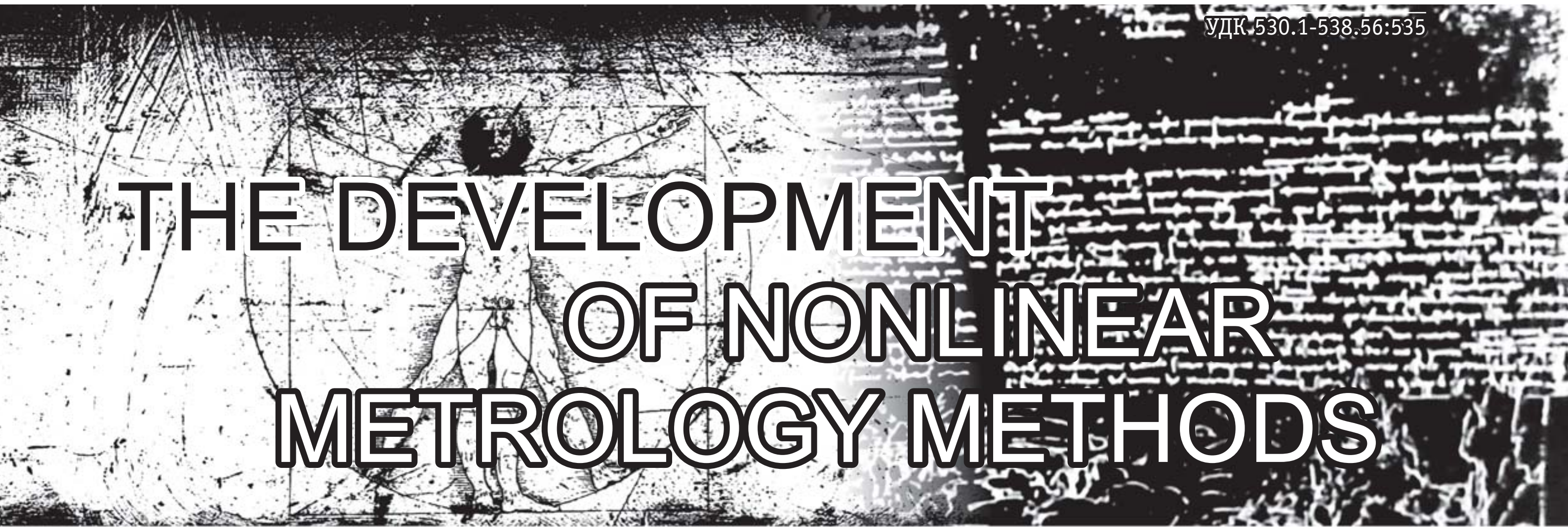

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THE DEVELOPMENT OF NONLINEAR METROLOGY METHODS

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У статті представлено результати дослідження у новій перспективній сфері — нелінійній метрології. У рамках теорії нелінійної метрології створено спеціальні моделі вимірювання, моделі аналізу результатів вимірення, засновані на ключових принципах та концепціях теорії динамічного хаосу та фрактальних уявлень щодо динаміки реальних систем. Наведено модель вимірення здоров'я людини, яка містить числовий портрет, ентропійну та часову шкали для оцінювання стану здоров'я людини взагалі, під час лікування або спортивної підготовки.
Ключові слова: *нелінійні метрології, нелінійна динамічна система, моделі вимірювання.*

В статье представлены результаты исследования в новой перспективной сфере — нелинейной метрологии. В рамках теории нелинейной метрологии созданы специальные модели измерения и модели анализа результатов измерения, основанные на ключевых принципах и концепциях теории динамического хаоса и фрактальных представлений о динамике реальных систем. Приведена модель измерения здоровья человека, которая содержит числовой портрет, энтропийную и временную шкалы для оценки состояния здоровья человека вообще, в процессе лечения или спортивной подготовки.
Ключевые слова: *нелинейные метрологии, нелинейная динамическая система, модели измерения.*

The article contains the results of research in a new perspective sphere of nonlinear metrology. The special measurement model and measurement results analysis model, that based on the main principles and concepts of dynamical chaos theory and fractal representations of the real systems behavior, are offered in the frame of nonlinear metrology theory. The model of human health measurement is represented here. It has the numerical portrait, entropy and time lines for evaluation of human health condition. It can be used for assess the state of human health during a treatment or an athletic training.
Keywords: *nonlinear metrology, nonlinear dynamical system, measurement model.*

INTRODUCTION

The nonlinear metrology is a new perspective scientific area. It unites the complex tasks of measurement of real nonlinear dynamical systems parameters. Most of the real-world systems are open, dissipative and nonlinear dynamical systems (NDS). The states of such systems are characterized by a group of dynamical variables (DV) $(X^1(t), X^2(t), \ldots, X^n(t))$ (generally for n-dimensional space). The DV values at any time t_i relate to the initial values $(X^1(t_0), X^2(t_0), \ldots, X^n(t_0))$ by the evolution function of dynamical system F [1]:

$$F(X^1(t_0), X^2(t_0), \ldots, X^n(t_0)) \to (X^1(t), X^2(t), \ldots, X^n(t)).$$

In general case the DV behavior during the time can be regular or chaotic according to the NDS properties and their initial conditions.

