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SIMPLIFIED METHODS OF COMPUTER-AIDED WIND TURBINES DESIGN

Розглядається проблема оптимального проектування вітрових двигунів різного типу. Запропонована нова комбінована цільова функція, яка одночасно включає параметри вітродвигуна і локальних вітрових характеристик. Показано, що використання спрощеної, комп’ютерно-орієнтовної методики проектування розрахунків дозволяє підвищити річну енерговіддачу вітродвигунів, розміщених в заданому географічному пункті.

Ключові слова: *вітровий двигун, оптимальне проектування, локальні вітрові умови, адаптований програмний продукт.*

Рассматривается проблема оптимального проектирования ветровых двигателей различного типа. Предложена новая комбинированная целевая функция, которая одновременно включает параметры ветродвигателей и локальные ветровые характеристики. Показано, что использование упрощенной компьютерно-ориентированной методики проектных расчетов позволяет увеличить годовую энергоотдачу ветродвигателей, установленных в определенном географическом пункте .

Ключевые слова: *ветряной двигатель, оптимальное проектирование, локальные ветровые условия, адаптированный программный продукт.*

The problem of optimal design of different types of wind turbines is considered. The new combined objective function, which includes wind turbine parameters and local wind characteristics, is used in optimization procedure. It is shown that the application of suggested simplified computer-aided design technique allows to increase annual wind energy output at a given wind turbine site.

Key words: *wind turbine, optimal design, local wind conditions, adapted software.*

1. Introductory remarks. The rapid growth of wind energy production in the world initiated the further development of the new and improvement of the existing methods of wind turbine (WT) design. There exist a lot of publications devoted to the subject of interest [1-20]. The suggested in such publications methods are characterized by different approaches to mathematical modeling and different level of accuracy in determination of basic WT parameters. At the same time to the problem of adaptation of these parameters to the local wind conditions such methods still pay less attention than it necessary to do. The presented in this paper method intends to cover this problem by the use of appropriate optimization technique and

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determination at WT parameters adapted to local wind characteristics, such as mean annual wind speed (MAWS) and its frequency distribution (WSFD). This problem has been discussed in several previous publications of authors [21-24] and here is suggested some new approaches to its solution.

It is known that design procedure consists of several components, and a major parts of them are as follows:

1) Theoretical analysis and calculations which includes the determination of all operational parameters, such as aerodynamic characteristic of rotating blades, power coefficient C_p and its behavior with respect to tip-speed ratio λ, formation of power curve (power of WT versus wind speed), loads and structural stresses analysis, calculation of dynamic characteristics, determination of the rotor blade geometry, nominal wind velocity V_N and nominal rotational velocity ω_N, annual wind energy output, efficiency factor and others. This part of design procedure is based on the principles of optimization of main WT parameters.
2) Structural design of all WT constituent parts, such as rotor, mechanical drive-train, electric generator, control system, supporting tower and many others.
3) Development of appropriate manufacturing, installation and maintenance technology of WT.
4) Accomplishment of thorough economic analysis of manufacturing and maintenance costs, as well as costs of produced energy. The design procedure is usually completed with preparation of corresponding technical documentation.

According to national standards the design procedure is usually subdivided into several stages. In Ukraine we usually follow the appropriate national standard – (ГОСТ 2.103 – 68 – in Russian). In Russian these stages are called: «Техническое предложение» → «Эскизный проект» → «Технический проект». The corresponding English definitions can be formulated as follows: «Conceptual» → «Preliminary» → «Detailed» designs. In the computer-aided design (CAD) the first stage is the most important. Actually, almost all theoretical analysis can be performed at the conceptual stage. Design procedure starts usually with selection of WT type, which is determined by the character of its exploitation. The application of approximate, engineering methods and simplified relations at this stage provides the opportunity to consider the large number of the alternative variants in order to obtain optimal solution.

At the final stages of design the more correct, and therefore, more complicated methods of WT design are used. It is known that computer-aided design (CAD) is erratic system which is based on the «dialog» between the designer and computer. Designer in this procedure relies upon his experience, and upon the statistical data which were accumulated during the analysis of existing prototypes. In this paper the appropriate statistical data will be presented for the use in the design procedure.

2. Methodical background. In this work we will consider two main types of WT which have either horizontal or vertical axis of rotation – HAWT or VAWT. First of all, we introduce the design variables, starting with power coefficient C_p which is given by relation

$$C_p = P_1/P_0\ ,. \qquad (1)$$

where P_0 is the power of undisturbed wind flow and P_1 – is the power extracted from the wind by WT. Tip-speed ratio λ is also very important nondimensional parameter

$$\lambda = \omega R / V_0\ . \qquad (2)$$

Here ω is rotational velocity of WT; R is length of the blade for HAWT and for VAWT it will be maximal distance of a blade section from axis of rotation. Power of the wind is proportional to cubed velocity.

$$P_0 = 0.5 \rho V_0^3 S_1, \qquad (3)$$

where ρ is air density, S_1 is rotor swept area and V_0 is wind velocity far upstream of the rotor.

The proposed here appropriate method of WT design will be based oh the use of special optimization technique. Problem of WT optimal design have been discussed in such publications as [25-30] and many others. In this paper we suggest new, combined objective function, which allows to determine WT parameters optimally adapted to local wind conditions at a proper site.

