\delta_{\varepsilon i}|)$. If the tolerance is given only for the zero-error $|\delta_{210}|$ then the modules $|\delta_{i0}|$ of individual δ_{i0} errors of opposite signs can be large, and their participation in the instantaneous and limited errors $|\delta_{r21r}|$ and $|\delta_{r21}|$ will be also significant.

Example 2. Given are permissible limited errors of zero $|\delta_{210}|_{max}$ and of transfer function $|\delta_{r21r}|_{max}$ of the variant B current powered bridge $4R_{10}$ of variable R_1,R_2 increments $\pm\varepsilon$, $f(\varepsilon)=2\varepsilon$. Accuracy of bridge elements has to be find.

Formulas of errors given in rows 10 and 11 of Table 4 now have to be applied. Many combinations of limited errors of bridge elements are possible. Acceptable in practice is the case of equal limited errors $|\delta_{i0}|=|\delta_0|$ of bridge constant resistances R_{30}, R_{40} and of sensor initial resistances R_{10}, R_{20} and also equal $|\delta_{\varepsilon1}| = |\delta_{\varepsilon2}| \equiv |\delta_\varepsilon|$ errors of sensor resistance increments ε_1, ε_2, than:

- without corrections of zero:

$$|\delta_{i0}|=0{,}25|\delta_{210}|_{max}\ ,\ |\delta_\varepsilon| \le |\delta_{r21r}|_{max} - |\delta_{i0}| \text{ if } |\varepsilon| \le 0{,}5;$$

$$\left|\delta_\varepsilon\right| \le \frac{1}{2|\varepsilon|}\left(\left|\delta_{r21r}\right|_{max} - \left|\delta_0\right|\right) \text{ if } |\varepsilon| \ge 0{,}5,$$

- with such correction: $\delta_{210}=0$. Limited errors are the same as above if all actual errors δ_{i0} are unknown.

It is also possible to find statistical parameters of the distribution of transfer function errors for a given bridge and known standard deviations $\bar{\delta}_{i0}$, $\bar{\delta}_{\varepsilon i}$ of resistances of the sensor set offered by producer.

Example 3. Limited errors of option E of $4R_{10}$ bridge circuit with single sensor of resistance $R_1 = R_{10}(1+\varepsilon_1)$.

Current supply case: the transfer function r_{21} has the initial sensitivity $t_0=0{,}25R_{10}$ and imbalance function $f(\varepsilon_1) = \dfrac{\varepsilon_1}{1+0{,}25\,\varepsilon_1}$,

From (15), (26) and cell 26b' of Table 4 the relative limited error of r_{21} is

$$\left|\delta_{r21}\right| \le \left|\delta_{210}\right| + \left|\delta_{r21\varepsilon}\right| = \sum_1^4\left|\delta_{i0}\right| + \frac{|\varepsilon_1|}{1+0{,}25\varepsilon_1}\left|\delta_{r21r}\right|$$

where: $\left|\delta_{210}\right| = \sum_1^4\left|\delta_{i0}\right|$,

$$|\delta_{r21r}| = \frac{\frac{1}{2}\left|1-\frac{1}{8}\varepsilon_1\right|\left|\delta_{10}\right| + \frac{1}{2}(1+\frac{5}{8}\varepsilon_1)\left|\delta_0\right| + \left|\delta_{\varepsilon1}\right|}{1+0{,}25\varepsilon_1}\ -$$

from row 25b) of Table 4 for $|\delta_{i+1,0}|=|\delta_0|$,

or

$$|\delta_{r21r}| = \left|\delta_0\right| + \frac{\left|\delta_{\varepsilon1}\right|}{1+0{,}25\varepsilon_1}$$

when $|\delta_{10}| = |\delta_0|$, $\varepsilon_1 \le 8$ - row 19.

