UDC 004.02, 004.8, 004.9; 519.6

V.I. Shynkarenko, T.M. Vasetska, E.Y. Boiko

REDUCING THE NUMBER OFEXPERT JUDGMENTS IN ANALYTIC HIERARCHY PROCESS BY SORTING AND SURVEY MANAGEMENT

Annotation. *Modification of the classical method of analytic the hierarchy process with elements of sorting for the ranking a large number of alternatives in multicriterial choice problems was developed. The ways to achieve the best consistency of the matrix of pairwise comparisons of alternatives were proposed.*

Keywords: *analytic hierarchy process, consistency, multicriterial choice, sorting.*

Introduction

The analytic hierarchy process (AHP) [1, 2, 3], which was proposed by T. Saaty, is one of the popular methods of multi-criteria choice.

The fact that this method is widely used is confirmed by the existence of a large number of modifications [4-7] and software that implements this method [2, 8-14].

However, this method has several disadvantages which significantly complicate its use.Most AHP modifications were developed to eliminate the problems associated with its use [7, 15, 16].Among the problems of the AHP should be noted following: the ability to compare a small number of alternatives (about 10), the complicated procedure of approval the matrices of paired comparisons, the need to redefine the matrices of paired comparisons at adding (removing) alternatives and others.This paper presents some possible solutions to the problems of AHP.

The Problems of Classic AHP

Let us consider the problems associated with AHP using and existing methods of their solving.AHP works very well on a small number of alternatives and criteria. But the analysis of the consistency of expert judgments causes many difficulties.There are transitive (order) [5] and cardinal (numerical) consistency of matrices [1].Author of AHP adheres to the numerical consistency. The matrix is consistent, when all its elements are in the next relation [1]:

$$a_{ij} = a_{ik} \cdot a_{kj}, \qquad (1)$$

where a_{ij} is judgment of preferences in pairwise comparisons of *i*-thand*j*-thalternatives. T. Saaty introduced the concept of consistency index (IC) to assess the degree of deviation from the ideal consistency matrix:

$$IC = \frac{\lambda_{max} - n}{n - 1}, \qquad (2)$$

where $\lambda_{\max}$- the maximum eigenvalues of the matrix of pairwise comparisons, n - the matrix size.

The ratio of the consistency index (CI) to the average random consistency index (RI) of the matrix of the same order is called the consistency ratio (CR):

$$CR = \frac{CI}{RI}. \qquad (3)$$

CR is a normalized measure of evaluation the consistency of the any dimension matrix. CR value less than or equal to 0.1 is considered as acceptable.

Under the ordinal consistency is understand the transitive preferences for any three alternatives (A, B, C), i.e., if $A \succ B$ and $B \succ C$, then $A \succ C$, where $\succ$-some preference relation.

Ideal consistency matrix is a matrix satisfying a cardinal and transitive consistency.

The experience of the practical use of AHP shows that to make consistency matrix of order 3..4 isquite difficult. For example, $a_{12} = 0{,}25$ and$a_{23} = 3$, and$a_{13} = 1$. Consistency ratio is more then 0.1 therefore the matrix of pairwise comparison is inconsistent. Expert was recommended to review the judgment and re-weigh the alternatives, which can take a long time and will not bring the desired result.

There are several approaches to make the consistency of matrices. In [5] it is proposed to achieve the transitive consistency if there is no cardinal consistency.But this approach greatly narrows the range of acceptable values, and some matrices are discarding.Forexample, considerthematrix $A_1 = \begin{bmatrix} 1 & 3 & 1 \\ 1/3 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$. CR is 0.068 that is less than 0.1. This matrix has an acceptable cardinal consistency, but there is no transitive consistency. In this case the statement is violated, that the first alternative is superior to the second in 3 times, and the second and third are equivalent, so the first must be superior to the third. But the first and third alternatives are equivalent too.

Another problem stems from the fact that when the cardinal and transitive consistency are going beyond the scale proposed by Saaty [1]. Forexample, $A_1 = \begin{bmatrix} 1 & 9 & 81 \\ 1/9 & 1 & 9 \\ 1 & 1 & 1 \end{bmatrix}$. Inthiscasethe first alternative is much superior the second, the second is much superior the third, consequently, the first must be superior to the third.

The degree of excellence was calculated by formula (1), is 81, andnot included tothe Saaty`s scale. The matrix is an inconsistent by replacing the 81 by 9.To solve this problem is proposed to normalize the calculated values in accordance with the scale of Saaty, which allows creating a quite consistent matrix [12].

It is required to ensure the consistency of matrices of large dimensions, when we have many alternatives. It is a difficult task. The problem of a limited number of alternatives is solved by using an absolute measurement scale [4], i.e, the so-called ideal model to which each alternative should be compared. In fact, with this formulation of the problem the expert ranks all the alternativesimmediately.But the problem of matrix consistency still remains.

The second approach is based on the fact that expert fills only a basic set of alternatives [1, 2], and other relations are calculated according to the formula (1).Thus, the expert primary forms the consistent matrix, but some matrices, which are consistent by Saaty (i.e. $OC < 0.1$), are eliminated, and theexpert can not sufficiently plausible assess the situation.