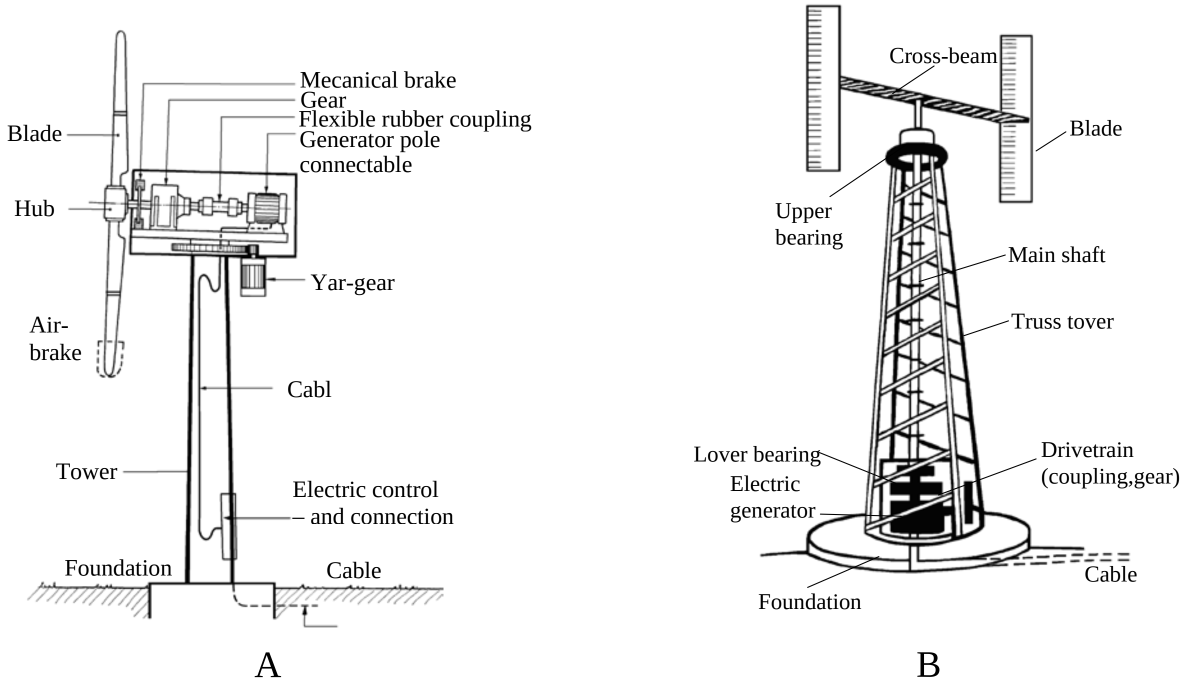

Fig. 1. Schematic view of HAWT-(left) and VAWT-(right)

3. Design of horizontal axis wind turbines. Optimal design procedure of HAWT will be subdivided on two consequtive stages. At the first stage of such procedure we will determine optimal geometry of the rotor blades which provides maximal value of power coefficient. It means that $C_p(\lambda)$will be considered as objective function. Geometrical constrains will be determined by relations (5) and (6). As a mathematical model in this case we will use the blade element theory (see, for example, [20] or [21]). According to this model, power coefficient of WT can be presented in the form:

$$C_p = 8\lambda\int_{\bar{r}_0}^{1}\left(1-\bar{v}_1\right)\cdot\bar{u}_1\,\bar{r}^2 d\bar{r}\,. \tag{4}$$

In this expression v_1 and u_1 are axial and circumferential induced velocities in the plain of rotation, $\bar{v}_1 = v_1/V_0$ and $\bar{u}_1 = u_1/V_0$; r is distance from axis of rotation, $\bar{r} = r/R$; $\bar{r}_0$ – determines the position of root chord on the blade (Fig. 2)

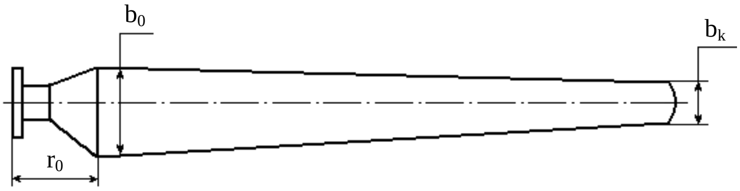

Fig. 2. Typical geometry of HAWT blade

The values of $\bar{v}_1$ and $\bar{u}_1$ depend on $\bar{r}$. Blade chord and twist angle variations along the blade depend on $\bar{r}$ as well, i.e. $\bar{b} = \bar{b}(\bar{r})$ and $\varphi = \varphi(\bar{r})$. It is known that the optimal shape of HAWT blade has curvilinear leading and trailing edges. At the same time, due to technological and other requirements both edges of contemporary blades are usually straight-lined. In such case the shape of the blade can be presented in the form:

$$\bar{b} = \frac{b}{R} = \bar{b}_k\left[\frac{\bar{b}_0}{\bar{b}_k} - \left(\frac{\bar{b}_0}{\bar{b}_k} - 1\right)\left(\frac{\bar{r}-\bar{r}_0}{1-\bar{r}_0}\right)\right]. \tag{5}$$

The relation $\bar{b}_0/\bar{b}_k$ is determined with the use of statistical data, $\bar{b}_0/\bar{b}_k \approx 4.5...5.0$ and $\bar{r}_0 \approx 0.15...0.20$. Geometry of the blade is also characterized by the shape of the airfoil section. Here we will use «Espero» airfoil section, which was suggested by Sabinin [7]. Aerodynamic coefficients of this section – the drag coefficient C_x and lift coefficient C_y as function of angle of attack α are determined experimentally and can be found in [21,24]. Relative thickness of airfoil $\tilde{\delta} = \delta/b$ is usually varies linearly along the blade

$$\tilde{\delta} = \frac{\delta(\bar{r})}{\bar{b}(\bar{r})} = \tilde{\delta}_0 - \frac{\tilde{\delta}_0 - \tilde{\delta}_k}{1-\bar{r}_0}\left(\bar{r}-\bar{r}_0\right). \tag{6}$$

Here we assume that $\tilde{\delta}_0 = 0.35$ and $\tilde{\delta}_k = 0.10$, when $\bar{r} = \bar{r}_0$ and $\bar{r} = 1$ respectively, and $\bar{r}_0 = 0.2$.

