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*Khmelnytsky National University, Khmelnytsky***A PROGRAM MODULE FOR FUNDAMENTAL MODEL OF SEARCHING THE ACTIVE OBJECT WITHIN PARTIAL INDETERMINANCY AS MULTISTAGE DIAGONAL 2×2 -GAME**

There is programmed the search of an active object, moving through a rectangular area, divided into identical frontiers vertically with their minimal horizontal division. To converge the partial indeterminacy in pre-evaluated probabilities of the object positioning there is solved the known continual antagonistic game on the square of those probabilities. The optimal behavior of searchers is in the diagonal 2×2 -game solution, evolving from frontier down to frontier through the area. Also the programmed search full code is presented.

Keywords: active object search, subarea minimal division, partial indeterminacy, diagonal game, multistage solution.

Introducing and stating the problem

A great many of activity domains has the arising problems with searching the objects of various nature. They are tasks of searching the lost people or kids, the trespassers of the state frontier, the operative military quest and some others [1, 2]. And modeling such search processes is an actual problem to be resolved as fast as possible. Nevertheless the search area is supposed to be closed and rectangular, some initial conditionals in those quest models are usually not strict and certain.

Analyzing investigations on search processes models

Questions on building models of physical objects search processes are obviously not so developed as questions of computer or Internet (web) search. Though the computer search is with information only, which may be processed and filtered by a single user. But when it is speaking on searching the physical and active object, then it is compulsory to regard its capability to conscious or non-conscious masking and hiding [3]. For instance, while searching the lost kid, it is necessary to mind his the most bad actions to be appeared. A problem in modeling the active object search is still to construct the program module for implementing the optimal search actions drill with having accepted the uncertain data on evaluated probabilities of the object local positions.

Formulating the paper goal and stating the tasks

May an active object move in the known and learned direction. The easiest way to figure out the area of the object straight movement is to take this area rectangular. Will break the area into rectangular stripes (frontiers), which are crossed successively by the object, beginning from the first down to $M \in \overline{1, M}$, the last. Each frontier should be divided into at least two subareas, and the investigation is going to be accomplished with this fundamental division. There is a search squad to locate

the moving object. And both this squad and the object have the pre-evaluated probabilities of positioning within every subarea, though those probabilities are given as nonzero measured segments. The started papering goal is to develop the program module for implementing the optimal search actions fundamental drill with having accepted the uncertain probabilities of the object local positions, where these segment probabilities are known for both the search squad and active object. For hitting this goal here are the tasks to model the search area fundamentally (within minimal frontier subarea division) with minimizing the location error and to construct the program module for getting instantly the search solution.

Modeling the search area within minimal frontier subarea division

To divide each frontier into subareas minimally is acceptable, when there is poor information about evaluating the object wants of to stay or move through every area frontier. Also if the frontier length is comparable with its depth (width), then it is rationally to divide each frontier into only two identical subareas, being obviously rectangular. The third motive of such fundamental division, could be called macrodivision, is connected with the search squad capacity or search resource volume. Low capacity or volume may cause to divide each frontier into subareas minimally. The fourth gap to divide minimally comes if it is hard to pre-evaluate segment probabilities of positioning within every subarea for three or more identical subareas within each frontier.

Consider the i -th frontier, $i = \overline{1, M}$. Here is known the probability segment $\left[p_i^{(1)}; p_i^{(2)} \right]$ of positioning within the first subarea of this frontier, and

$$\mu_R \left(\left[p_i^{(1)}; p_i^{(2)} \right] \right) \neq 0 \quad \forall i = \overline{1, M} \quad (1)$$

by

$$p_i^{(1)} > 0 \text{ and } p_i^{(2)} < 1 \quad \forall i = \overline{1, M}. \quad (2)$$

The probability segment of positioning within the second subarea of the i -th frontier is determined at once, as the object can move through only one of those two subareas. Thus there are the prior data in the set of probability segments $\left[p_i^{(1)}; p_i^{(2)} \right]_{i=1}^M$ of positioning within first subareas.