AHP Modification

The matrices of order 3 were investigated. Using a scale of Saaty it is possible to form 4912 matrices, of which only 1495 will be consistent (i.e., will have the consistency ratio less than 0.1).The software, which provides the specific interface for setting matrices of paired comparisons, was developed.The expert submitted three ways to specify matrices - graphic [6], semantic and numeric (Saaty scale)simultaneously.The ratio of the alternatives is represented as a triangle the vertices of which are the alternatives (Fig. 1).

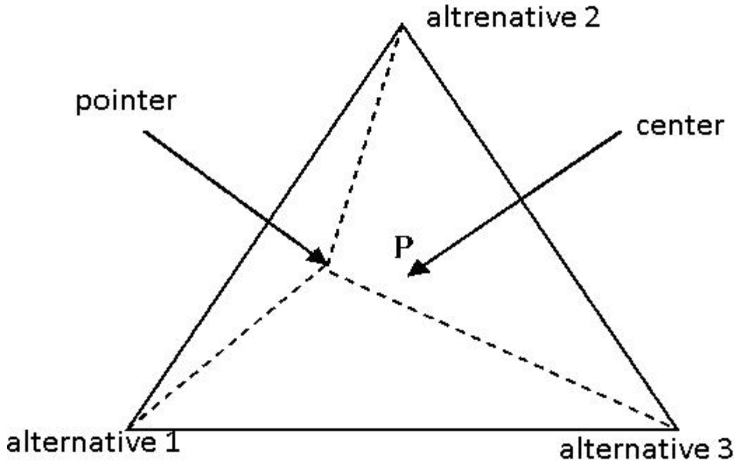

Figure 1 -Graphic representation of a matrix of pairwise comparisons by Saaty.

The distance from the point to the indicating vertices of the triangle shows the preferred alternative, concerning the others.In the center of the triangle is the point of the matrix filled with ones (i.e, the case when all the alternatives have equal importance). The problem of minimizing the function was solved for all consistent matrices:

$$F = \sum_{i=1}^{N}\sum_{j=1}^{N}(a_{ij} - \frac{s_{ij} - r_i}{s_{ij} - r_j})^2 , \qquad (4)$$

$$r_i = (x_i - x)^2 + (y_i - y)^2 , \qquad (5)$$

$$s_{ij} = (x_i - x_j)^2 + (y_i - y_j)^2 , \qquad (6)$$

where N - number of alternatives, a_{ij} - an element of the matrix of paired comparisons, s_{ij} - the distance between the vertices corresponding to the alternatives i and j, r_i - the distance between the vertex of the appropriate alternative i and the pointing point.It is also assumed that $s_{ij} = 1$, as this parameter is used to normalize the values.

The distribution of pointing points corresponding to the consistent matrices of paired comparisons (CR<0.1) was obtained.As seen in Figure 2, there is a certain regularity of points distribution. Expert determines the appropriate degree of preference alternatives using the developed control element.

The developed system is proposed to group alternatives by 3..4 in the group and to fill the corresponding matrix. The experiment was conducted: the four consistent matrices of dimension 3 (A_1, A_2, A_3, A_4)

 ISSN 1562-9945

formed the matrix of dimension 4 (A_5). All possible combinations of values of the matrix elements of dimension 3 were used.

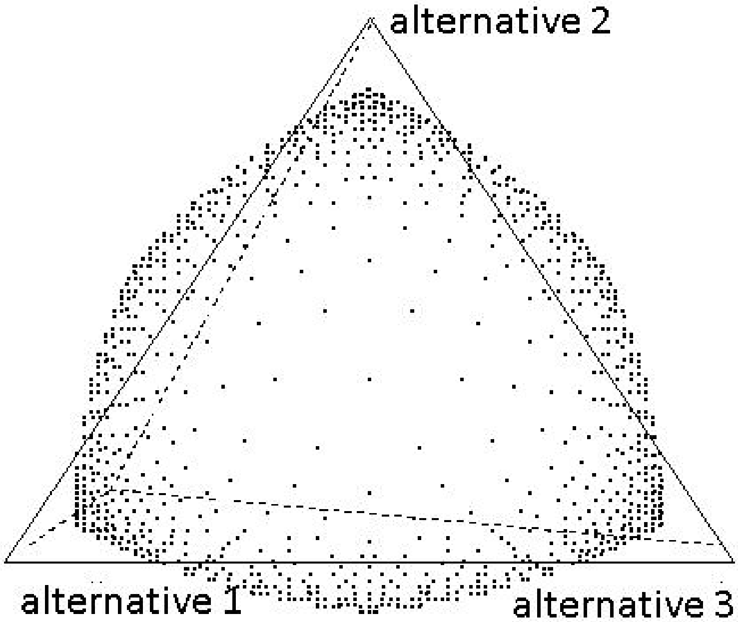

Figure 2 -Distribution of pointing points for the consistent matrices

During the inspection for consistency derived matrices of the order 4 it was proved that if the matrix is formed from the 4 consistent matrices of order 3, it also will be consistent.It solves the problem of redefining matrices of paired comparisons while adding (removing) alternatives.