Analysis of the measured quantity values of DVs by the standpoints of the classical metrology approaches is possible only in case of regular or stationary behavior of system. If NDS behavior is classified like the «dynamical chaos», measurement, process and analyze of the measured quantity values are able only with using new methodological basis.

The classical models of measurement, process and analyze of the measured quantity values are based on two key physical positions:

- measured physical quantity can be represented by a single value, the values of physical quantities in transition or dynamical processes can be described by mathematical equations, that also ensures the uniqueness of the physical quantity value;
- physical quantities of systems are ergodicity values and, as a consequence, measured quantity values are ergodicity values too and their allocation is random [2].

However, DV of NDS can't be characterized by a single value and DV behavior can't be described by deterministic equation. The examples of successful description of real systems behavior by equations (a recovery of evolution function for dynamical system F) are very rare events. The specific metrological approaches, measurement models and methods for evaluation of measurement uncertainty must be developed for measurements in the NDS. For solving this problem the measurement model [3] and the measurement results analysis model [4] for NDS are created. These models base on the principles and methods of fractal analysis and dynamical chaos theory. Entropy analysis of the measurement results for NDS is made.

The task of the article is review of main results of research in the sphere of nonlinear metrology.

1. The measurement model

The model for measurement of DV in NDS [3] contains: the scheme of measurement experiment; the method for assessment of necessary and sufficient volume of information; the method for identification of the system behavior and for choosing the mathematical tools for measurement results processing; the method of measured quantity values evaluation.

The measurement model is destined for obtaining information about one of DVs set — X. If behavior of measurement system is chaotic, that system's phase portrait is a strange attractor with clear boundaries. The strange attractor projection on the axis of X values is equal to the interval $[X_{\min}, X_{\max}]$ that contains all possible true quantity values of DV. The purpose of measurement is to evaluate this interval. The main difference between DV of NDS and random variable is that one DV is characterized by interval of all possible true quantity values $[X_{\min}, X_{\max}]$.

According to the postulate that it is impossible to get the true quantity value of X during the measurement, the interval of true quantity values $[X_{\min}, X_{\max}]$ must be determined only with measurement uncertainty too. Therefore, applying the measurement model gives an interval $U(X) > [X_{\min}, X_{\max}]$ that contains all measured quantity values x_i of X_i (here X_i is a state of X) and theirs measurement uncertainties u_i. The interval $U(X)$ is equivalent to measurement uncertainty of all possible DV states. For calculation of $U(X)$ the group of m identical measuring instruments forms m time series of measured quantity values:

$$x_i^1(t_i), \ldots, x_n^1(t_n); x_i^2(t_i), \ldots, x_n^2(t_n); \ldots; x_i^m(t_i), \ldots, x_n^m(t_n), \quad (1)$$

here $x_i^1(t_i)$, $x_i^2(t_i)$, $x_i^m(t_i)$ — the measured quantity values of state X_i in the time moment t_i, that are got by measuring instrument №1, №2, №m respectively; n — number of X states.

Evaluation of measurement results y_i is based on knowledge about sources of uncertainties and the type A measurement uncertainty values. It can be described by next way [7]:

$$(y_1 - u_1, y_1 + u_1); (y_2 - u_2, y_2 + u_2); \ldots; (y_n - u_n, y_n + u_n). \quad (2)$$

The measured quantity values (1) in the phase space are displayed like areas u_i (Figure 1, a), and the measurement results of all possible states of X look like the projection of all u_i on the phase plane $U(X)$ (Figure 1, b).

For calculation of $U(X)$ the minimum $(y_{\min} - u_{\min}$, $y_{\min} + u_{\min})$and the maximum $(y_{\max} - u_{\max}, y_{\max} + u_{\max})$ values of the measurement results (2) are chosen. In this case all possible values of DV X are located in the interval:

$$U(X) = (y_{\min} - u_{\min}, y_{\max} + u_{\max}). \quad (3)$$

For classification of DV behavior the measurement model uses the method of fractal analysis of time series (2) [6]. For this the fractal dimension D of time series (2) is defined by Hurst method. If D=1,5 the DV behavior is random. In a case when 1<D<1,5 or 1,5<D<2 the DV behavior is chaotic. The fractal analysis of time series lets select the correct mathematical tools for measurement results processing.

