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FUZZY MODEL FOR DECISION-MAKING SUPPORT IN THE DETERMINING OF THE CHARACTERISTICS OF THE FINISHED PRODUCT

Abstract.The possibility of the use of fuzzy mathematics in hierarchy analysis technique is considered. A variant of hierarchy analysis technique on the basis of linguistic estimates in the determining of the characteristics of the finished product is proposed.

Key words: multicriterion problem, hierarchy analysis technique, alternatives, vectors of priorities, instruments of fuzzy mathematics, linguistic variables.

Introduction

When making managerial decisions and predicting possible results, a decision maker normally deals with a complicated problem of interdependent components, which is to be analyzed. The hierarchy analysis technique is a closed logical structure, which, by means of simple and valid rules, provides solving of multicriterion problems including both qualitative and quantitative factors, with quantitative factors having different dimensionality.

The aim of hierarchy analysis technique is justification of selecting the one best alternative among proposed ones, their characteristics being vectors with heterogeneous as well as fuzzy components [1].

The objective of research

The validity of the selecting of the finished product characteristics based on the combined usage of hierarchy analysis technique and fuzzy mathematics is analyzed in this research.

Presentation of main material

The primary stage in applying hierarchy analysis technique (HAT) is structuralizing of the choice problem as a hierarchy. The simplest way is creating a hierarchy from the top (objective) through intermediate levels (criteria) towards the lowest level which is generally a set of alternatives [1].

The questions of a peculiarity of antecedent factor ranking are developed in the article [2]. This method is convenient to use when estimating technical and economic characteristics choosing those of them that have the greatest effect on decision-making in production of paving tile. Experts revealed 3 factors-criteria, that influence considerably a choice of a type of paving tile in the course of production and must be involved in further calculations. After hierarchical reproduction of a problem, priorities of criteria are established and each alternative is estimated according to criteria.

In our instant, the analysis of three types of paving tile: A, B, C was carried out according to their desirability. As a basis of comparison three main criteria were selected: price, quality, technical characteristics. As a result of expert evaluation, the decision was made about the dominance of the price of the product and its technical characteristics over its quality. Therefore the element (2.1) of the matrix A is instantiated 5, i.e. a_{21}=5. This automatically presupposes that a_{12}=1/5.

By designating P as price, Q as quality, T as characteristics – criteria, we are able to record the matrix of comparison in the following way:

$$A= \begin{array}{c} \\ P \\ Q \\ T \end{array} \begin{array}{ccc} P & Q & T \\ \left[\begin{array}{ccc} 1{,}00 & 5{,}00 & 5{,}00 \\ 3{,}00 & 1{,}00 & 3{,}00 \\ 0{,}20 & 0{,}33 & 1{,}00 \end{array}\right] \end{array}$$

In similar way the elements of the matrices A_p, A_q, A_t are determined based on the judgements of a decision maker. Then the priority vectors on given matrices are calculated. Relative weights are calculated by way of average values of the elements of the corresponding lines of the normed matrix N, the elements of which are determined by division of the elements of each column of the dual comparison matrix into the sum total of the elements of the same column. Required relative weights W_p, W_q, W_t, of the criteria are calculated by way of average values of the elements of the corresponding lines of the normed matrix A. The sum total of the elements of this matrix`s first column equals 4.20; of the second column – 6.33; of the third – 9.

The relative weights of alternative decisions corresponding to the tiles A, B, C are calculated within each criterion P, Q, and T using comparison matrixes - A_p (sum total of the column elements equals correspondingly 9; 4.3; 1.5), A_q (sum total of the column elements equals correspondingly 1.5; 4.3; 9), A_t (sum total of the column elements equals

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correspondingly 9; 5.3; 1.5) and A_l (sum total of the column elements equals correspondingly 1.5; 4.3; 9). As a result we obtain the following normed matrix:

		Tile A	Tile B	Tile C
$N_{p=}$	Tile A	0.111	0.077	0.130
	Tile B	0.333	0.231	0.217
	Tile C	0.556	0.692	0.652

Average values of the elements in the lines $W_{p\ tile\ A}$ =0.106; $W_{p\ tile\ B}$ = 0.260; $W_{p\ tile\ C}$ = 0.633. By analogy, we create $N_{q,}$ for which values - $W_{q\ tile\ A}$ = 0.106, $W_{q\ tile\ B}$ =0.260, $W_{q\ tile\ c}$ = 0.633; for N_t: $W_{t\ tile\ A}$ = 0.633, $W_{t\ tile\ B}$ =0.260, $W_{t\ tile\ c}$ = 0.106. Values $W_{p\ tile\ A}$, $W_{p\ tile\ B}$, $W_{p\ tile\ C}$ provide relative weights for types of tile according to its price.