$$A_1 = \begin{bmatrix} 1 & a_{12} & a_{13} \\ 1/a_{12} & 1 & a_{23} \\ 1/a_{13} & 1/a_{23} & 1 \end{bmatrix};\ A_2 = \begin{bmatrix} 1 & a_{12} & a_{14} \\ 1/a_{12} & 1 & a_{24} \\ 1/a_{14} & 1/a_{24} & 1 \end{bmatrix};\ A_3 = \begin{bmatrix} 1 & a_{23} & a_{24} \\ 1/a_{23} & 1 & a_{34} \\ 1/a_{24} & 1/a_{34} & 1 \end{bmatrix};$$

$$A_4 = \begin{bmatrix} 1 & a_{13} & a_{14} \\ 1/a_{13} & 1 & a_{34} \\ 1/a_{14} & 1/a_{34} & 1 \end{bmatrix};\ A_5 = \begin{bmatrix} 1 & a_{12} & a_{13} & a_{14} \\ 1/a_{12} & 1 & a_{23} & a_{24} \\ 1/a_{13} & 1/a_{23} & 1 & a_{34} \\ 1/a_{14} & 1/a_{24} & 1/a_{34} & 1 \end{bmatrix}.$$

The expert must fill four matrix of dimension 3.At each step, the system will request only those coefficients which have not been inputyet.Thus, in the first matrix the expert inputs three values, in the second two and only one in the third, the fourth matrix is filled automatically. When filling each of the following matricesthe field of assessments of possible values is narrowed, as those values that were cut off do not match the consistent matrices (Fig. 3). In the figure the points on the second and third triangles, those are marked in areas, corresponding to consistent matrices.

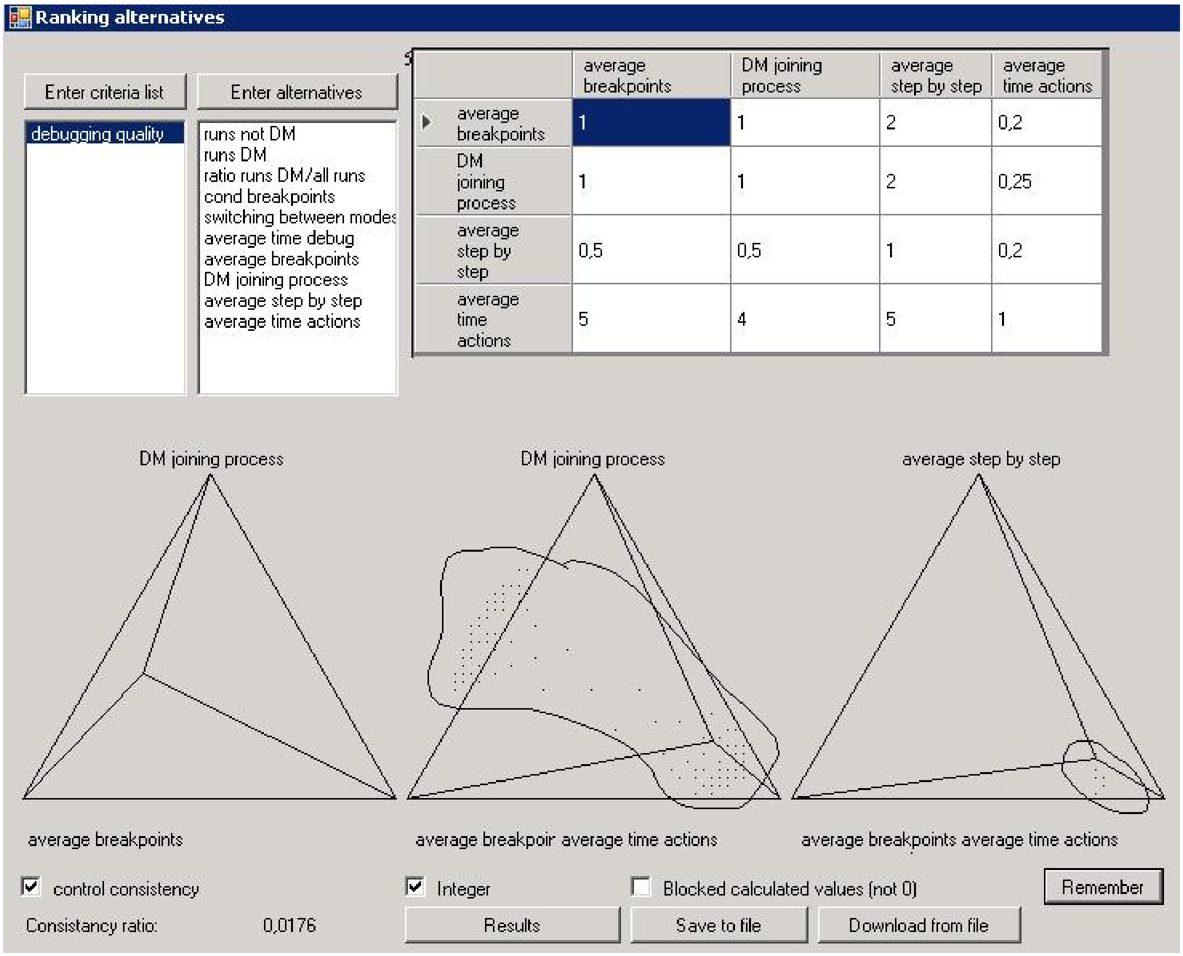

Figure 3-Filling the matrices of pairwise comparisons with control of consistency