Voltage supply:

$$\left|\delta_{k21}\right| \le \left|\delta_{210}\right| + \left|\delta_{k21\varepsilon}\right| = \sum_1^4\left|\delta_{i0}\right| + \frac{0{,}25\varepsilon_1}{1+0{,}5\varepsilon_1}\left|\delta_{k21k}\right|$$

where

$$\left|\delta_{k21k}\right| = \frac{0{,}25\varepsilon_1\left(\left|\delta_{10}\right|+\left|\delta_{20}\right|\right)+\left|\delta_{\varepsilon1}\right|}{1+0{,}5\varepsilon_1} \xrightarrow{|\delta_{10}|=|\delta_{20}|}$$

$$\xrightarrow{|\delta_{10}|=|\delta_{20}|} \frac{0{,}5\left|\varepsilon_1\right|\left|\delta_0\right|+\left|\delta_{\varepsilon1}\right|}{1+0{,}5\varepsilon_1}$$

After comparison of denominators one can see that last error formula is more nonlinear as function of ε_1.

Example 4. **Limited errors of the 4R bridge with single industrial Pt 100 sensors**

The good example of the broadly variable resistance sensors are platinum sensors Pt 100 of A and B classes commonly used in industrial temperature measurements. Tolerated differences from their nominal parameters are given in standard EN 60751÷A2 1997. They are expressed in ^{0}C or as permissible resistance values in ohms - see $|\Delta|$ of class A and class B on Fig 3. Characteristic of class A sensors is determined up to 650^0C and for less accurate class B – up to 850^0C. Initial limited errors $|\delta_{10}|$ of both classes are 0,06% and 0,12% respectively.

On the base of nominal characteristic of Pt 100 sensors the maximum limited error

$$|\delta_{R1}|_{max} \equiv |\delta| = |\delta_{10}| + |\delta_{\varepsilon1}|$$

for $\varepsilon\to\infty$ of both classes is calculated as ratio of tolerances $|\Delta|$ and increments of sensor resistance [8 – 10], i.e. as

$$|\delta|=\frac{\left|\Delta_i(T)\right|-\left|\Delta_{i0}\left(0^0C\right)\right|}{R_i(T)-R_{i0}\left(0^0C\right)}=\left|\delta_{i0}\right|+\left|\delta_{\varepsilon i}\right| .$$

Obtained values are given on Fig 2.

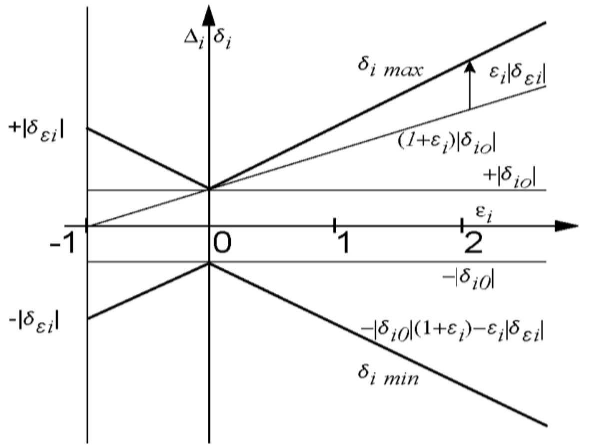


Fig. 2. Areas of possible values of absolute $\Delta_i=R_i-R_{i0}$ (6) and relative errors $\delta_i=\Delta_i/R_{i0}$ (7a) of the variable resistance $R_i=R_{i0}(1+\varepsilon_i)$ with given limited relative errors $|\delta_{i0}|$, $|\delta_{\varepsilon i}|$

They are only slightly changing and could be approximated by the single value and related to the maximum or mean value of the temperature range of each sensor. In the full range of positive

Celsius temperatures the limited error $|\delta|$ doesn't exceed 0,2% of ε for class A and $|\delta| \leq 0{,}5\%$ for class B.