Drill for minimizing the location error

As the active object, trespassing the area through its frontiers subareas, knows the set of probability segments of positioning within them, and the object knows that these segments are learned by the search squad, it will move most cautiously, calculating the best search actions of the squad. If the active object is a lost kid or man, then the searchers should mind the most unlucky attempts of the lost to be appeared. Such conflict process may be modeled as the multistage diagonal 2×2 -game for optimal search-hide [3, p. 13; 4, p. 81], where the matrix elements change (evolve) from stage to stage (from frontier to frontier) due to changing probability segments in the set $\left[p_i^{(1)}; p_i^{(2)} \right]_{i=1}^M$.

It is known, that the diagonal game has the identical optimal behaviors for both players [3, p. 13; 4, p. 81; 5, 6]. Namely, in the i -th frontier diagonal $v_{kj}^{(i)}_{2 \times 2}$ -game both searcher and hider have the optimal strategy

$$Q_{opt}^{(i)} = \begin{bmatrix} q_{opt}^{(i)} & 1 - q_{opt}^{(i)} \end{bmatrix} = \begin{bmatrix} v_{22}^{(i)} & 1 - v_{22}^{(i)} \end{bmatrix} \quad (3)$$

by $V_i = v_{kj}^{(i)}_{2 \times 2} \quad \forall i = \overline{1, M}$. This optimal strategy must hold the optimal behavior in the $i-1$ -th frontier, $i = \overline{2, M}$. Generally the $i+1$ -th frontier probability

segment $\left[p_{i+1}^{(1)}; p_{i+1}^{(2)} \right]$ should be corrected, driven with

$$V_i = \begin{bmatrix} v_{11}^{(i)} & 0 \\ 0 & v_{22}^{(i)} \end{bmatrix} = \begin{bmatrix} y_{opt}^{(i)} q_{opt}^{(i-1)} / \left(y_{opt}^{(i)} q_{opt}^{(i-1)} + 1 - y_{opt}^{(i)} \right) & 0 \\ 0 & 1 - y_{opt}^{(i)} q_{opt}^{(i-1)} / \left(1 - y_{opt}^{(i)} q_{opt}^{(i-1)} + y_{opt}^{(i)} q_{opt}^{(i-1)} \right) \end{bmatrix} \quad (7)$$

$$by \quad q_{opt}^{(i)} = v_{22}^{(i)} = \frac{1 - y_{opt}^{(i)} \quad 1 - q_{opt}^{(i-1)}}{1 - y_{opt}^{(i)} \quad 1 - q_{opt}^{(i-1)} + y_{opt}^{(i)} q_{opt}^{(i-1)}} = \frac{1 - \frac{p_i^{(2)}}{1 + p_i^{(2)} - p_i^{(1)}} \quad 1 - q_{opt}^{(i-1)}}{\left(1 - \frac{p_i^{(2)}}{1 + p_i^{(2)} - p_i^{(1)}} \right) \quad 1 - q_{opt}^{(i-1)} + \frac{p_i^{(2)}}{1 + p_i^{(2)} - p_i^{(1)}} q_{opt}^{(i-1)}} = \frac{0}{1 - p_i^{(1)} \quad 1 - q_{opt}^{(i-1)} / \left(1 - y_{opt}^{(i)} \quad 1 - q_{opt}^{(i-1)} + y_{opt}^{(i)} q_{opt}^{(i-1)} \right)} \quad \forall i = \overline{2, M}. \quad (8)$$

the optimal strategy (3) by $i = \overline{1, M-1}$.