It is proposed to modify the method of analytic hierarchy process to simplify the filling of matrices of pairwise comparisons for a large number of alternatives. The essence of the modification is to dividealternativesinto groups, apply AHP to each of the groups and rank alternatives in the groups.Thereafter, to perform the rearrangement and apply AHP for each of the groups.To perform these activities as long as the positions of the alternatives in groups no longer change.Finally, to extend the definition of the general matrix of pairwise comparisons based on the automatically calculated values assessments and rank the alternatives according to the weights.Since regrouping leads to the sorting of alternatives, this method is called AHP with sorting (AHPS).

The initial grouping reduces to determining the number of groups of 4 alternatives and the number of groups of 3 alternatives.Consider the table of partitions for different number of alternatives (Table 1).

Table 1.

The Grouping

Number of alternatives	Grouping
3	3
4	4
5	3 3
6	3 3
7	3 4
8	4 4
9	3 3 3
10	3 3 4
11	4 4 3
12	4 4 4
13	3 3 3 4
14	3 3 4 4

Number of alternatives	Grouping
15	3 4 4 4
16	4 4 4 4
17	3 3 3 4 4
18	3 3 4 4 4
19	3 4 4 4 4
20	4 4 4 4 4
21	3 3 3 4 4 4
22	3 3 4 4 4 4
23	3 4 4 4 4 4
24	4 4 4 4 4 4
25	3 3 3 4 4 4 4

It is necessary that the group had four or three alternatives, with preference given to four.Based on the analysis partition table for the different number of alternatives the following dependencies were derived for determining the number of groups from 4 (7) and 3 (8) alternatives:

$$k_4 = \begin{cases} N/4, & if \quad \mathrm{mod}(N,4) = 0, \\ 0, & if \quad N = 3 \;\; or \;\; N = 5, \\ N/4 - (3 - \mathrm{mod}(N,4)), & in\ other\ cases. \end{cases} \qquad (7)$$

$$k_3 = \begin{cases} 0, & if \quad \mathrm{mod}(N,4) = 0, \\ 1, & if \quad N = 3, \\ 2, & if \quad N = 5, \\ (N - k_4 * 4)/3, & in\ other\ cases, \end{cases} \qquad (8)$$

where N -the number of alternatives.

The alternatives regrouping must be performed in groups after ranking of alternatives.For this purpose in each group the value l is calculated as integer part from the dividing number of alternatives in the group for 2 (without rounding) according to:

$$l = [M_k / 2]. \qquad (9)$$

The new group is formed of the latest elements of the current and the first elements of the following groups as follows:

$$G'_k = \{G_{k,M_k - p}\} \cup \{G_{k+1,1}, G_{k+1,2}\},$$
$$G_k = G_k / \{G_{k,M_k - p}\},\; G_{k+1} = G_{k+1} / \{G_{k+1,1}, G_{k+1,2}\} \;\forall p \in [1,l], \forall k \in [1,Q] \qquad (10)$$

where M_k -numberofelementsin k -thgroup; G'_k -set of alternatives in k -th new group, G_k - set of alternatives in k -th group, $G_{i,j}$ -j-th alternative in$i$-thgroup, $Q = k_3 + k_4$-number of groups. When addingalternativesin new group such alternatives are removed from the previous group. If after applying the AHP and the ranking the positions of alternatives were changed in each group, it is necessary to return the alternatives that stand in these positionsand re-ranking.

When returning the alternatives in appropriate positions according to the formula (9) for each group, the integer part from dividing the alternatives number in the group for 2 is determined. The new group is formed from elements of the current and the following groups as follows:

$$G_k = \{G_{k,1,} G_{k,2,}\} \cup \{G'_{k,p}\},\ \forall p \in [1,l],\ \forall k \in [1,Q], \quad (11)$$

$$G_{k+1} = \{G'_{k,M_k-p}\} \cup \{G_{k+1,M_k-p}\}, \forall p \in [1,l],\ \forall k \in [1,Q], \quad (12)$$

where M_k — numberofelementsin k -thgroup; G'_k -set of alternatives in k -th group on the previous step, $G_{i,j}$ — p -th alternative in$k$-thgroup, $Q = k_3 + k_4$-number of groups.

Let us consider the the work of AHPS on example of indicators ranking of program debugging (Table 2) for assessment of student work. Debugging indicators will be alternatives; criterion of assessing in this problem isone -the quality of debugging.