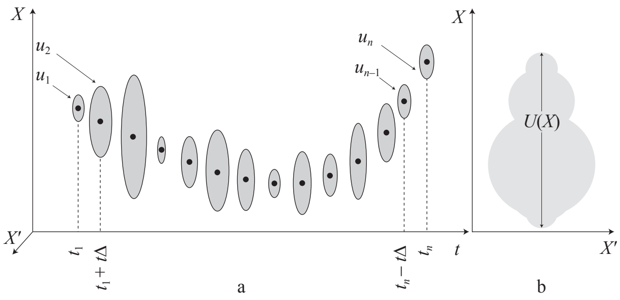

Figure 1. Measurement results: a — the measured quantity values of DV X at different time moments t_i, where Δt — an interval between measurements; *b* — the measurement results of all possible states X

A fractal dimension D is used also for determination the necessary and sufficient number of measurement experiments:

$$n_{\min} \geq 10^{2+0,4D}.$$

Using the measurement models for NDSs [3] allows researching any random process with one metrological base, extending the Guide to the expression of uncertainty in measurement [7] for such complex systems like open, dissipative and chaotic NDS.

2. The measurement results analysis model

In the metrological theory a measurement equation is used like a tool for analysis of measurement results. A necessary condition for creation of measurement equation is stability of system. Stability is ability of system to save settings or dynamic under small perturbations and it is required condition for the analysis and prediction of the DV behavior. It's well known that there are some definitions for stability. Creating the measurement equation asks for the Lyapunov stability when the two random trajectories of the system phase portrait are close to each other at any time. The trajectories of chaotic NDS diverge exponentially, so chaotic NDS are not stabile by Lyapunov. For these systems the measurement equation can't be created and it is necessary to develop new alternative analysis tools.

In dynamical systems theory, along with the Lyapunov stability, the Lagrange stability is considered. The Lagrange stability asks for a location of all measured DV X values within a certain phase space area. In the case of dissipative chaotic NDS such area is a strange or chaotic attractor. If the system phase portrait is a strange attractor, the NDS is stabile by Lagrange. In this case, all possible values X_i of X locate in the interval $U(X)$ (3).

Thus, if the NDS is not stabile by Lyapunov that its description by the measurement equation is an impossible task, but if the NDS is stabile by Lagrange that its dynamic can be analyzed and predicted with using $U(X)$ (3) — the measurement results of all possible states X_i of DV X.

The measurement results analysis model [4], instead of researching the measurement equation, proposes to research the key NDS parameters. The model provides making a number of successive operations: determination of the attractor embedding dimension, the phase portrait restoration, the definition of local (the Lyapunov exponents, the time of prediction) and general (Kolmogorov-Sinay entropy) parameters of NDS.

The most important part of the model is a restoration of a phase portrait. The restoration method was proposed by F. Takens [8] and consists in the construction of the state vectors of a system using the time series of measured quantity values (1):

$$\vec{x}(t_i) = \left(x_1(t_i), x_2(t_i - \tau), \ldots, x_M(t_i - (M-1)\tau)\right), \qquad (4)$$

here τ — the time-step delay of state vector components; M — the embedding dimension of phase portrait.

The Takens method is the established and widely used tool for restoration of a phase portrait. But from the metrological point of view it has the drawbacks. The method uses measured quantity values (1) like initial dates and doesn't use the measurement uncertainty. Since the true value of DV X_i locates in the interval $y_i - u_i \leq X_i \leq y_i + u_i$ (2) the analysis model proposes instead of one state vector $\vec{x}(t_i)$ (4) to use two vectors (Figure 2):

$$\left.\begin{array}{c} \vec{Y}(y_i - u_i, t_i) = \\ = \begin{bmatrix} Y_1(y_i - u_i, t_i), Y_2(y_{i-1} - u_{i-1}, t_i - \Delta t), \ldots \\ \ldots, Y_M(y_{i-M+1} - u_{i-M+1}, t_i - (M-1)\Delta t) \end{bmatrix}, \\ \vec{Y}(y_i + u_i, t_i) = \\ = \begin{bmatrix} Y_1(y_i + u_i, t_i), Y_2(y_{i-1} + u_{i-1}, t_i - \Delta t), \ldots \\ \ldots, Y_M(y_{i-M+1} + u_{i-M+1}, t_i - (M-1)\Delta t) \end{bmatrix}, \end{array}\right\} \qquad (5)$$

here $i = M+1, \ldots, n$.