However, elements of the i -th frontier diagonal V_i -game are taken from within segment $\left[p_i^{(1)}; p_i^{(2)} \right]$, including boundary ends. So, here is partial indeterminacy in taking the elements of the i -th frontier diagonal V_i -game matrix. For resolving this indeterminacy there should be solved the antagonistic $\left[p_i^{(1)}; p_i^{(2)} \right]$ -indeterminacy game with the kernel

$$K \quad x, y = \max \left\{ \frac{x}{y}, \frac{1-x}{1-y} \right\} \quad (4)$$

on the square

$$\left[p_i^{(1)}; p_i^{(2)} \right] \times \left[p_i^{(1)}; p_i^{(2)} \right], \quad (5)$$

where y is a suggested value of the probability of positioning within the i -th frontier first subarea, and x is a really existing value of the probability [4, p. 130; 7, 8]. The kernel (4) values mean the maximal disbalance in suggesting the value of the probability. The lower this disbalance the higher class of suggestion has been made. Then in the antagonistic $\left[p_i^{(1)}; p_i^{(2)} \right]$ -indeterminacy game with the kernel (4) on the square (5) the second player, minimizing the kernel (4) value, holds the optimal strategy [4, p. 132]:

$$y_{opt}^{(i)} = \frac{p_i^{(2)}}{1 + p_i^{(2)} - p_i^{(1)}} \quad \forall i = \overline{1, M}. \quad (6)$$

Having taken the elements of the i -th frontier diagonal V_i -game as the second player optimal strategy (6) in the antagonistic $\left[p_i^{(1)}; p_i^{(2)} \right]$ -indeterminacy game with the kernel (4) on the square (5), there is the drill to minimize the location error in searching the active object: on the i -th frontier the search squad should hold at its optimal strategy (3) in the V_i -game with

The optimal strategy (3) with (8) holds the optimal behavior within the $i-1$ -th frontier, where

$$q_{opt}^{(1)} = v_{22}^{(1)} = 1 - y_{opt}^{(1)} = 1 - p_1^{(1)} / \left(1 + p_1^{(2)} - p_1^{(1)} \right), \quad (9)$$

by making the product $y_{opt}^{(i)} q_{opt}^{(i-1)}$ with maintaining the

unit sum of the diagonal elements in (7), $i = \overline{2, M}$.

This prompts that the fraction (8) and (9) of the search squad should be directed in the first subarea of the i -th frontier $\forall i = \overline{2, M}$ and the first frontier respectively. The rest searchers will work in the second subarea of the corresponding frontier. Such direction corresponds to the optimal resource distribution and minimizes the location error of the being searched one.

Module for solving the multistage

diagonal $v_{kj}^{(i)}$ $_{2 \times 2}$ -game

One of the best programmable mathsoft environments is Matlab from “The Mathworks, Inc” [9, 10].

There is extra-comfortable enhancement to write codes not only as scripts with F5-running them after, but also as Matlab-functions.

These Matlab-functions may be defined with several input arguments and may return into Matlab Workspace as many output arguments as needed. In the prob-

lem of searching the active object within partial indeterminacy over M -frontiered area with subarea minimal division there should be input the single argument

– the set $\left[p_i^{(1)}; p_i^{(2)} \right]_{i=1}^M$ of the probability segments.

Having been named “sao_smd” (searching the active object with subarea minimal division), the being constructed Matlab-function first line is on fig. 1.

The set $\left[p_i^{(1)}; p_i^{(2)} \right]_{i=1}^M$ is in the matrix variable

“PS” (probability segments), where the first column contains the probabilities $p_i^{(1)}$, and the second – the probabilities

$p_i^{(2)}$. The output argument is the optimal strategies

$Q_{opt}^{(i)}$, having been assigned as the matrix variable “SSOP” (search squad optimal proportion).