Table 2.

Software debugging indicators

№	Name of indicators
1	Number of runs the program without debugging
2	Number of runs the program in debug mode (DM).
3	Number of different use conditional breakpoints.
4	Number of switching between runs to the DM and without debugging
5	The ratio of the number of runs in the PO to the total number of program runs
6	Average time of debugging
7	Average number of breakpoints for each run debugging
8	Number of runs in debug mode with joining process.
9	The average number of step by step operations.
10	The average time between the user's actions in the DM

When the expert has inputted all the necessary information: alternatives and criteria, the system divides them into groups according to formula (10).Expert is proposed to fill a matrix of pairwise comparisons for each group using matrices of graphical control or a table. The result of filling the general matrix of pairwise comparisons of alternatives is given in Table 3.

Table 3.

Filling a matrix of pairwise comparisons for the first three groups

№ alter-ve	1	2	3	4	5	6	7	8	9	10
1	1	1	3							
2	1	1	4							
3	0.33	0.25	1							
4				1	0.25	0.33				
5				4	1	1				
6				3	1	1				
7							1	0.5	0.33	1
8							2	1	2	3
9							3	0,5	1	4
10							1	0.33	0.25	1

Thenaccordingtothealgorithmthe evaluation of alternativesingroupsbyAHPandsortingalternativesingroups (step1 inTable 4)are carried out.Each step in the table 4 has two columns: the alternatives order in groups before the application of AHP in each group and the overall ranking ("before"), and after ranking ("after"). Blank lines in the table are the boundaries of the groups.The alternatives position in groups was changed; therefore, new groups areformed (step 2 tab. 4).

Table 4.

The order of alternatives in groups for step-by-step AHPS

1step		2step		3 step		4step		5 step		6 step		Ranking		
before	after	before	after	before	after	before	after	before	after	before	after	№ al	Rank 1	Rank 2
1	2	3	3	2	2	2	2	1	1	2	2	2	0.283	0.285
2	1	5	5	1	1	1	1	3	3	6	6	1	0.281	0.271
3	3	6	6	3	3	9	9	6	6	10	10	3	0.167	0.17
				5	5			5	5			6	0.062	0.066
4	5	4	4			5	6			1	1	5	0.061	0.058
5	6	8	8	6	6	6	5	2	2	3	3	4	0.058	0.056
6	4	9	9	4	4	4	4	4	4	4	4	8	0.029	0.029
				8	8	8	8	8	8	8	8	9	0.023	0.025
7	8			9	9			7	7			7	0.019	0.019
8	9					3	3			5	5	10	0.014	0.016
9	7					7	7			7	7			
10	10					10	10			9	9			

Groups are created automatically; the expert fills the offered matrices (Table 5).In Table 5 and later (Table 6) the alternatives assessments of the first group at this step are highlighted with light gray, and alternatives of the second group-the dark gray.The values that are inputted by expert on the second step are bold.Alternatives in new groups are assessed by AHP and sorted in accordance with estimates.The alternatives positions in groups have not changed (step 2 Table 4).

Table 5.

Filling a matrix of pairwise comparisons at the 2 step of sorting

№ alt-ve	1	2	3	4	5	6	7	8	9	10
1	1	1	3							
2	1	1	4							
3	0,33	0,25	1		**6**	**8**				
4				1	0,25	0,33		**3**	**6**	
5			**0,167**	4	1	1				
6			**0,125**	3	1	1				
7							1	0,5	0,33	1
8				**0,33**			2	1	2	3
9				**0,167**			3	0,5	1	4
10							1	0,33	0,25	1

But as the general matrix of pairwise comparisons is not full and not enough data to calculate the remaining values, then the regrouping alternatives is executed (step 3 Table 4).Alternatives return to the previous group and from each of the following group the alternative is added to the previous one. Thus two groups of four alternativesin each are obtained.The result of filling the matrix of pairwise comparisons in step 3 is shown in Table 6.

Table 6.

Filling a matrix of pairwise comparisons in step 3 of sorting

	1	2	3	4	5	6	7	8	9	10
1	1	1	3		**9**					
2	1	1	4		**7**					
3	0,33	0,25	1		6	8				
4				1	0,25	0,33		3	6	
5	**0,11**	**0,14**	0,167	4	1	1				
6			0,125	**3**	1	1		**4**	**7**	
7							1	0,5	0,33	1
8				0,33		**0,25**	2	1	2	3
9				0,167		**0,147**	3	0,5	1	4
10							1	0,33	0,25	1

The alternatives positions have not changed, but the data for the calculation of other valuesare still insufficiently, so the system generates the thre new groups(step 4 Table 4). Expert inputs the necessary assessment values of alternatives for each group (in bold in Table. 7).

Table 7.