By analogy, values $W_{q\ tile\ A}$, $W_{q\ tile\ B}$, $W_{q\ tile\ c}$ are relative weights, concerning with the quality of products, and $W_{t\ tile\ A}$, $W_{t\ tile\ B}$, $W_{t\ tile\ c}$ concern with technical characteristics. If the columns of the normed matrix are identical it means that primary comparison matrix is coherent. If a dual comparison matrix is not coherent, it is advisable to calculate an index of coherence for it, which provides information on the rate of coherence violation.

In our case, the matrix A_p is not coherent since the matrix N_p columns are different. We determine the value $n_{max.}$ because $\overline{w_1} = 0{,}106$; $\overline{w_2} = 0{,}260$; $\overline{w_3} = 0{,}633$, therefore:

$$A_{P_{\overline{w}}} = \begin{bmatrix} 1 & 0{,}33 & 0{,}2 \\ 3 & 1 & 0{,}33 \\ 5 & 3 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0{,}106 \\ 0{,}260 \\ 0{,}633 \end{bmatrix} = \begin{bmatrix} 0{,}320 \\ 0{,}790 \\ 1{,}946 \end{bmatrix}$$

Hence, n_{max} *=3,055*. For *n=3*, *CI=0,027681; RI=0,66; CR=0,041941*.Since *CR<0,1*, the level of the matrix incoherence is acceptable. The estimation of alternatives is based upon calculating of the weight index.

tile A:	F tile A=	0,24	optimal
tile B:	F tile Б=	0,13	
tile C:	F tile B=	0,14	

On the basis of these calculations the tile A obtains the highest combined weight and, therefore, is the most optimal choice of a decision maker.

Ranking (regulation according to preference) of all variants is represented graphically in the Fig. 1.

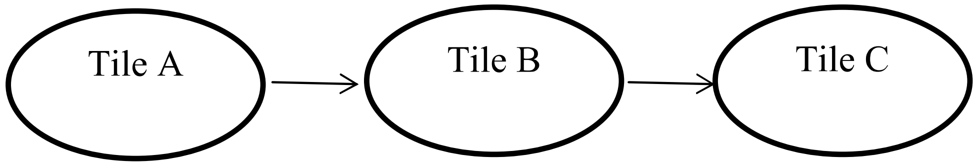

Figure 1 – Regulation of variants according to preference according to hierarchy analysis technique.

Within hierarchy analysis technique there are no general rules for forming of decision-making model structure. The framework of the technique appliance does not depend on the scope of activity, in which a decision is made. It makes the technique multipurpose, its usage allows to organize the system of decision-making support [3].

The theory of hierarchy analysis technique has a number of disadvantages, gaps and erroneous assumptions. One of these is that the scales of the preference strengths estimating (measuring) of variants according to each criterion are established as ratio scales, being non-related to each other and to criteria priorities. Thus, there exists an obvious necessity of developing, based on the theory of criteria importance, valid and effective methods of solving multicriterion problems with hierarchical criterion structure and realization of these methods in computer systems of decision-making support.

For solving this problem we shall use instruments of fuzzy mathematics [4-5]. Hierarchy analysis technique, being a technique of solving multicriterion problems in complex conditions with hierarchical structures comprising nonformalized elements, is used as an indirect method for determination of fuzzy set adjectives. Utility function is not regarded as a probability function, but as a fuzzy value, fuzzy set adjectives being considered as subjective estimations of decision makers (DM).

Formally, the problem of multicriterion choice can be represented as a triple of objects [1, 3]:MS=<G, C, A>, where G is a goal of a multicriterion choice problem; C – criterion set; A – alternative set.

When decreasing the number of criteria, included in the system of preferences, the complexity of dual comparison matrices in hierarchy analysis technique increases considerably. As a means of the indicated problem it is possible to use decomposition-aggregate approach to solving multicriterion problems, when a complex criterion, being a subgoal of a general multicriterion choice problem, is subdivided into less complex ones [6].

If each criterion is regarded as a linguistic variable, the alternative set is - $A = \{A_1, A_2, \ldots, A_n\}$, the criterion set - $C = \{C_1, C_2, \ldots, C_m\}$. For each *i*-criterion the linguistic variable is L_i with termal set $T = \{T_{i1}, T_{i2}, \ldots, T_{it}\}$, where the semantics of each term $\mu_{ij}(x_i) : X_i \to [0,1]$, $X_i \in [x_i^{inf}, x_i^{sup}]$, - of basic set of linguistic variables for an *i*-criterion x_i^{inf}, x_i^{sup} - lower and upper limit of a basic set correspondingly.