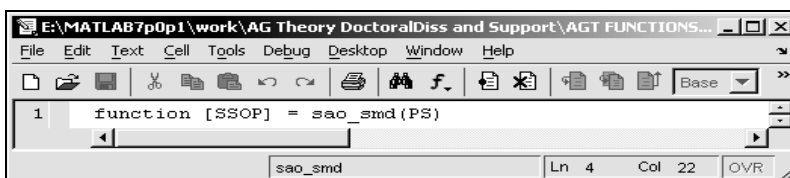


Fig. 1. Matlab-function “sao_smd” header code with input and output arguments

Calculations start from the second line (fig. 2), where the general correctness of the input “PS” is checked. The lines 3 – 5, 6 – 8, 9 – 11 are coded with additionally controlling the correctness of the input. In the line 12 there is calculated $v_{11}^{(1)}$ as $y_{opt}^{(1)}$. And the line

13 gives $Q_{opt}^{(1)}$. The following rest optimal strategies

$Q_{opt}^{(i)}$ are calculated within the lines 14 – 17. At last, the variable “SSOP” is finally passed to the output.

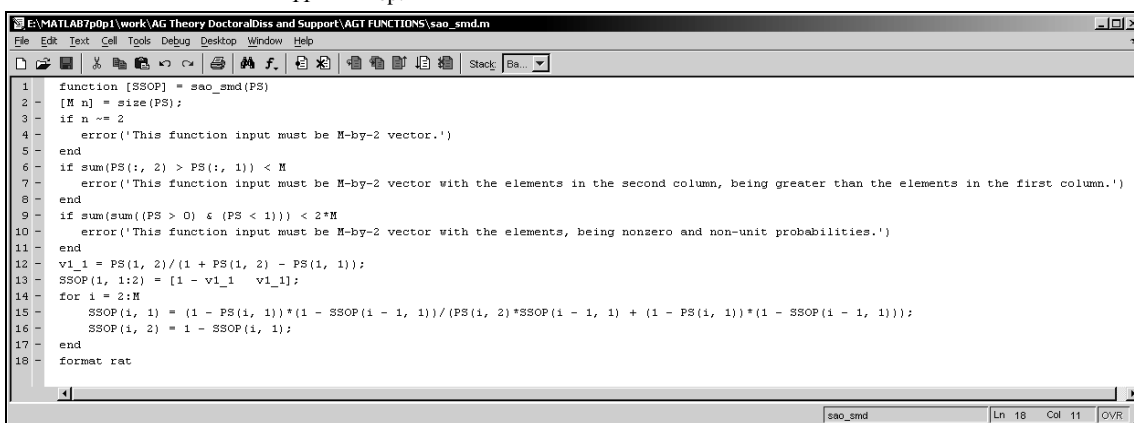


Fig. 2. Matlab-function “sao_smd” full code

It ought to be underlined that for getting calculated $Q_{opt}^{(i)}$ in convenient view there has been supported

the fractional format of the data $Q_{opt}^{(i)}$ presentation in Matlab Command Window [11]. The corresponding coding is in the line 18 of “sao_smd”, and the importance of this will be cleared in the next section.

Exemplifying the Matlab-function “sao_smd” application

Suppose, that there are 6 frontiers with subarea minimal division. The set $\left[p_i^{(1)}; p_i^{(2)} \right]_{i=1}^6$ is

$$\left\{ \begin{array}{l} 0.2; 0.3, 0.3; 0.4, 0.25; 0.35, \\ 0.4; 0.5, 0.3; 0.4, 0.1; 0.2 \end{array} \right\}. \quad (10)$$

Before running “sao_smd”, the set (10) should be typed in Matlab Command Window properly (fig. 3). Actually, there is typed the first line of the “sao_smd” code, only instead of “PS” there is typed the set (10) as it is on

the screenshot (fig. 4). After having run “sao_smd” just by pressing the Enter key, there are instant results in Matlab Command Window (fig. 5). These results may be saved into a mat-file for using them further (fig. 6).