Filling a matrix of pairwise comparisons in step 4 of sorting

	1	2	3	4	5	6	7	8	9	10
1	1	1	3		9				9	
2	1	1	4		7				9	
3	0,33	0,25	1		6	8	9			9
4				1	0,25	0,33		3	6	
5	0,11	0,14	0,167	4	1	1		3		
6			0,125	3	1	1		4	7	
7			0,11				1	0,5	0,33	1
8				0,33	0,33	0,25	2	1	2	3
9	0,11	0,11		0,167		0,14	3	0,5	1	4
10			0,11				1	0,33	0,25	1

The assessments of alternatives for each group are marked with the corresponding color (1st - dark gray, 2nd - gray, 3rd - light gray).

Aftersortingofalternativesingroupsthealternativespositioninthesecon dgroupwaschanged, thusthe newgroupsareformed (step5 in Table 4).The result of filling the matrix of pairwise comparisons is presented in Table 8.Aftersortingofalternativesingroupsthealternativespositioninthesecondg roupwaschanged, thusthe newgroupsareformed (step5 in Table 4).The result of filling the matrix of pairwise comparisons is presented in Table 8.

Table 8.

Filling a matrix of pairwise comparisons in step 5 of sorting

	1	2	3	4	5	6	7	8	9	10
1	1	1	3		9	6			9	
2	1	1	4	5	7		8	9	9	
3	0,33	0,25	1		6	8	9			9
4		0,2		1	0,25	0,33	5	3	6	
5	0,11	0,14	0,167	4	1	1		3		
6	0,167		0,125	3	1	1		4	7	
7		0,125	0,11	0,2			1	0,5	0,33	1
8		0,11		0,33	0,33	0,25	2	1	2	3
9	0,11	0,11		0,167		0,14	3	0,5	1	4
10			0,11				1	0,33	0,25	1

Thepositionofalternatives in groups has not changed, at the ranking of alternatives in each group by AHP. The expert has input enough data for further calculations.

The system filledthe missing values of the estimatesautomatically; the resulting matrix of pairwise comparisons is presented in Table 9 (marked in gray values that were calculated with system, as the average number of all possible values obtained on the basis of the formula (1)).The consistency ratio of the matrix is 0.164, but the calculated evaluation of alternatives does not match the scale of Saaty, and the result can not be checked for validity.

Table 9.

Pre-filled matrix of pairwise comparisons

1	1	3	18	9	6	51	60	9	186
1	1	4	5	7	20	8	9	9	202
0,333	0,25	1	6	6	8	9	22	27	9
0,055	0,2	0,167	1	0,25	0,33	5	3	6	12
0,11	0,143	0,167	4	1	1	3	3	4	9
0,167	0,05	0,125	3	1	1	3	4	7	15
0,019	0,125	0,11	0,2	0,33	0,33	1	0,5	0,33	1
0,0167	0,11	0,045	0,33	0,33	0,25	2	1	2	3
0,11	0,11	0,037	0,167	0,25	0,143	3	0,5	1	4
0,005	0,005	0,11	0,083	0,11	0,067	1	0,33	0,25	1

Two approaches can be applied to make estimates appropriate to the Saaty scale to solve this problem. At the first pass the transitivity of estimates of maximum excellence is eliminated. All values of pairwise comparisons that exceed 9 are replaced with 9 (as the maximum possible value at very strong superiority). The result is shown in Table 10 (the assessments values calculated by the system are marked with grey). For this matrix, the consistency ratio is equal to 0.129, which is slightly more than an acceptable level.

Table 10.

The matrix of pairwise comparisons with the equation estimates the maximum superiority

1	1	3	9	9	6	9	9	9	9
1	1	4	5	7	9	8	9	9	9
0,33	0,25	1	6	6	8	9	9	9	9
0,11	0,2	0,167	1	0,25	0,33	5	3	6	9
0,11	0,143	0,167	4	1	1	3	3	4	9
0,167	0,11	0,125	3	1	1	3	4	7	9
0,11	0,125	0,11	0,2	0,33	0,33	1	0,5	0,33	1
0,11	0,11	0,11	0,33	0,33	0,25	2	1	2	3
0,11	0,11	0,11	0,167	0,25	0,143	3	0,5	1	4
0,11	0,11	0,11	0,11	0,11	0,11	1	0,33	0,25	1

Then the expert has an opportunity to review the automatically calculated estimates. All assessments completed automatically by rows are selected: $a_{1,4}, a_{1,7}, a_{1,8}, a_{1,10}$, $a_{2,6}, a_{2,10}$, $a_{3,4}, a_{3,8}, a_{3,9}$, $a_{4,1}, a_{4,3}, a_{4,10}$, $a_{5,7}, a_{5,9}, a_{5,10}$, $a_{6,2}, a_{6,7}, a_{6,10}$, $a_{7,1}, a_{7,5}, a_{7,10}$, $a_{8,1}, a_{8,3}$, $a_{9,3}, a_{9,5}$, $a_{10,1}, a_{10,2}, a_{10,4}, a_{10,5}, a_{10,6}$.For the formation of new groups the alternative with the lowest number of unfilled estimates is selected and group of 3-4 alternatives ,with which it is linked, is formed: for the alternative 2 this

 ISSN 1562-9945

are 6 and 10.Because the 10th alternative is already in the group, the estimations related with it are not counted and the next alternative is selected for the formation of groups: this 4-th alternative and the associated with it 1-st and 3-rd, which in turn are associated with 8-th alternative. Thus, the group of 4 alternatives is formed: 1, 3, 4, 8. And the last group will include those alternatives that are not included in the previous: 5, 7, 9.