Limited errors $|\delta_{r21}|$, $|\delta_{r21\varepsilon}|=|\delta_{r21}-\delta_{210}|$,$|\delta_{r21r}|$, of the 4R bridge transfer function r_{21} with the single industrial sensor of A or B class has be calculated from formulas of table 4. It was assumed that all limited errors $|\delta_{i0}|$ of constant bridge arms are equal and not higher that the sensor initial error $|\delta_{10}|$, bridge balance is at 0^oC, and current of supply source is stable enough or ratio of output signal and this current is measured.

Maximum temperature range $(0 - 600)^0C$ is taken for calculations and for it the relative increment of sensor resistance is: $\varepsilon_{max}=2{,}137$.

As example numerical formulas of limited errors $|\delta_{r21}|$ or $|\delta_{r21}|$ of the class A are also estimated. Limited errors of the class B sensor bridge have been similarly estimate.

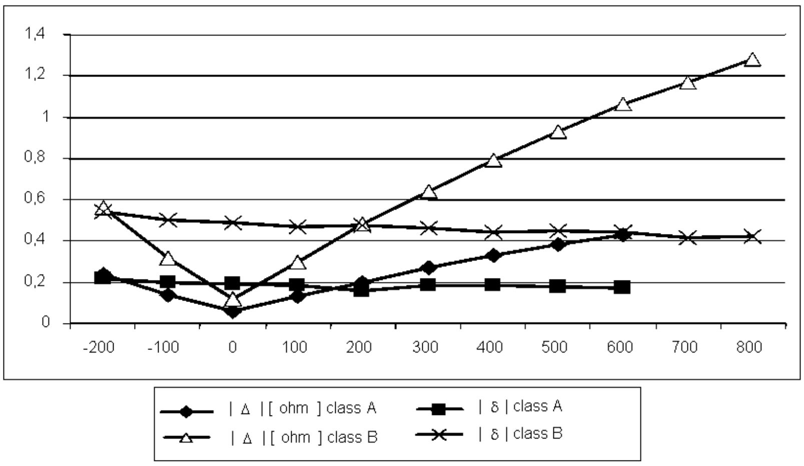


Fig. 3. Tolerances $|\Delta|$ and maximum limited relative errors $|\delta|$ of temperature sensors Pt100 type A and B evaluated from their standard characteristic [7 – 9]

For clarifying considerations the lead resistances are taken as negligible. Five different cases of measuring circuit are considered, i.e.: bridge without any adjustments, outer and internal zero setting, negligible initial errors only of constant arms or of the sensor arm as well. All results are presented in Table 5.

Ratio of limited errors of the bridge without adjustments and Pt sensor is 1,7 for class A and 2,9 for class B. If errors of the bridge resistances are negligible (line 5) limited error is only slightly higher than for the sensor, but if also the initial sensors error is adjusted, then the bridge transfer function r_{21} error is even smaller then of the sensor itself ! (line 1). Results for examples 2, 3 and 4 are between 1 and 6.

For comparison relative limited errors of the output voltage of the bridge C including two similar Pt100 sensors of class A or B in opposite arms are calculated from line 19 of table 4. For linearity of the output signal resistance increments of both sensors should be equal. Limited errors of both sensors and temperature range 0-600^oC are the same as before. The output signal of this bridge is twice higher that for the single sensor bridge E.

The error related to this signal doesn't exceed: for the class A sensors: 0,51% - without null correction and 0,39% - if it is corrected, and 0,97% or 0,73% respectively for the class B. These values are slightly higher than in Table 4.

From (26) it follows that for lower temperature ranges the bridge limited errors and then also uncertainty type B used for calculation the accuracy of measurement results become higher.