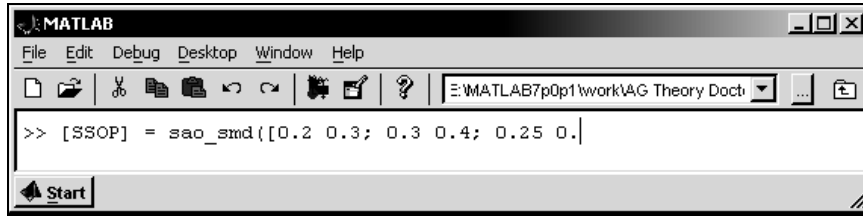


Fig. 3. Starting to type in Matlab Command Window the input argument (10) of “sao_smd” before running it

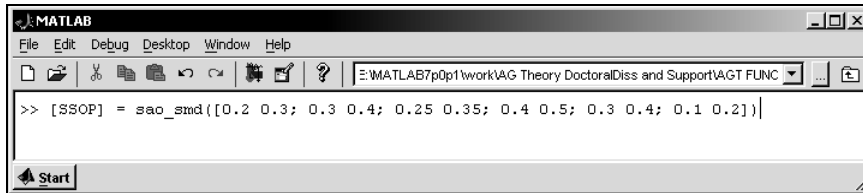


Fig. 4. The typed in Matlab Command Window line with “sao_smd” Matlab-function, being ready to be run by pressing the Enter key

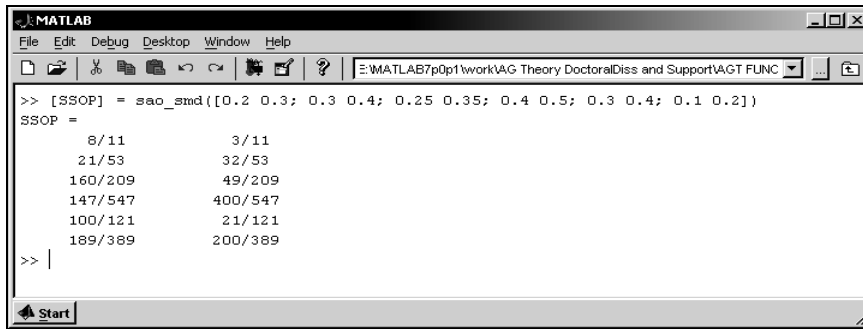


Fig. 5. The calculated set $Q_{opt}^{(i)}$ as matrix variable “SSOP” in Matlab Command Window over data (10)

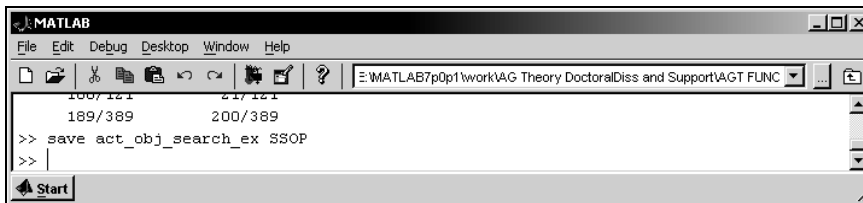


Fig. 6. Exemplifying how to save into a mat-file the calculated optimal behaviors $Q_{opt}^{(i)}$

As it is seen from fig. 5,

$$Q_{opt}^{(1)} = \begin{bmatrix} q_{opt}^{(1)} & 1 - q_{opt}^{(1)} \end{bmatrix} = \begin{bmatrix} 8 & 3 \\ 11 & 11 \end{bmatrix}, \quad (11)$$

$$Q_{opt}^{(2)} = \begin{bmatrix} q_{opt}^{(2)} & 1 - q_{opt}^{(2)} \end{bmatrix} = \begin{bmatrix} 21 & 32 \\ 53 & 53 \end{bmatrix}, \quad (12)$$

$$Q_{opt}^{(3)} = \begin{bmatrix} q_{opt}^{(3)} & 1 - q_{opt}^{(3)} \end{bmatrix} = \begin{bmatrix} 160 & 49 \\ 209 & 209 \end{bmatrix}, \quad (13)$$