The distance between the vectors (5) characterizes the uncertainty of the restored state vector $\vec{Y}(t_i)$ in point of time t_i:

$$S(u_i, t_i) = \left|2\sqrt{u_i^2 + u_{i-1}^2 + \ldots + u_{i-M+1}^2}\right|. \qquad (6)$$

The vector field limited by state vectors (5) forms the phase portrait that contains the uncertainty of the restored state vector $\vec{Y}(t_i)$ (6) (Figure 2).

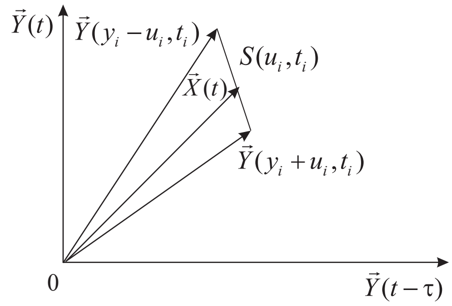

Figure 2. The restored vector field in point of time t_i for $M = 2$

The restored NDS's phase portrait is the object for analysis of the measurement results and for prediction of future behavior of NDS. Using restored phase portrait the formulas for determination of NDS local and general parameters, that contain the measurement uncertainties of DV, are represented in the analysis model.

3. The entropy analysis

Also in metrology for evaluation of measurement results the probabilistic information theory is applied. This theory uses its key elements — the amount of information I and the Shannon entropy H like the quantities characterizing the measurement uncertainty. In the terms of the information theory the sense of measurement is a reduction of the interval of knowledge uncertainty about measured value (Figure 3). The amount of information obtained from measurements is given by next formula:

$$I = H_{before} - H_{after}, \quad (7)$$

here: H_{before} — the Shannon entropy of DV X before measurement; H_{after} — the Shannon entropy of DV X after measurement.

According to information theory, when the number of measurement experiments increases the value of Shannon entropy decreases $H_{after} \to H_{\min} \to 0$, it's got the maximum amount of information about the measured DV X and the uncertainty area (Figure 3, b) tends to the point (Figure 3, c) matching the true value of the measured DV X.

In case of the measurement of DV in NDS the situation is different. The multiple measurements of DV also lead to a decrease of the Shannon entropy value $H_{after} < H_{before}$. A long-term measurements and consideration of all the factors, that influence on the measurement result, reduce the entropy values to certain minimum value $H_{after} \to H_{\min}$. However, the minimum value of the Shannon entropy doesn't tend to zero $H_{\min} \neq 0$. The amount of information received during measurement in NDS is limited to some uncertainty area (Figure 3, b). Increasing the number of measurement experiments and an observation time system also doesn't let to reduce this area. The reason of such situation is next. The measurement uncertainty in the case of NDS depends on the factors that are the causes of the type A and type B uncertainties that can be considered or excluded, but also on a complicated behavior of DV.

The measurement model, the measurement results analysis model and the entropy analysis will allow to create and to use the modified Concept of expression of uncertainty in measurement in case of real-world, open, dissipative, chaotic systems of different origins.

4. The practical application

Successful metrological provision of scientific and industrial problems is the important key for their solutions. On the other hand, the quality of a measurement model and a measurement results analysis model are depended on a research profoundness of observed processes and systems.

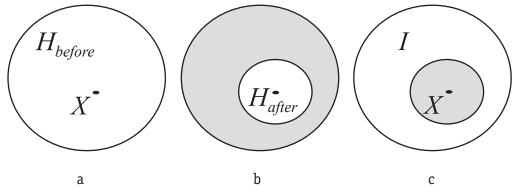

Figure 3. The visualization of information sense of measurement: *a* — the uncertainty area before measurement; *b* — the uncertainty area after measurement (white area); *c* — the area without uncertainty that is equal to the amount of received information (white area)

The results of research of real physical, biological, social and even financial systems often allow us to classify them like open dissipative NDS. Physicists, chemists and biologists more often use the synergistic approaches, methods of dynamical chaos theory and fractal analysis for study of various dynamical systems. However, using the modern methods for study of real NDS researchers have not had the adequate metrological approaches and measurement models for such difficult systems.

The examples of real chaotic physical NDS are the electrical circuits, lasers and acoustic beams in the far field. In 1983 Professor of California University L. Chua first ever demonstrated the regime of chaotic oscillations in an electrical circuit that consisted of two capacitors, a coil, linear and non-linear resistors of negative resistance. The experiment confirmed the assumption that even the simplest electrical circuits may have a chaotic behaviour.