Table 5

Limited errors of few cases of the current supplied 4R bridge
with the single resistor sensor, e.g. Pt 100 type A or B

<table>
<tr><th rowspan="3">No</th><th rowspan="3">Particular causes of $4R_{i0}$ bridge</th><th rowspan="3" colspan="2">Limited errors $\left|\delta_{r21}\right|$ (or $\left|\delta_{r21\varepsilon}\right|$)
when: $R_{i0}=R_{10}$,
and $|\delta_{20}|=|\delta_{30}|=|\delta_{40}|\equiv|\delta_0|$</th><th rowspan="3">Class</th><th colspan="3">$\left|\delta_{21}\right|$, $\left|\delta_{21\varepsilon}\right|$, $\left|\delta_{r21\varepsilon}\right|$ in % for Pt100</th></tr>
<tr><th rowspan="2">Arbitrary increments ε_1 and $|\delta_0|=|\delta_{10}|$</th><th colspan="2">0-600°C (ε_1=2,137)</th></tr>
<tr><th>$\left|\delta_{r21}\right|$</th><th>$\left|\delta_{r21\varepsilon}\right|=0{,}72\left|\delta_{r21}\right|$</th></tr>
<tr><td rowspan="2">1</td><td rowspan="2">**Sensor accuracy**</td><td rowspan="2" colspan="2">$\left|\delta_{R1}\right|=\frac{\left|\Delta_{R1}\right|}{R_1}\le\left|\delta_{10}\right|+\frac{\left|\varepsilon_1\right|}{1+\varepsilon_i}\left|\delta_{\varepsilon 1}\right|$</td><td>A</td><td>$<0{,}06+\frac{\left|\varepsilon_1\right|}{1+\varepsilon_i}0{,}12$</td><td>0,14 (0,44°C)</td><td></td></tr>
<tr><td>B</td><td>$<0{,}12+\frac{\left|\varepsilon_1\right|}{1+\varepsilon_i}0{,}34$</td><td>0,35 (1,1°C)</td><td></td></tr>
<tr><td rowspan="2">2</td><td rowspan="2">**Bridge without adjustments**</td><td rowspan="2" colspan="2">$\left|\delta_{r21}\right|\le\frac{(1+\varepsilon_1)\left|\delta_{10}\right|+\left(3+2\varepsilon_1+\frac{1}{4}\varepsilon_1^2\right)\left|\delta_0\right|+\left|\varepsilon_1\right|\left|\delta_{\varepsilon 1}\right|}{\left(1+\frac{1}{4}\varepsilon_1\right)^2}$</td><td>A</td><td>$\le\frac{0{,}24+0{,}18\varepsilon_1+0{,}12\left|\varepsilon_1\right|+0{,}015\varepsilon_1^2}{(1+0{,}25\varepsilon_1)^2}$</td><td>$\left|\delta_{r21}\right|<0{,}41$</td><td>(0,33)</td></tr>
<tr><td>B</td><td>$\le\frac{0{,}48+0{,}36\varepsilon_1+0{,}34\left|\varepsilon_1\right|+0{,}03\varepsilon_1^2}{(1+0{,}25\varepsilon_1)^2}$</td><td>$\left|\delta_{r21}\right|<0{,}90$</td><td>(0,72)</td></tr>
<tr><td rowspan="2">3</td><td rowspan="2">**Null setting outsight the bridge**</td><td rowspan="2" colspan="2">$\left|\delta_{r21\varepsilon}\right|=\left|\delta_{r21}-\delta_{210}\right|\le$
$=\frac{\left|\varepsilon_1\right|\left[\left|\frac{1}{2}\right|1-\frac{\varepsilon_1}{8}\left|\left|\delta_{10}\right|+\left(\frac{1}{2}+\frac{5}{16}\varepsilon_1\right)\left|\delta_0\right|+\left|\delta_{\varepsilon 1}\right|\right]}{(1+\frac{1}{4}\varepsilon_1)^2}$</td><td>A</td><td>$\le\frac{0{,}18\left|\varepsilon_1\right|+0{,}0113\varepsilon_1^2}{(1+0{,}25\varepsilon_1)^2}$</td><td>0,143</td><td>0,139</td></tr>