$$Q_{opt}^{(4)} = \begin{bmatrix} q_{opt}^{(4)} & 1 - q_{opt}^{(4)} \end{bmatrix} = \begin{bmatrix} 147 & 400 \\ 547 & 547 \end{bmatrix}, \quad (14)$$

$$Q_{opt}^{(5)} = \begin{bmatrix} q_{opt}^{(5)} & 1 - q_{opt}^{(5)} \end{bmatrix} = \begin{bmatrix} 100 & 21 \\ 121 & 121 \end{bmatrix}, \quad (15)$$

$$Q_{opt}^{(6)} = \begin{bmatrix} q_{opt}^{(6)} & 1 - q_{opt}^{(6)} \end{bmatrix} = \begin{bmatrix} 189 & 200 \\ 389 & 389 \end{bmatrix}. \quad (16)$$

For the pre-evaluated segment probabilities (10) the searchers should be shared precisely if there are 25926970201 persons in squad staff, where 25926970201 is the least common multiple of denominators of the probabilities $q_{opt}^{(i)}$. Naturally, it cannot be even imagined to use this number. However, if each frontier has its own subsquad, then the number of searchers within each subsquad should be the following: the first, second, third, fourth, fifth and sixth subsquad should contain $11n_1$, $53n_2$, $209n_3$, $547n_4$, $121n_5$, $389n_6$ searchers, where $n_i \in \mathbb{N} \quad \forall i = 1, 6$ respectively. Then in the first subarea of the first, second, third, fourth, fifth and sixth frontier there must be directed

$8n_1, 21n_2, 160n_3, 147n_4, 100n_5, 189n_6$ searchers by $n_i \in \square \quad \forall i = \overline{1, 6}$ respectively.

Concluding and looking for further investigation

The model of optimal searching the active object within the rectangular area with minimal subarea division, taking the partial indeterminacy $\left[p_i^{(1)}; p_i^{(2)} \right]_{i=1}^M$, is based on the second player optimal strategy in the multistage diagonal 2×2 -game with the kernel, which elements evolve from stage (frontier) to stage (next frontier). By that the kernel elements, beginning from the second frontier, are defined by weighting them with the optimal strategy probabilities on the previous frontier. The probabilities in $Q_{opt}^{(i)}_{i=1}^M$ prompt how to separate (share) the search resource (search squad) optimally (or most rationally as possible) for searching the active object. The constructed program module, taking $\left[p_i^{(1)}; p_i^{(2)} \right]_{i=1}^M$ as input argument, returns $Q_{opt}^{(i)}_{i=1}^M$, helping the decision-maker to share the search squad as fast as possible [12, 13]. Further investigation should be connected with more than minimal subarea division. This will generalize the considered search model.

List of references

1. Катеринчук І.С. Рекомендації щодо вибору раціональних режимів функціонування системи протидії незаконній міграції на державному кордоні / І.С. Катеринчук, А.Б. Мисик, М.Ю. Цибровський // *Честь і закон*. – Х. : АБВ МВСУ, 2007. – № 1. – С. 58-64.
2. Катеринчук І.С. Математична модель визначення імовірних маршрутів руху порушників державного кордону / І.С. Катеринчук, С.М. Ширококов, М.Ю. Цибровський /

Збірник наукових праць. – Хмельницький: Видавництво Національної академії Державної прикордонної служби України імені Б. Хмельницького, 2005. – № 13, Ч. I. – С. 48-53.