In 2005 the group of scientists at the Max Planck Institute of Quantum Optics, investigating the chaotic behaviour of the quantum world, have been able to give the first ever demonstration of quantum chaos during atom ionisation. The experiment based on a display of classical photoeffect was fulfilled. During the experiment a laser beam forced rubidium to emit the electrons in a strong magnetic field. As a result, the electrons, whose behavior should be random, had a chaotic behaviour. The experiment proved that there is a link between chaos and fluctuations of photostream.

The scientists deal with dissipative and chaotic NDS during a solving of various hydroacoustic problems too. In 1990s in the ocean acoustics the phenomenon of ray chaos in inhomogeneous waveguides was described. It has been shown that at large distances (the thousands kilometers) the acoustic beams start to behave chaotically. This chaotic behavior must be taken into account in metrological assurance of hydroacoustic measurements.

The research results allow us to propose a model of human health measurement [9]. A living organism can be represented like an open and self-organizing dissipative NDS. In the general case, the biophysical condition of human can be represented like an attractor. The different external random or periodic disturbances influence on this attractor. If we accept the model that health characterizes an organism's stability then for quantitative evaluation of health it is necessary to measure the recovery time of the steady state. Experimental medicine during a long time has used for evaluation of health the recovery time after physical exercises. The blood tension, heart rate, brain activity indicators and other characteristics of body, changing in time, can be considered like DV of such DNS.

The model of human health measurement contains:

- the range of DV values $X(t)$ (a pulse, a blood pressure) in stable $[X_{y}^{\min}(t), X_{y}^{\max}(t)]$ and excited $[X^{\min}(t), X^{\max}(t)]$ states;
- the normalized Shannon entropy $\|H_S\|$ in the stable and excited $\|H\|$ states;
- the prediction time for DV behavior in the stable t_{fS} and excited t_f states.

As the main indicator of a health condition we choose a time T that DV spends for relaxation after stop of external normalized influence. It's proposed the numerical portrait [7] and the entropy and time lines [8] that can be used for evaluation of health condition:

$$\left.\begin{array}{c}\left[X_{y}^{\min}(t), X_{y}^{\max}(t)\right], \|H_{i0}\|, t_{fy};\\ \left[X^{\min}(t), X^{\max}(t)\right], \|H\|, t_f;\\ T\end{array}\right\}, \qquad (7)$$

$$\left.\begin{array}{c}0, \ldots, \|H_S\|, \ldots, 1;\\ 0, \ldots, T_S, \ldots, \infty.\end{array}\right\} \qquad (8)$$

As the datum points for the entropy and time scales (8) it's proposed the norm of entropy H_S and the norm of relaxation time T_S of a healthy organism. These values are individual characteristics of an organism and change their value over time. The model of human health measurement can be used for assess the state of human health during a treatment of the patients and during an athletic training.

Conclusion

Till the last time some of the described DNS and their DV have been considered like «immeasurable» variables from the point of view of classical metrology. The development of nonlinear metrology methods, that have the measurement model and measurement results analysis model and use of entropy analysis, will allow metrological science to solve these and similar difficult measurement tasks that exist today and will appear in the future. The use of these models will allow examine any random processes standing on single position.

REFERENCES

1. Schuster H 1984 Deterministic chaos (Physik-Verlag Weinheim).
2. Machekhin Yu. Physical models for analysis of measurement results // Measurement Techniques. — 2005. — № 48. — P. 155—161.
3. Machehin Yu. P., Kurskoy Yu. S. Measurment model for nonlinear dynamic systems parameters // Systems of information processing. — 2012. — № 99.— P. 169—175.
4. Machehin Yu. P., Kurskoy Yu. S. Measurements analysis in nonlinear dynamical systems // Systems of information processing. — 2012. — № 105. — P. 117—122.
5. Machehin Yu. P., Kurskoy Yu. S. Entropy analysis of dynamic variables // Systems of information processing. — 2013. — № 106. — P. 100—104.
6. Crownover R. Introduction to Fractals and Chaos // Jones & Bartlett Publishers. — 1995.
7. BIPM, IEC, IFCC, ISO, IUPAC, IUPAP and OIML 1995 Guide to the Expression of Uncertainty in Measurement, 2nd edn.
8. Takens F. Detecting strange attractor in turbulence // Lecture Notes in Mathematics. — 1981. — Vol. 898 (Berlin: Springer).
9. Machehin Yu. P., Kurskoy Yu. S. Fractal-entropy analysis of measurement results in nonlinear dynamical systems // Measurement Techniques. — 2014. — Vol. 6. ■