<tr><td>B</td><td>$\le\frac{0{,}46\left|\varepsilon_1\right|+0{,}023\varepsilon_1^2}{(1+0{,}25\varepsilon_1)^2}$</td><td>0,38</td><td>0,34</td></tr>
<tr><td rowspan="2">4</td><td rowspan="2">**Null setting in the bridge**</td><td rowspan="2" colspan="2">$\left|\delta_{r21\varepsilon}\right|\le\frac{\left|\varepsilon_1\right|\left[\frac{1}{2}\left|\delta_{10}\right|+\left(\frac{1}{2}+\frac{\varepsilon_1}{4}\right)\left|\delta_0\right|+\left|\delta_{\varepsilon 1}\right|\right]}{(1+\frac{1}{4}\varepsilon_1)^2}$</td><td>A</td><td>$\le\frac{0{,}18\left|\varepsilon_1\right|+0{,}015\varepsilon_1^2}{(1+0{,}25\varepsilon_1)^2}$</td><td>0,193</td><td>0,14</td></tr>
<tr><td>B</td><td>$\le\frac{0{,}46\left|\varepsilon_1\right|+0{,}03\varepsilon_1^2}{(1+0{,}25\varepsilon_1)^2}$</td><td>0,48</td><td>0,34</td></tr>
<tr><td rowspan="3">**5**</td><td rowspan="3">**Only errors of sensor increments** (setting δ_{i0})</td><td colspan="2">Sensor</td><td rowspan="2">A</td><td rowspan="2">$\le\frac{0{,}18\left|\varepsilon_1\right|+0{,}015\varepsilon_1^2}{(1+0{,}25\varepsilon_1)^2}$</td><td rowspan="2">0,081 0,11</td><td rowspan="2">0,08</td></tr>
<tr><td>alone</td><td>in the circuit</td></tr>
<tr><td>$\left|\delta_{R1\varepsilon}\right|\le\frac{\left|\varepsilon_1\right|}{1+\varepsilon_i}\left|\delta_{\varepsilon 1}\right|$</td><td>$\left|\delta_{r21\varepsilon}\right|\le\frac{\left|\varepsilon_1\right|}{\left(1+\frac{1}{4}\varepsilon_1\right)^2}\left|\delta_{\varepsilon 1}\right|$</td><td>B</td><td>$\le\frac{0{,}46\left|\varepsilon_1\right|+0{,}03\varepsilon_1^2}{(1+0{,}25\varepsilon_1)^2}$</td><td>0,23 0,22</td><td>0,11</td></tr>
<tr><td rowspan="2">**6**</td><td rowspan="2">**Negligible bridge resistance errors** $|\delta_0|$</td><td rowspan="2" colspan="2">$\left|\delta_{r21}\right|\le\frac{(1+\varepsilon_1)\left|\delta_{10}\right|+\left|\varepsilon_1\right|\left|\delta_{\varepsilon 1}\right|}{\left(1+\frac{1}{4}\varepsilon_1\right)^2}$
($|\delta_{20}|=|\delta_{30}|=|\delta_{40}|\to 0$)</td><td>A</td><td>$\le\frac{(1+\varepsilon_1)0{,}06+\left|\varepsilon_1\right|0{,}12}{\left(1+0{,}25\varepsilon_1\right)^2}$</td><td>$\left|\delta_{21}\right|=0{,}19$</td><td>(0,14)</td></tr>
<tr><td>B</td><td>$\le\frac{(1+\varepsilon_1)0{,}12+\left|\varepsilon_1\right|0{,}34}{\left(1+0{,}25\varepsilon_1\right)^2}$</td><td>$\left|\delta_{21}\right|=0{,}47$</td><td>(0,34)</td></tr>
</table>