3. Теория игр: [учеб. пособие для ун-тов] / Л.А. Петросян, Н.А. Зенкевич, Е.А. Семина. – М.: Высшая школа, Книжный дом "Университет", 1998. – 304 с.
4. Воробьев Н.Н. Теория игр для экономистов-кибернетиков / Н.Н. Воробьев. – М.: Наука, Главная редакция физико-математической литературы, 1985. – 272 с.
5. Оуэн Г. Теория игр: пер. с англ.; 2-е изд. / Г. Оуэн. – М.: Едиториал УРСС, 2004. – 216 с.
6. Суздаль В.Г. Теория игр для флота / В.Г. Суздаль. – М.: Воениздат, 1976. – 318 с.
7. Романюк В.В. Модель визначення оптимального рішення проектувальника у задачі про розрахунок повздовжньої стійкості двох елементів будівельної конструкції при дії на них нормованого стискаючого зусилля / В.В. Романюк // *Проблеми трибології*. – 2010. – № 1. – С. 42-56.
8. Романюк В.В. Моделювання дії нормованого одиничного навантаження на три колони однакової висоти у будівельній конструкції і знаходження оптимальної площі кожної опори / В.В. Романюк // *Проблеми трибології*. – 2010. – № 3. – С. 18-25.
9. Штовба С.Д. Проектирование нечетких систем средствами MATLAB / С.Д. Штовба. – М.: Горячая линия – Телеком, 2007. – 288 с.
10. Дьяконов В. Математические пакеты расширения MATLAB. Специальный справочник / В. Кружлов, В. Дьяконов. – СПб.: Питер, 2001. – 480 с.
11. Романюк В.В. Моделювання реалізації оптимальних змішаних стратегій в антагоністичній грі з двома чистими стратегіями в кожного з гравців / В.В. Романюк // *Наукові вісті НТУУ "КПІ"*. – 2007. – № 3. – С. 74-77.
12. Юдин Д.Б. Математические методы управления в условиях неполной информации / Д.Б. Юдин. – М.: Сов. радио, 1974. – 400 с.
13. Трухаев Р.И. Модели принятия решений в условиях неопределённости / Р.И. Трухаев. – М.: Наука, 1981. – 258 с.

Надійшла до редколегії 16.09.2010

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ПРОГРАМНИЙ МОДУЛЬ ДЛЯ ЕЛЕМЕНТАРНОЇ МОДЕЛІ ПОШУКУ АКТИВНОГО ОБ'ЄКТА В УМОВАХ ЧАСТКОВОЇ НЕВИЗНАЧЕНОСТІ ЯК БАГАТОЕТАПНОЇ ДІАГОНАЛЬНОЇ 2×2 -ГРИ

В.В. Романюк

Програмується пошук активного об'єкта, котрий рухається через прямокутну область, поділену вертикально на ідентичні рубежі з їх мінімальним горизонтальним розділенням. Для усунення часткової невизначеності у попередньо оцінених імовірностях позиціонування об'єкта розв'язується відома континуальна гра на квадраті цих імовірностей. Оптимальною поведінкою пошуковців є розв'язок діагональної 2×2 -гри, що еволюціонує від рубежу до рубежу по області. Також представляється повний код запрограмованого пошуку.

Ключові слова: пошук активного об'єкта, мінімальний поділ підобластей, часткова невизначеність, діагональна гра, багатоетапний розв'язок.

ПРОГРАММНЫЙ МОДУЛЬ ДЛЯ ЭЛЕМЕНТАРНОЙ МОДЕЛИ ПОИСКА АКТИВНОГО ОБЪЕКТА В УСЛОВИЯХ ЧАСТИЧНОЙ НЕОПРЕДЕЛЕННОСТИ КАК МНОГОЭТАПНОЙ ДИАГОНАЛЬНОЙ 2×2 -ИГРЫ

В.В. Романюк

Программируется поиск активного объекта, который движется через прямоугольную область, поделённую вертикально на идентичные рубежи с их минимальным горизонтальным разделением. Для устранения частичной неопределённости в предварительно оценённых вероятностях позиционирования объекта решается известная континуальная игра на квадрате этих вероятностей. Оптимальным поведением разыскивающих является решение диагональной 2×2 -игры, что эволюционирует от рубежа до рубежа по области. Также представляется полный код запрограммированного поиска.

Ключевые слова: поиск активного объекта, минимальный раздел подобластей, частичная неопределённость, диагональная игра, многоэтапное решение.