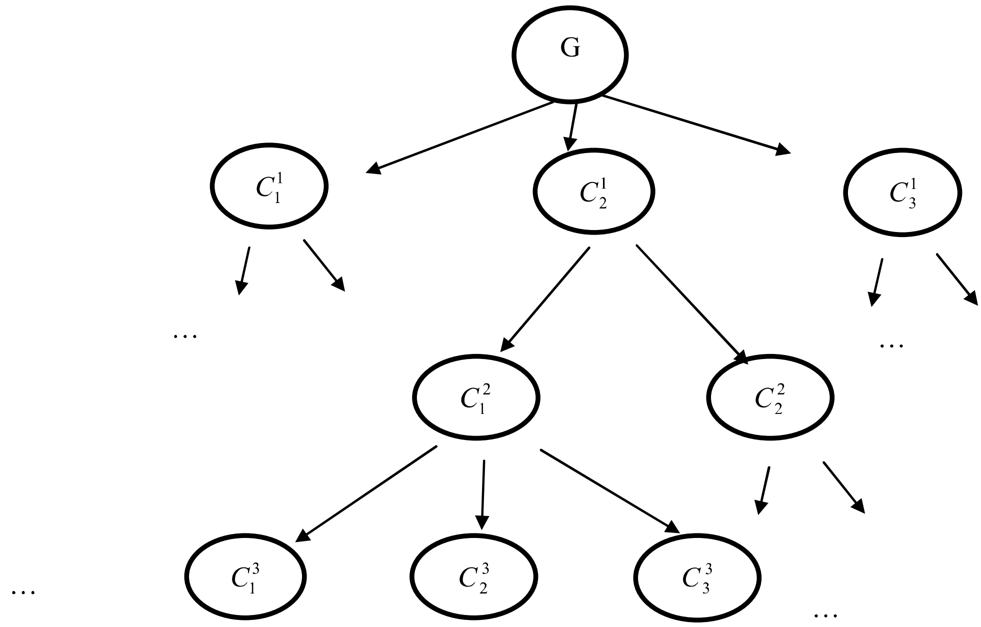

Figure 2 – The hierarchy of the multicriterion choice preference system of a decision maker for paving tile (*G – choice of product*, C_1^1 *- a way of tile paving;* C_2^1 *- method of production;* C_2^3 *- attractiveness;* C_1^2 *- subgoal–vibropressing;* C_2^2 *- subgoal– vibrocasting;* C_1^3 *- subgoal – quality;* C_2^3 *- subgoal –price;* C_3^3 *- subgoal –characteristics*)

Designations of the terms of the linguistic variable L_i are qualitative characteristics of tha criterion C_i, in our case: L_1 = (quality= {not bad, satisfactory, high}), L_2 = (price = {low, medium, high}), L_3 = (characteristics {not bad, satisfactory, excellent}).

To estimate the effectiveness of an alternative according to a separate criterion, a linguistic variable E is introduced, it has a termal set $S = \{S_1, S_2, \ldots, S_t\}$, its capacity coincides with T_i, and it is also invariant relating to each criterion C_i. Semantics of the terms - $\mu_{S_k}(y) : Y \to [0,1]$; где $Y \in [0,1]$ - basic set of a variable E, $k = \overline{1,t}$ - numbers of terms.

Consolidating of estimates on different levels (Fig.2) is exercised according to the approach in the research [6], after convolution of criteria within a subgoal, and an optimum alternative for a subgoal is selected according to $A^*_{пц} = \max\limits_{d \in [1,n]} \delta^d_{s''+(m-s'')}$, where d is a number of an alternative. After transforming of all subgoals into a criterion on the k – level, the level, where convoluted criteria are placed, is destroyed and a number of levels is decreased by 1.

Then the global priority vector is determined. Aggregation (consolidation) of alternative estimates after transformation of a preference system structure of a decision maker with the aim of selecting the best alternative using the principles of optimum choice, is performed in the following way: $A^*_G = \max\limits_{d \in [1,n]} \delta^d_G$, where δ^d_G - estimate of the alternative d according to a goal of multicriterion choice G [6].

After performing calculations we find, that evaluating the alternatives, the optimum way is production of paving tile by means of vibropressing, quality, price, characteristics, a way of paving and attractiveness reaching a predetermined level.

Conclusions

The proposed variant of hierarchy analysis technique, involving applying of linguistic estimates of the quality of alternatives separately by criteria and different convolutions of local priorities at the level of aspects, allows to include semi-structured data in the process of multicriterion analysis. It is shown that hierarchy analysis technique may be used effectively not only for decision making in non-formalized spheres and solving of ranking problems of a final set of complex objects represented in the form of hierarchical structure, but also for decision mak-

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ing in problems where formalization of all fuzzy notions by means of fuzzy sets is required.

The approach is performed based on the data of a particular enterprise, the model proposed allows to perform calculations using modern informational technologies.

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