The variation of a setting angle (twist angle) φ with respect to $\bar{r}$ and the induced velocities $\bar{v}_1$ and $\bar{u}_1$ are obtained from expressions

$$\frac{\bar{v}_1}{1-\bar{v}_1}=\frac{\bar{b}\cdot l}{\pi}\cdot\frac{C_y\cos\beta+C_x\sin\beta}{8\bar{r}\sin^2\beta}, \tag{7}$$

$$\frac{\bar{u}_1}{\lambda\bar{r}+\bar{u}_1}=\frac{\bar{b}\cdot l}{\pi}\cdot\frac{C_y\sin\beta-C_x\cos\beta}{8\bar{r}\sin\beta\cos\beta}, \tag{8}$$

$$\beta=arctg\frac{1-\bar{v}_1}{\lambda\bar{r}+\bar{u}_1}, \tag{9}$$

$$\varphi=\beta-\alpha\,. \tag{10}$$

Here l is number of blades, β is the angle between the local velocity vector $\bar{W}$ and plane of rotation. The combination $\bar{b}\cdot l/\pi=\sigma$ is called solidity factor. The chord $\bar{b}$ should be substituted from (5).

It is suggested that angle φ in each blade section should provide angle of attack α_0, which corresponds to maximal value of airfoil efficiency $K=C_y/C_x$. For “Espero” airfoil the values of α_0 for different $\tilde{\delta}$, as well as corresponding values of C_x and C_y are given in the Table 1.

Table 1. Values of α_0, C_x and C_y for a different thickness $\tilde{\delta}$

$\tilde{\delta}=\delta/b$	α_0°	$C_y(\alpha_0)$	$C_x(\alpha_0)$
0.100	4.35	0.830	0.00623
0.125	4.10	0.880	0.00680
0.150	3.60	0.940	0.00700
0.200	1.40	1.000	0.00870
0.220	-0.50	0.960	0.01220
0.240	-1.00	0.980	0.01620
0.300	-1.50	0.946	0.08232

According to (5) and (7) – (10) we can write

$$C_p=f_1(\bar{r})\cdot f_2(\lambda\bar{r},\alpha,C_x,C_y,\sigma). \tag{11}$$

As far as $\alpha=\alpha_0$ and corresponding values of C_x and C_y are known, we can see that C_p at a given section of blade depends only on one parameter - rotor solidity factor $\sigma=\bar{b}\cdot l/\pi$. Hawing in mind expression (5) for $\bar{b}(\bar{r})$ we can see that behavior of C_p with respect to λ, i.e. $C_p(\lambda)$ is determined only by the value of $\bar{b}_k$. The typical dependence $C_p(\lambda)$ is shown in Fig. 3.

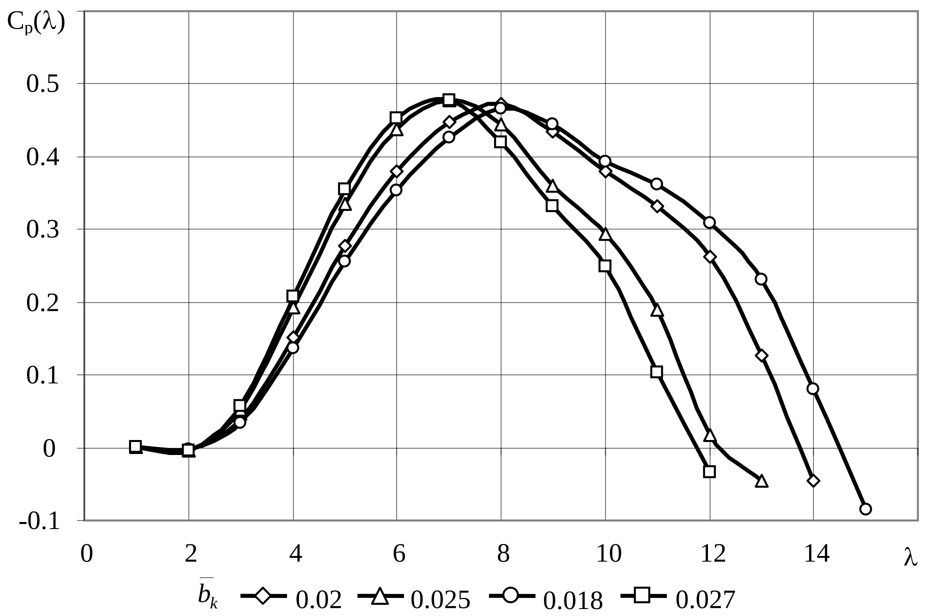

Fig. 3. Power coefficient C_p as a function of $\bar{b}_k = b_k/R$ (i.e. different solidity factors)

In this case $\bar{b}_k = b_k / R$ should be selected as variation parameter when optimization problem, aimed at obtaining maximum value of $C_p = C_{pm}$, is considered. Initial value of $\bar{b}_k$ in this procedure can be obtained with the use of expression [24].

$$\bar{b}_k = \frac{16\pi}{9} \cdot \frac{G}{\lambda_m \cdot l \cdot C_y} \sqrt{\frac{1}{\lambda_m^2 \cdot \bar{r}^2 + 4/9}} \tag{12}$$