General conclusions

Two methods of describing the accuracy measures of the arbitrary imbalanced sensor bridges are presented together and compared., i.e.:

- one component accuracy measure related to initial sensitivity of the bridge transfer functions, given before in [3], [6] - [9],

- the new double component one of separately defined measures for zero and transfer function increment.

The second one is similar as used for the broad range instruments, e.g. digital voltmeters. Accuracy measures of bridge arms are defined for initial resistances and for their increments. Then this methods are independent from the sensor characteristic to the measured quantity.

This methods are discussed using on few examples of 4R bridges of equal initial resistances, supplied by current or voltage source and with single, double and four element sensors.

Given formulas allow to find accuracy of the 4R bridge or uncertainty of measurements with bridge circuits if actual or limited values of errors or standard statistical measure of their resistances and sensors are known.

Formulas of general and particular cases of the bridge may be used for computer simulation of the accuracy of various sensor bridges and measured objects of the X twoport equivalent circuit in different circumstances.

Systematic errors could be calculated also as random ones for set of sensor bridges in production or in exploitation. If all correlation coefficients are negligible obtained values should be smaller than limited errors.

Similar formulas as presented in this papers, could be formulated for any types of impedance sensor circuits as DC and AC bridges of single and double supply, active bridges linearized by feedback or multipliers, Anderson loop and impedance converters with DSP processing. Given in this paper methods of the simplification of their accuracy description could be also applied in many industrial measurements.

A unified approach as given above to the accuracy description of unbalanced bridges and other circuits of broadly variable parameters, introduced in [3], [6 – 9], is not found so far in literature.

The first type method of describing the 4R bridge accuracy was also used by author for two-parameter bridge measurements [3], [5 – 7].

The presented method is also valuable for accuracy evaluation in testing any circuit from its terminals as twoport, which is commonly used in diagnostics and in impedance tomography.

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ОПИС ТОЧНОСТІ ШИРОКО НЕВРІВНОВАЖЕНИХ МОСТІВ ОПОРУ ДАТЧИКА

З.Л. Варша

Після короткого введення в таблиці 1 наведені коефіцієнти передачі ненавантажених чотирьохплечих мостів довільних змінних опорів плеча, що живляться джерелами струму або напруги. Знайдено їхні формули поширення похибок і введено дві раціоналізовані форми мір щодо точності, пов'язані з початковою чутливістю моста й подвійної складової форми через суму нульової помилки, також збільшують помилку коефіцієнтів передачі моста. У таблиці 3 наведені обидві форми мір коефіцієнтів передачі широко застосовуваного моста - подібних початкових опорів плеча в балансі й різних варіантів їхніх з'єднаних збільшень. Оскільки приклад обмежується похибками для деяких мостів опору із платиною Pt100, обчислені й проаналізовані індустріальні датчики класу А і В. Представлений підхід обговорювався й був виявлений як універсальне рішення для всіх мостів, а також для будь-яких інших ланцюгів, використовуваних для параметричних датчиків.

Ключові слова: *точність, функція передачі, похибка, типова статистична характеристика, неврівноважені мости опору датчика.*

ОПИСАНИЕ ТОЧНОСТИ ШИРОКО НЕУРАВНОВЕШЕННЫХ МОСТОВ СОПРОТИВЛЕНИЯ ДАТЧИКА

З.Л. Варша

После краткого введения в таблице 1 приведены коэффициенты передачи ненагруженных четырехплечих мостов произвольных переменных сопротивлений плеча, питаемых источниками тока или напряжения. Найдены их формулы распространения погрешностей и введены две рационализированные формы мер по точности, связанные с начальной чувствительностью моста и двойной составляющей формы через сумму нулевой ошибки, и увеличивающие ошибку коэффициентов передачи моста. В таблице 3 приведены обе формы мер коэффициентов передачи широко применяемого моста - подобных начальных сопротивлений плеча в балансе и различных вариантов их соединённых приращений. Поскольку пример ограничивается погрешностями для некоторых мостов сопротивления с платиной Pt100, вычислены и проанализированы индустриальные датчики класса А и В. Представленный подход обсуждался и был обнаружен в качестве универсального решения для всех мостов, а также для любых других цепей, используемых для параметрических датчиков.

Ключевые слова: *точность, функция передачи, погрешность, типовая статистическая характеристика, неуравновешенные мосты сопротивления датчика.*