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USING SEMIDEFINITE OPTIMIZATION TO FIND EFFECTIVE TOPOLOGY OF TRUSS-LIKE ELASTIC STRUCTURES

Abstract. The paper considers a convex formulation of the truss topology optimization problem and solving it via semidefinite programming regarding to the minimalcompliance of the truss, volume and buckling constraints.

Keywords: topology, semidefinite optimization, buckling, stiffness matrix, stress constraints.

Introduction

The effectiveness of rod systems such as trusses, which are widely used in building, largely depends on the correct constructive scheme or, in general, topology. Often it is very difficult to solve this problem, so usually it is necessary to consider several options. It is not the fact that among them there will be an option that provides the usage of the minimal amount of the material. Therefore, in these cases the optimization algorithms that allow to solve such problems in different formulations become actual.

In the field of theory of structures the optimization problems, including the optimization of truss-like systems, have been regarded many times. For example, in [1] the author proposes to use a genetic algorithm to find an optimal topology and minimize the mass of the system. But, as he notes himself, the performance of the algorithm drops dramatically even with a slight increase in the number of structural elements. In the paper [2] an approach to optimize the design of single-span single-storey frames, based on the search for such a topology, which would allow to minimize the potential energy of the frame under a load, is presented. Unfortunately, the author does not provide a complete methodology for finding of such topology. Thus, despite the existence of interesting developments in this field, many of them are purely mathematical problems which find rarely application in the engineering practice.

Problem formulation

In the computational mechanics a topology optimization can be considered as a mathematical programming problem [3]. In [4] this problem has been solved as a problem of minimization of displacements of structure nodes or - according to scalar quantities - minimization of a potential energy of an elastic deformation. Using a

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slightly modified mathematical formulations from [4], the author of this article has provided an algorithm that takes into account all of the basic requirements (strength, buckling, deformability) which are applicable to truss-like structures, and validated the obtained results in the **ANSYS** simulation software. The algorithm represents a beam system as a graph, where the set of edges E represents the beams and the set of vertices Y represents the connections of the beams. Since the structure of the system has to be detected, initially it appears as a complete graph, and the problem eventually reduces to the finding of a such minimally required set of edges and their sections which provides the minimal total mass of the structure for given external loads and constrains.

Governing equations and optimization variables

Considering the prototype of a construction as a complete graph, which has n=|Y| vertices and m=|E| edges, let's define such variables:

$f \in R_{+}^{m}, E \in R_{+}^{m}, L \in R_{+}^{m}, v \in R_{\geq 0}^{m}$ - axial forces, Young's modulus, lengths and volumes of beams;

$A \in R^{3n \times m}$ - the matrix of the system, where a_i^T is its column;

$K \in R^{3n \times 3n}$ - the element stiffness matrix of the system;

$F \in R^{3n}$ - external loads, which are applied to the nodes of the construction;

$u \in R^{3n}$ - nodes displacements.

If the system is in equilibrium then the law of conservation of forces in a matrix form can be written as:

$$Af + F = 0. \qquad (1)$$

Axial forces, which appear in the beams according to the theory of Euler-Bernoulli, are described as:

$$f_i = \frac{-E_i v_i}{L_i^2} a_i^T u. \qquad (2)$$

The nodal displacements are described by the matrix system:

$$K \cdot u = F, \qquad (3)$$

where the stiffness matrix can be written as [6]:

$$K = \sum_{i=1}^{m} \frac{E_i v_i}{L_i^2} a_i a_i^T. \qquad (4)$$

The work of external loads (and the energy of elastic deformation of the construction) we can represent as:

$$W = \frac{1}{2} F^T u . \tag{5}$$

Thus, to solve the optimization problem it is necessary to minimize two quantities: W and $\sum_{i=1}^{m} v_i$.

Semidefinite optimization problem

The problem of the topology optimization of a truss-like system can be written as follows:

$$\begin{aligned} &\text{minimize}_{u,v} W \\ &\text{s.t.} Ku = F \\ &\sum_{i=1}^{m} v_i \le V \\ &v \in R^m_{\ge 0} \end{aligned} . \tag{6}$$

But this formulation of the minimization problem is not convex, so we need to transform it according to the equations (1)-(5). We will choose the form of a semidefinite programming because available software for solving semidefinite programs is efficient and reliable. It can be shown that for a positive semidefinite matrix

$$\begin{pmatrix} \Omega & F^T \\ F & K \end{pmatrix} \ge 0 \tag{7}$$

there is a vector *u*, which allows to write an equivalent system:

$$\begin{aligned} &Ku = F \\ &K \ge 0 \\ &\Omega \ge W \end{aligned} . \tag{8}$$

Moreover, the numerical experiments show, that the ratio $v_1 : v_2 : ... : v_m$ remains constant regardless of the quantity V, so we can limit the sum $\sum_{i=1}^{m} v_i$ to 1. This fact can significantly speed up the overall solution of the problem. In view of the above we can redefine the problem (6) as follows:

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$$\begin{aligned}
&\text{minimize}_{\Omega,v}\Omega\\
&\text{s.t.}\sum_{i=1}^{m} v_i \le 1\\
&v_i \ge 0 \forall i = 1\ldots m\\
&\begin{pmatrix} \Omega & F^T \\ F & \sum_{i=1}^{m}\frac{E_i v_i}{L_i^2} a_i a_i^T \end{pmatrix} \ge 0
\end{aligned} \quad . \qquad (9)$$

In this form the problem becomes convex. The solution of this problem determines the optimal topology design and the relationship between the volumes of the beams. The next step consists in the selection of the minimal total volume of material V (and the cross-sectional areas, respectively), which would be carried out under the conditions of the strength (10) and the overall stability of the system (11):

$$\frac{f_i L_i}{V v_i} \le \gamma R \forall i = 1\ldots m, \qquad (10)$$

where γ is a resistance factor, R is a nominal resistance.

The overall stability of the system is defined as:

$$\det K_\tau > 0, \qquad (11)$$

where K_τ is a tangent stiffness matrix of the system.

The last step is to verify the model, using the **ANSYS** simulation software.

Arch truss topology optimization

Let's consider an arch truss with a complete graph which has been depicted in the figure 1. Node 1 is a freely supported end, node 7 is a roller bearing. A force $|F|=-10^5$N has been applied to the node 4. Coordinates of vertices (in meters) have been shown in Table 1.

Young's modulus is $2\cdot10^{11}$ Pa, shear modulus is $7.81\cdot10^{10}$ Pa, nominal resistance is $2.1\cdot10^{8}$ Pa, resistance factor is 0.9. The tube section with inner and outer diameters ratio d/D=0.95 has been used. The modelling has been carried out in the MATLAB environment with package CVX [8].

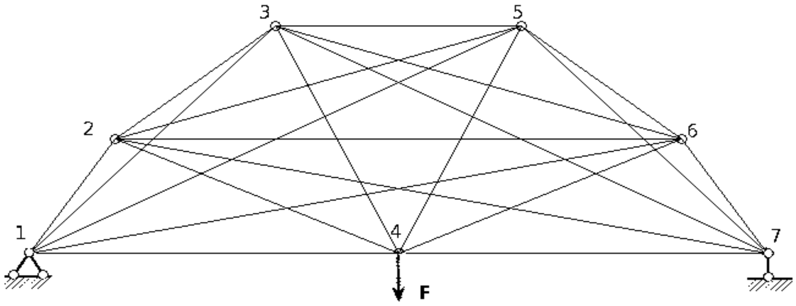

Figure 1 - A complete prototype graph of the truss

Table 1

Coordinates of vertices

	1	2	3	4	5	6	7
X axis	0	0.7	2	3	4	5.3	6
Y axis	0	1	2	0	2	1	0

In the figure 2 the optimized truss topology is depicted. In the table 2 the outer diameters of the tubes are presented. The verification of the model in the **ANSYS** simulation software has shown that the truss would loose its stability under the load of |F| > 97774 N (the calculations have been done according to the load of 100000 N). Thus, the solution of the semidefinite optimization problem in MATLAB is consistent with the results of the model validation in ANSYS.

Table 2

The diameters of the tubes

Beam	1-2	1-4	2-3	2-4	3-4	3-5	4-5	4-6	4-7	5-6	6-7
D, м	0.078	0.059	0.083	0.046	0.068	0.087	0.068	0.046	0.059	0.083	0.078

Conclusions

A problem of finding of an optimal topology of truss-like elastic structures using semidefinite programming has been presented. In contrast to the approaches [1, 2, 4, 6, 7] this solution of the problem takes into account such criteria as the strength and buckling. Verification of the model has been performed in ANSYS.

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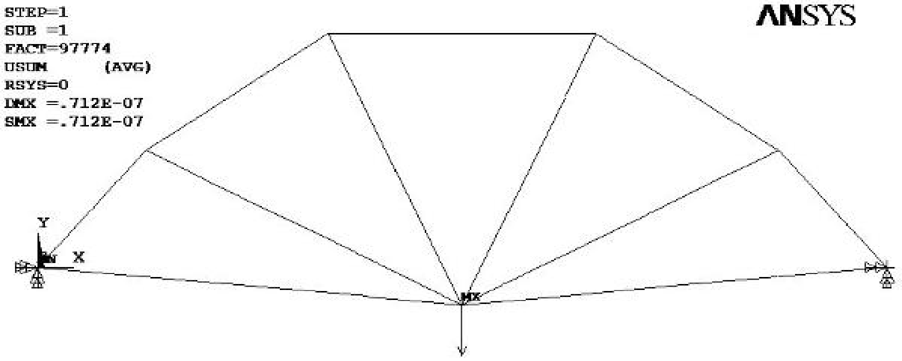

Figure 2 - Buckling of the truss

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