Here λ_m corresponds to maximum value of C_p. for HAWT, and usually $\lambda_m \approx 6 \div 7$, C_y – corresponds to the tip blade section (for $\tilde{\delta}_k = 0.1$ we have $C_y = 0.8$ – see Table 1) in this case when $\bar{b} = \bar{b}_k$ we have $\bar{r} = 1$, G is correction factor, $G \approx 0.7$. We will suggest that HAWT has three blades, i.e. $l = 3$. The calculation of C_p as a function of λ and its optimization will be performed with the use of expressions (4)…(10).
The consecutive steps of this procedure are presented below:

1) Assume value of $\bar{b}_k$.
2) Assume λ for a given $\bar{b}_k$ within the range $\lambda = 1 \ldots \lambda_k$, where $\lambda = \lambda_k$ corresponds $C_p(\lambda) = 0$ (autorotation regime).
3) For a given λ assume $\bar{r}$ within the range from 0.2 to 1.0;
4) Determine $\tilde{\delta} = \tilde{\delta}(\bar{r})$ from (6).
5) Determine $\alpha = \alpha_0$, and corresponding values of C_x and C_y as a function of $\bar{r}$ – see the Table 1.
6) Determine $\bar{b}(\bar{r})$ from (5).
7) With the of (7), (8), (9) conduct the iteration procedure to obtain $\beta(\bar{r})$, $\bar{v}_1(\bar{r})$ and $\bar{u}_1(\bar{r})$. This procedure is given below (*a, b, c, d* points)
   a) Assume $\bar{u}_1 = \bar{v}_1 = 0$ and obtain β from (9);
   b) Obtain $\bar{u}_1$ and $\bar{v}_1$ from (7) and (8);

   c) Obtain β from (9) and return to position b;
   d) Stop the iteration process when difference between consequent and previous values of $\bar{v}_1$ and $\bar{u}_1$ are small enough (less than it was assumed). Hence, we will have $\bar{v}_1(r_i)$, $\bar{u}_1(r_i)$ and $\varphi(r_i)$, which are obtained from (10);

8) Hawing the $\bar{v}_1(\bar{r})$ and $\bar{u}_1(\bar{r})$ for a given $\bar{b}_k$ and different λ, determine $C_p(\lambda)$ from (4).
9) Assume new value of $\bar{b}_k$ and repeat the steps (2) – (8).
10) By comparing $C_p(\lambda)$ for different $\bar{b}_k$ select the optimal variant.

In Fig. 3 is shown C_p as a function of λ for different $\bar{b}_k$ and its maximum values. Value of $\bar{b}_k$ at the optimal variant should be selected with the use of two factors: value of C_{pm} and the width of so-called «peak zone». Wide peak zone provides more stable performance of WT when C_p deviates from its maximum position (it does not drop very rapidly). It can be shown that twist angle variation along the blade, i.e. $\varphi(r)$, depends on λ. At the final position of optimization procedure (when $\bar{b}_k$ and total solidity factor σ_Σ is fixed) the twist angle φ as a function of $\bar{r}$ can be also fixed for $\lambda=\lambda_m$ (λ_m corresponds to $C_p=C_{pm}$). Total solidity factor σ_Σ is determined as relation of blade area to the swept area of WT.

As a result of presented above computation we obtain: geometry of the rotor blades for a selected HAWT, thickness $\tilde{\delta}(\bar{r})$, chord $\bar{b}(\bar{r})$ and angle $\varphi(\bar{r})$ variations along the blades; power coefficient as a function of tip speed ratio – $C_p(\lambda)$.

Torque coefficient C_M and the thrust coefficient C_k is determined from the relations

$$C_M = C_p/\lambda\,, \tag{13}$$

$$C_k = 8\int_{\bar{r}_0}^{1}(1-\bar{v}_1)\bar{v}_1\bar{r}\,d\bar{r}\,. \tag{14}$$

Here $\bar{v}_1(\bar{r})$ is obtained earlier (position d in the iterative procedure).

It is necessary to emphasize that rotor geometry, its aerodynamic and power characteristics are obtained with the use of optimization procedure.

There exist several modification of the blade element theory. One of them is presented in [31]. System of equations (7)…(10) have been transformed to the form

$$\sin\beta - 2\lambda\bar{r}\sin^2\beta - \frac{\sigma}{4\bar{r}}\left[\left(C_y+\lambda\bar{r}C_x\right)\cdot\sin\beta+\left(\lambda\bar{r}C_y-C_x\right)\cos\beta\right]=0 \tag{15}$$

$$\beta=\alpha+\varphi \tag{16}$$

When $\alpha=\alpha_0$ fixed we have two unknown variables - β an φ. By solving this equations we can obtaine $\bar{v}_1$ and $\bar{u}_1$ with the use of relations

$$\bar{v}_1 = \frac{\sigma C_t}{\sigma C_t + 8\bar{r}\, sin^2 \beta} \tag{17}$$

$$\bar{u}_1 = \frac{C_n}{C_t} \cdot \bar{V}_1, \tag{18}$$

where

$$C_n = C_y\, cos\, \alpha + C_x\, sin\, \alpha \tag{19}$$

$$C_t = C_y\, sin\, \alpha - C_x\, cos\, \alpha \tag{20}$$

With the aid of relations (15)…(20) have been performed optimization of HAWT blade. At the beginning of the procedure was obtained the blade geometry with curvilinear leading and trailing edges and then it was transformed to straight-lined edges. The results are shown in Fig.4 and Fig.5