Expert, using the control element to fill the matrices of pairwise comparison, refines the estimates that were calculated automatically (see Table 11, changes in bold).

Table 11.

The matrix of of pairwise comparisons with the equated estimates of the maximum excellence, complemented by the expert

1	1	3	**5**	9	6	9	9	9	9
1	1	4	5	7	**7**	8	9	9	9
0,33	0,25	1	**2**	6	8	9	5	9	9
0,2	0,2	**0,5**	1	0,25	0,33	5	3	6	9
0,111	0,143	0,167	4	1	1	**1**	3	**3**	9
0,167	**0,143**	0,125	3	1	1	3	4	7	**2**
0,111	0,125	0,11	0,2	1	0,333	1	0,5	0,33	1
0,111	0,11	0,2	0,33	0,33	0,25	2	1	2	3
0,111	0,11	0,111	0,167	**0,333**	0,143	3	0,5	1	4
0,111	0,111	0,11	0,111	0,111	**0,5**	1	0,33	0,25	1

The alternatives ranking in groups based on the new estimates are performed.The consistency ratio of the general matrix of the comparison alternatives is a valid 0.1. The alternatives ranking is presented in Table 4, column "Ranking", corresponding ranks of alternatives "Rank1".

Consider the second variant of definition the general matrix of pairwise comparisons.If to analyze the pre-calculated matrix of pairwise comparisons (Table 9), it can be seen that all values greater than 9 have a very wide range. Consequently, for all values calculated automatically it is necessary to perform the normalization on a scale of Saaty.The normalization will be done only for large values (greater than 9), in the second half of the scale of Saaty (beginning with 5). Previously, all values, that are larger then 81, are replaced with 81. The normalization coefficient and value corresponding to the current value on the scale of Saaty are calculated as follows:

$$norm = (maxR - minR)/(maxS - minS), \quad (13)$$

$$zS = minS + [(z - maxS) / norm], \quad (14)$$

where*norm* – normalization coefficient;maxR, minR – maximum and minimum values on the current scale;maxS, minS – maximum and minimum values on the scale of Saaty;z – current value of assessment;zS– normalized value of assessment.

Here are the appropriate ranges of values calculated on a scale of Saaty: values from 9 to 23 are replaced with 5, between 24 and 38 – 6, from 39 to 53 – 7, from 54 to 68 – 8, from 69 to 83 – 9.As a result of this replacement the matrix was obtained (Table 12) which hasconsistency ratioequal 0.11 that is slightly greater than allowable.

Table 12.

The matrix of pairwise comparisons with the norm

1	1	3	5	9	6	6	7	9	9
1	1	4	5	7	5	8	9	9	9
0,33	0,25	1	6	6	8	9	5	6	9
0,2	0,2	0,167	1	0,25	0,33	5	3	6	5
0,11	0,14	0,167	4	1	1	1	3	4	5
0,167	0,2	0,125	3	1	1	1	4	7	5
0,167	0,125	0,11	0,2	1	1	1	0,5	0,33	1
0,14	0,11	0,2	0,33	0,33	0,25	2	1	2	3
0,11	0,11	0,167	0,167	0,25	0,14	3	0,5	1	4
0,11	0,11	0,11	0,2	0,2	0,2	1	0,33	0,25	1

To improve the consistency is proposed to form groups of the alternatives with estimations that should be reconsidered.It is proposed to use the approach described above. The groups are formed: alternatives 2, 6 and 10 - the first group, alternatives 1, 3, 4, 8 - the second group and alternatives 5, 7, 9 - the third group (Table 4, step 6, column "before").Expert refines assessment of alternatives for each group using the control element (Table 13, changes in bold). The alternatives in groups are ranked, and the ranking of all the alternatives on the basis of re-calculated the ranks is executed (Table 4, "Ranking - Rank2"). And the consistency ratio of the general matrix (Table 13) is equal to 0.1.

Conclusions

The developed system for ranking of alternatives allows essentially reduce the expert's assessment work of a large number of alternatives, as well as reduce the occurrence of inconsistent judgments.

This article presents the application of AHP for ranking indicators of debugging.In this approach the matrix of pairwise comparisons (dimension of 10) with a total of 45 expert judgments was formed.The proposed modified AHP allowsreducing the number of expert judgments to 29.Control of consistency of paired comparisons matrix helps to get the consistent matrix in the result.

Table 13.