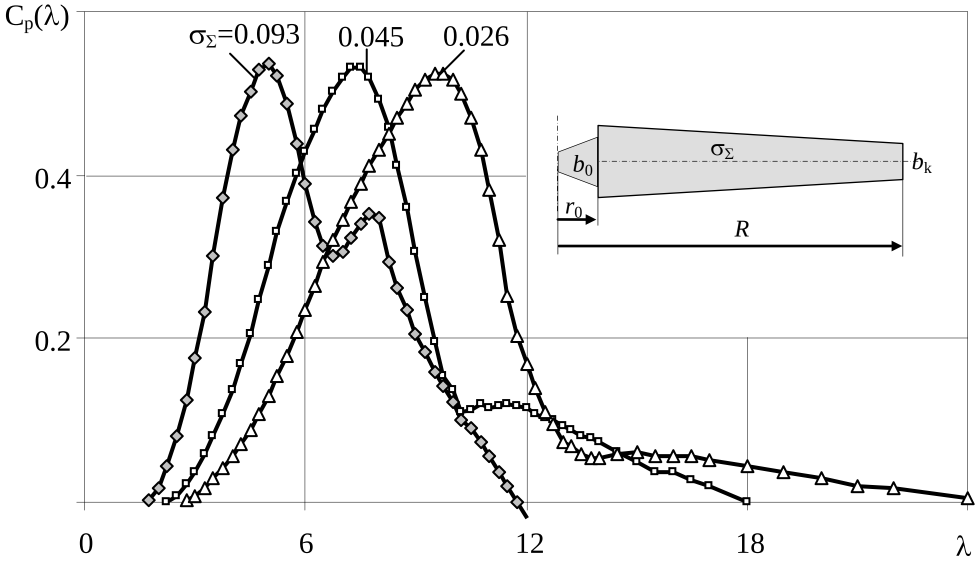

Fig. 4. Power coefficient Cp as a function of tip-speed ratio λ for different σΣ

Here σ_Σ is total blade solidity (relation of blade area to the WT swept area). The twist angle variation along the blade is presented in Fig.5.

Fig.4 shows that dependence $C_p(\lambda)$ for optimal blade can be non monotonous. The second stage of optimization is based on the use of annual WT energy output per unit of the swept area - $\overline{E} = E/S_1$.

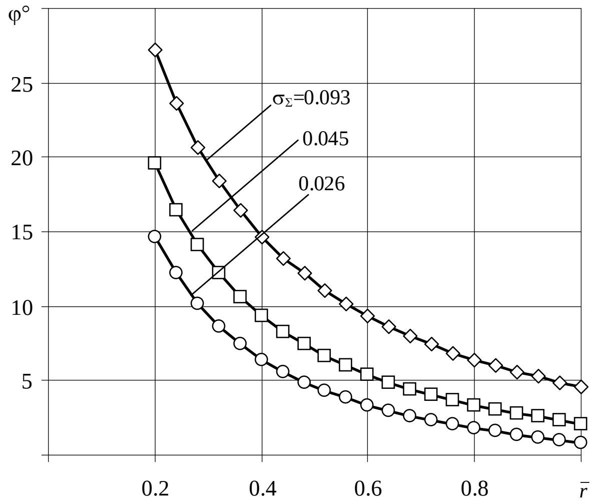

Fig. 5. Twist anale φ as a function of $\bar{r} = r$

The dependence of $\bar{E}$ from the nominal wind velocity V_N, i.e. $\bar{E}(V_N)$ will serve as objective function, which is necessary to maximize at the intended WT site.

$$\bar{E} = \bar{P} \cdot T = \frac{\rho \eta T}{2000} \int_{V_S}^{V_N} V^3 C_p(V) f(V) dV + \frac{\rho V_N^3 C_{pm} T}{2000} \int_{V_N}^{V_k} f(V) dV \tag{21}$$

Here $\bar{P}$ (kW) is mean annual WT power density; ρ is air density; $\eta = \eta_1 \cdot \eta_2$ (η_1 is efficiency of drive-train, $\eta_1 \approx 0.9$, and η_2 is efficiency of electric generator, $\eta_2 \approx 0.9$); $T = 8760$ hour (annual time); $f(V)$ - is wind speed frequency distribution at the proper WT site; V_N - nominal wind velocity; V_s - starting velocity; V_k - cut out velocity (wind storm velocity); nominal WT power P_N will be given and serves as initial dimensional parameter in the design procedure

$$P_N = \frac{\rho V_N^3}{2} \cdot C_{pm} \cdot \eta \cdot S_1 \tag{22}$$

Nominal velocity V_N is the variation parameter and is determined as a function of mean annual wind speed V_c

$$V_N / V_c == 1.5...2.5 \tag{23}$$

If nominal velocity V_N and λ_m is fixed, we can obtaine $C_p(V)$, when $C_p(\lambda)$ is given. In this case we have

$$\lambda = \frac{\omega R}{V} = \frac{(\omega R)_N}{V_N} \cdot \frac{V_N}{V} = \lambda_m \frac{V_N}{V} \tag{24}$$

As far as λ_m and V_N is known, we will have C_p as a function of V.

The constrains at this stage will include, first of all, parameters, which have been obtained at the first stage, namely: the optimal (for a given type of WT) function $C_p(\lambda)$ with fixed value of $C_p = C_{pm}$ and corresponding to it value of $\lambda = \lambda_m$.

Constrains includes several local wind characteristics: wind velocity density function (WVDF) *f(V)* and mean annual wind speed $V_{\tilde{n}}$ at the intended WT site. Apart from the named constrains we also have: $0 < V < V_k; V_s < V_N < V_k;$ V_k≈25…30m/s; V_S≈3…4m/s.

The optimal design procedure includes the following steps:

1. **Select nominal velocity with the use of (23);**
2. **Transform function $C_p(\lambda)$, obtained at the first stage, to $C_p(V)$ with the use of (24);**
3. **Determine $\overline{P}$ from (21);**
4. **Select new value of V_N and repeat procedure from the step 1 to step 3;**
5. **Calculate the function $\overline{P}(V_N)$ and fix the value of V_N, where $\overline{P}$ has maximum.**

4. Design of vertical axis wind turbine. Optimal design of VAWT is based on the use of other calculation technique as compared it to HAWT. There exist several specific methods which can be applied for this purpose. By working within framework of conceptual design, we intend to use simple and straightforward methods. One of them have been suggested by Templin in 1974 [32]. It is based on single stream tube approach when the induced velocity V_1 is supposed to be constant across the rotor. This approximate theory allows to obtain a closed solution. Detailed description of the method is presented, for example, in [21]. Plane view of the flow near such rotor is shown in Fig.6.

It is known that there exist two categories of VAWT – lift-driven rotors (like Darrieus WT), and drag-driven rotors (like Savonius WT). Here we will consider the first category. It will be also supposed that such WT have electric generator and are connected to external electricity grid. The schematic presentation of VAWT rotor is given in Fig. 7

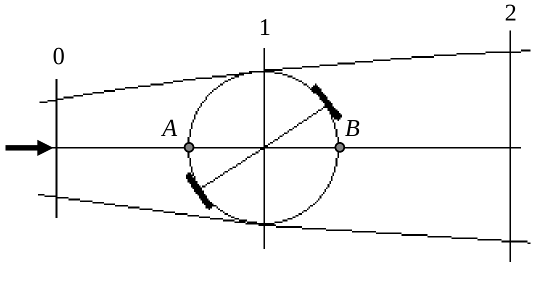

Fig.6. Plane view of the flow near the vertical rotor

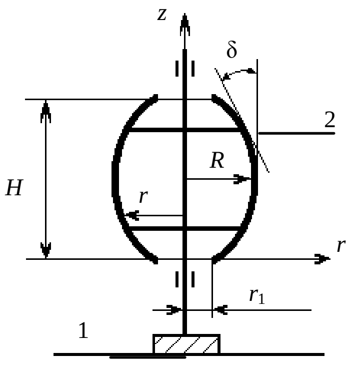

Fig. 7. Schematic presentation of VAWT rotor.

Here 1 is central shaft and 2 is curvilinear blade.

In the case when top and bottom radiuses equal to zero, i.e. $r = r_1 = 0$, we will have classic WT rotor, which have been introduced by Darrieus, see, for example [11].

On the other hand, when $r_1=R$ we will have so-called H-rotor with the straight-lined and vertically suspended blades, and $\delta=0$. Curvilinear blades of the rotor, shown in Fig.7, can be approximately represented by parabola. With the use of r, z as coordinate system the rotor shape will be described by expression

$$\overline{r} = 1 - \frac{4\left(1-\overline{r}_1\right)}{\overline{H}^2}\left(\overline{z} - \frac{\overline{H}}{2}\right)^2 , \tag{25}$$

where $\overline{r} = r/R$; $\overline{r}_1 = r_1/R$; $\overline{z} = z/R$; $\overline{H} = H/R$. Value of $\overline{H}$ is usually selected by designer and according to statistics, $\overline{H} \approx 1.5 - 2.0$. The swept area of the rotor S_1 is given by

$$S_1 = \frac{2}{3}R^2\overline{H}\left(2\overline{r}_1 + 1\right) \tag{26}$$

The second stage of optimal design is oriented on other objective function and corresponding constrains. As an objective function we suggest mean annual values of wind power density $\overline{P} = P_c / S_1 = f_1(V_N)$ or energy output S_1, $\overline{E} = E / S_1 = f_2(V_N)$, per unite of swept area S_1 which is necessary to maximize. There exists simple interrelation between these two parameters:

$$\overline{E} = \overline{P} \cdot T(kWh) \tag{37}$$

$$C_p = \frac{3\cdot\sigma\cdot\overline{V}_1^{\,3}\lambda_1}{4\overline{H}\left(2\overline{r}_1 + 1\right)}\int_0^{\overline{H}} d\overline{z}\int_0^{2\pi} C_t \cdot \frac{\overline{W}^2}{\overline{V}^2}\cdot\frac{\overline{r}}{\cos\delta}d\theta, \tag{27}$$

where W is total velocity ahead of airfoil section of the blade, and then

$$\left(\overline{W} / \overline{V}_1\right)^2 = \sin^2\theta\cos^2\delta + \left(\lambda_1\overline{r} + \cos\theta\right)^2, \tag{28}$$

$$\lambda_1 = \frac{\omega R}{V_1} = \frac{\lambda}{\overline{V}_1}, \tag{29}$$

$$V_1 = \frac{1}{1+G}, \tag{30}$$

$$G = \frac{3\cdot\sigma}{16\overline{H}\left(2\overline{r}_1 + 1\right)}\int_0^{\overline{H}} d\overline{z}\int_0^{2\pi}\frac{\overline{W}^2}{\overline{V}^2}\left(C_n\sin\theta - \frac{C_t\cos\theta}{\cos\delta}\right)d\theta, \tag{31}$$

$$C_t = C_y\sin\alpha - C_x\cos\alpha, \tag{32}$$

$$C_n = C_y\cos\alpha + C_x\sin\alpha, \tag{33}$$

$$\alpha = arctg\frac{\sin\theta\cos\delta}{\lambda_1\overline{r} + \cos\theta}. \tag{34}$$