The matrix of pairwise comparisons with the norm, is supplemented by expert

1	1	3	**4**	9	6	**6**	**9**	9	**9**
1	1	4	5	7	**5**	8	9	9	**9**
0,33	0,25	1	**3**	6	8	9	**5**	**6**	9
0,2	0,2	**0,33**	1	0,25	0,33	5	3	6	**5**
0,11	0,14	0,167	4	1	1	**2**	3	**2**	**5**
0,167	**0,2**	0,125	3	1	1	**3**	4	7	**2**
0,167	0,125	0,11	0,2	**0,5**	**0,33**	1	0,5	0,33	1
0,11	0,11	**0,2**	0,33	0,33	0,25	2	1	2	3
0,11	0,11	**0,167**	0,167	**0,5**	0,14	3	0,5	1	4
0,11	**0,11**	0,11	**0,2**	**0,2**	**0,5**	1	0,33	0,25	1

The developed method allows getting a rough picture of ranking of alternatives in a few steps of sorting.Subject to consistent judgments (assessments) the matrix of pairwise comparisons is filled mainly automatically. Expert only corrects some calculatedassessments.

This method also allows allocating groups of alternatives that are meaningless to compare with each other, as the result of the comparison is obvious - strong superiority, and it does not affect the result of ranking.The expert receives the maximum consistent matrix for this assessment (even if the ratio of consistency is more than 0.1). The completion of the system for multi-criteria ranking is perspective.

REFERENCES:

1. Саати Т. Л. Принятие решений. Метод анализа иерархий. / Т. Л. Саати — М.: Радио и связь, 1993. — 316 с.
2. Saaty T. L. Relative Measurement and Its Generalization in Decision Making Why Pairwise Comparisons are Central in Mathematics for the Measurement of Intangible Factors The Analytic Hierarchy/Network Process / T. L. Saaty // Royal Academy of Sciences, Spain, Series A. Mathematics, 2008 – November – C. 251–318.
3. Саати Т. Л. Принятие решений при зависимостях и обратных связях: Аналитические сети. / Т. Л. Саати — М.: Издательство ЛКИ, 2008. — 360 с.
4. Ногин В.Д. Упрощенныйвариант метода анализаиерархий на основенелинейной свертки критериев. / В.Д. Ногин // Журнал

вычислительной математики и математической физики. – 2004. – т. 44. – № 7. – С. 1259–1268.

5. Блюмин С. Л. Модели и методы принятия решений в условиях неопределенности / С. Л. Блюмин, И. А. Шуйкова // – Липецк: ЛЭГИ, 2001. – 138 с.
6. Кузнецов В. Г., Шинкаренко В.И. Инвариантно-согласованный метод анализа иерархий в задачах планировании энергосберегающих мероприятий системы электроснабжения железнодорожного транспорта / В. Г. Кузнецов, В. И. Шинкаренко // Системные технологии. Региональный межвузовский сборник научных работ. – Выпуск 6 (77). – Днепропетровск, 2011.
7. Колесникова С. И. Модификация метода анализа иерархий для динамических наборов альтернатив / С. И. Колесникова // Математические основы интеллектуальных систем, 2009. – №4(6). – С.102 – 109.
8. Абакаров А. Ш. Программная система поддержки принятия рациональных решений “MPRIORITY1.0”./ А. Ш. Абакаров, Ю. А. Сушков // Электронный научный журнал "Исследовано в России", 2005 – С. 2130-2146.
9. SuperDecisionsSoftware: [Электронный ресурс]. – Режим доступа: http://www.superdecisions.com/.
10. Expert Choice for Collaborative Decision Making: [Электронный ресурс]. – Режим доступа: http://expertchoice.com/.
11. Евтушенко Г. Л. Разработка веб-приложения «Система поддержки принятия решений NooTron 3.0» / Г. Л. Евтушенко, А. С. Воюев, А. О. Градовский, Н. А. Грачев, В. Г. Макаров, Ю. С. Штефан // Системные технологии. Региональный межвузовский сборник научных работ. – Выпуск 2 (91).– Днепропетровск, 2014. – С. 136 – 144.
12. Ломакин В. В. Алгоритм повышения степени согласованности матрицы парных сравнений при проведении экспертных опросов / В. В. Ломакин, М. В. Лифиренко // Фундаментальные исследования – 2013 – №11. – С.1798–1803
13. Теплякова Г. Л. Модель построения рейтинга кафедр интегрированным многокритериальным методом МВС+МАИ / Г. Л. Теплякова // Системні технології. Регіональний

міжвузівський збірник наукових праць. – Випуск 3 (86). – Дніпропетровськ, 2013. – С. 135 – 141.
14. Михалёв А. И. Интеграция методов многокритериального анализа и их применение в системе поддержки принятия решений / А. И. Михалёв, В. И. Кузнецов, Н. Н. Ковалик, Г. Л. Теплякова // Системні технології. Регіональний міжвузівський збірник наукових праць. – Випуск 4 (75). – Дніпропетровськ, 2011. – С. 140 – 152.
15. Подиновский В. В. О некорректности метода анализа иерархий / В. В. Подиновский, О. В. Подиновская // Проблемы управления. – 2011. – № 1. С. 8-13.
16. Михалевич М. В. Замечания в дискуссии Дж. Дайера и Т. Саати / М. В. Михалевич // Кибернетика и системный анализ. – 1994. – № 1. – С. 97-102.