Solidity factor σ is given by relation

$$\sigma = \frac{b\cdot l}{\pi R} = \frac{\overline{b}\cdot l}{\pi} \tag{35}$$

Here b is chord of airfoil section of the blade, and $\bar{b} = b/R$. As well as in the case of HAWT, optimal design of VAWT will be also subdivided in two stages. Let's consider the first stage (optimization of Darrieus H-rotor). For this rotor power coefficient C_p has the form

$$C_p = \frac{\sigma \cdot \bar{V}_1^3 \lambda_1}{4\bar{H}} \int_0^{\bar{H}} d\bar{z} \int_0^{2\pi} \left(C_t \cdot \frac{W^2}{V^2} \right) d\theta \tag{36}$$

The objective function at this stage will be $C_p(\lambda)$, which is necessary to maximize. The variation parameter will be solidity factor σ. The necessary constrains will be as follows:

- Geometry of blade airfoil section and its aerodynamic characteristics – $C_x(\alpha)$ and $C_y(\alpha)$ are given;
- Tip-speed ratio λ varies within the range from λ_1=1 to $\lambda=\lambda_{1k}$ where λ_{1k} corresponds to autorotation regime (when $C_p \rightarrow 0$);
- Power coefficient, $C_p \geq 0$;
- Number of blades, l=2…4.

Optimization procedure consist of several steps. Here we present such procedure.

1. Assume value of solidity factor σ within the range from σ=0.05 to σ=0.5;
2. Assume value of λ_1, starting from λ_1=1 and up to $\lambda_1=\lambda_{1k}$;
3. Select the azimuthal angle within the rouge from θ=0 to θ=2π;
4. Determine the angle of attack with the use of (34);
5. Determine C_t and C_n with the use of (32) and (33);
6. Determine $\left(\bar{W}/\bar{V}_1\right)^2$ from (28);
7. Determine G from (31);
8. Determine $\bar{V}_1$ from (30);
9. Determine C_p for a given λ_1 with the use of (27);
10. Assume new value of λ_1 and repeat the procedure from the step (26) to step (33);
11. Determine λ from (26);
12. Obtain the function $C_p(\lambda)$ for a given value of σ;
13. Assume the new value of σ and repeat the procedure from step 1 to step 12;
14. Obtains the function $C_p(\lambda)$ for the next value of σ;
15. Perform analysis of $C_p(\lambda)$ for a different values of σ and select an optimal variant, having in mind maximum value of C_p and the width of a peak zone.

Power coefficient C_p as a function of λ for vertical-axis rotor is shown in Fig.8 for different solidity factors σ. It can be seen that $C_p(\lambda)$ for σ=0.15, which have C_{pm}=0.42 and λ_m=4, can be recommended as optimal variant because it has high value C_{pm} and wide peak zone.

The second stage of optimal design is oriented on other objective function and corresponding constrains. As an objective function we suggest mean annual values of wind power density $\bar{P} = P_c / S_1 = f_1(V_N)$ or energy output S_1, $\bar{E} = E / S_1 = f_2(V_N)$, perunite of swept area S_1 which is necessary to maximize. There exists simple interrelation between these two parameters:

$$\bar{E} = \bar{P} \cdot T(kWh) \tag{37}$$

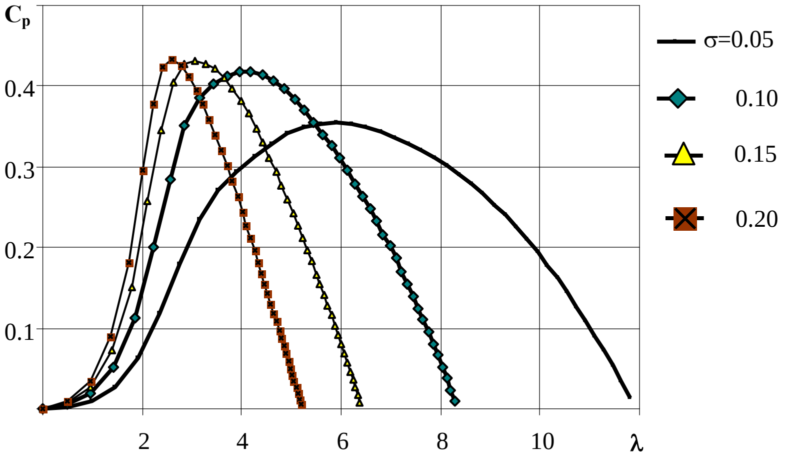

Fig. 8. Variation of C_p with respect to λ for VAWT with different values of solidity factor σ

Here T is annual time, *T=8760* hours

$$\bar{P} = \frac{\rho\eta}{2000}\int_{V_s}^{V_N} V^3 C_p(V) f(V) dV + \frac{\rho V_N^3 C_{pm}\eta}{2000}\int_{V_N}^{V_k} f(V) dV \tag{38}$$

In this expression V_s and V_N are correspondingly starting and nominal velocities, and V_k is cut-out velocity (wind storm velocity); $\eta=\eta_1,\eta_2$, where η_1 and η_2 are drive-train and electric generator efficiencies respectfully. The variation parameter at the second stage is the nominal wind velocity V_N, which depends on mean local wind parameter k_E.

$$V_N = V_c \cdot k_E \tag{39}$$

Here $k_E = 1.5...2.5$

When V_N is selected, we can transform function $C_p(\lambda)$, obtained at the first stage, to function $C_p(V)$, having in mind [24]

The constrains at this stage will include, first of all, parameters, which have been obtained at the first stage, namely: the optimal (for a given type of WT) function $C_p(\lambda)$ with fixed value of $C_p = C_{pm}$ and corresponding to it value of $\lambda=\lambda_m$. Constrains includes several local wind characteristics: wind velocity density function (WVDF) *f(V)* and mean annual wind speed $V_{\tilde{n}}$ at the intended WT site. Apart from the named constrains, we also have: $0 < V < V_k; V_s < V_N < V_k;$ $V_k \approx 25 \ldots 30$m/s; $V_S \approx 3 \ldots 4$m/s.

The optimal design procedure includes the following steps:

1. Select nominal velocity with the use of (39);
2. Transform function $C_p(\lambda)$, obtained at the first stage, to $C_p(V)$ with the use of (24);
3. Determine $\overline{P}$ from (38);
4. Select new value of V_N and repeat from the step from 1 to step 3;
5. Calculate the function $\overline{P}(V_N)$ and fix the value of V_N, where $\overline{P}$ has maximum.

It will be end the of optimal design procedure. We obtained optimal geometrical, aerodynamic and power characteristics of horizontal-axis and vertical-axis wind turbines. The corresponding software system, which is supposed to be used in computer – aided design, is worked out.

As an example of suggested approximate method application the optimal design procedure was conducted for HAWT and VAWT, which have both nominal power $P_N = 800kW$. It is supposed that HAWT is installed near Borispol (Ukraine), where mean anneal wind speed $V_c = 4.3m/s$, and VAWT is installed at the offshore territory of Sivash lake, where $V_c = 6m/s$. The result of calculation is shown in Fig.9 and Fig.10. It can be seen that nominal wind speed, providing maximum power density $\overline{P}(kW/m^2)$, and maximum annual energy output $\overline{E}(kWh/m^2)$, in case of HAWT is equal $V_N \approx 8.5m/s$ and case of VAWT $V_N \approx 12m/s$.

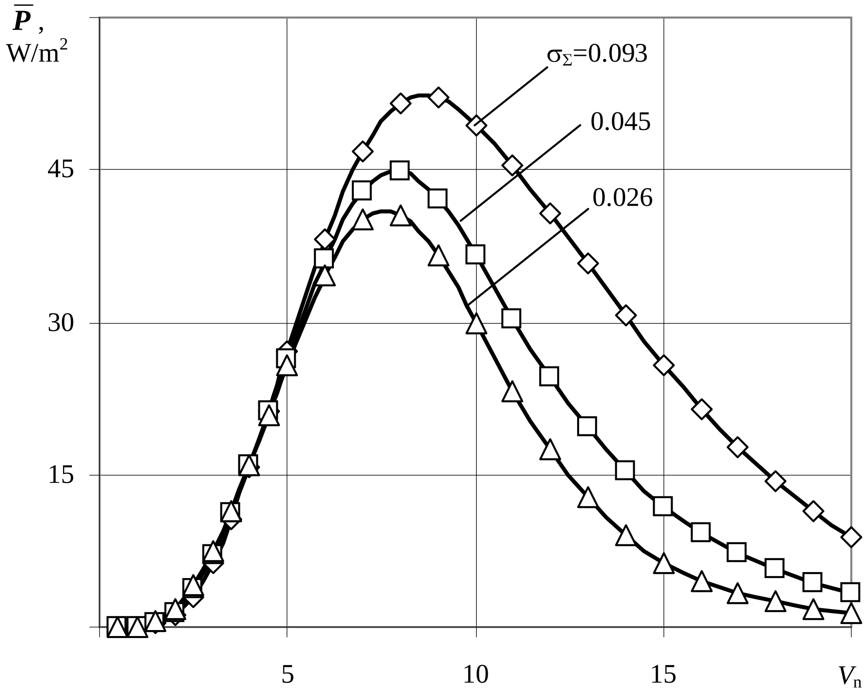

Fig. 9. Mean annual power density as a function of nominal velocity for HAWT with different solidity factors σ_Σ

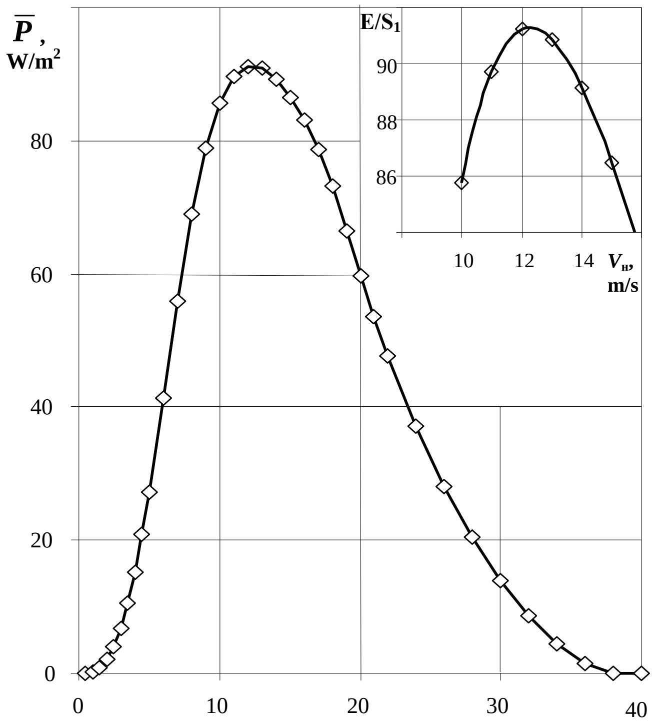

Fig. 10. Mean annual wind energy output as a function of nominal velocity for VAWT

The calculation is performed for nominal power $P_N = 800kW$ (anneal wind speed $V_c = 5m/s$). In this case HAWT has $V_N \approx 8m/s$, ω_N = 20 rpm, R = 29m. In the of VAWT we have $V_N \approx 9m/s$, ω_N = 9 rpm, R = 29m, H = 48m.

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