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PECULIARITIES OF RISKS ASSESSMENT BASED ON FUZZY MODELS

Abstract: *The article describes the methodology for the risk assessment based on fuzzy models. In the models of risk management, triangular norms can extend the probabilistic models to fuzzy models and thereby make possible their use in conditions of poor statistics. The effectiveness of this methodology for the design of fuzzy controllers, decision support systems based on fuzzy models of risk assessment was shown. There are some practical examples that demonstrates that fuzzy measures and integrals are good techniques to solve different complex problems.*

Keywords: *risks assessment; fuzzy models; fuzzy measures; fuzzy integrals.*

Introduction

Nowadays, risk management is seen as a key direction of applied management, so much attention is paid to researching of risk areas and the main types of risks, finding effective methods of their evaluation, control and monitoring. Efficient solving of any problems mainly depends on the accuracy and validity of decisions at all stages of problem solving and, regardless of the complexity of tasks, impossible without risk.

The general scheme of risks management is presented on figure 1.

The first phase is the collection of data, usually denoted as Risk Analysis, i.e. identification of hazards present in the workplace and work environment as well as the exposed workers, and identification of potential consequences of the recognized hazards – risks, i.e. the potential causes of injury to workers, either a work accident or an occupational disease. This is followed by the Risk Assessment phase, which includes the risk evaluation, the ranking of the evaluated risks and their classification in acceptable or unacceptable. At the end of this phase, the unacceptable safety and health risk situations are identified. The last phase is Risk Control that includes designing/planning safety control measures to eliminate or at least to reduce risks, followed by the implementation of safety control measures. Part of the risks could be transferred to insurance companies [1].

The main cause of risk is the uncertainty of the environment, which is caused by factors such as lack of full and reliable information on the environment; limited capabilities for processing information about the process or system; chance of occurrence of adverse events; conflicts; breach of contractual obligations; political decisions.

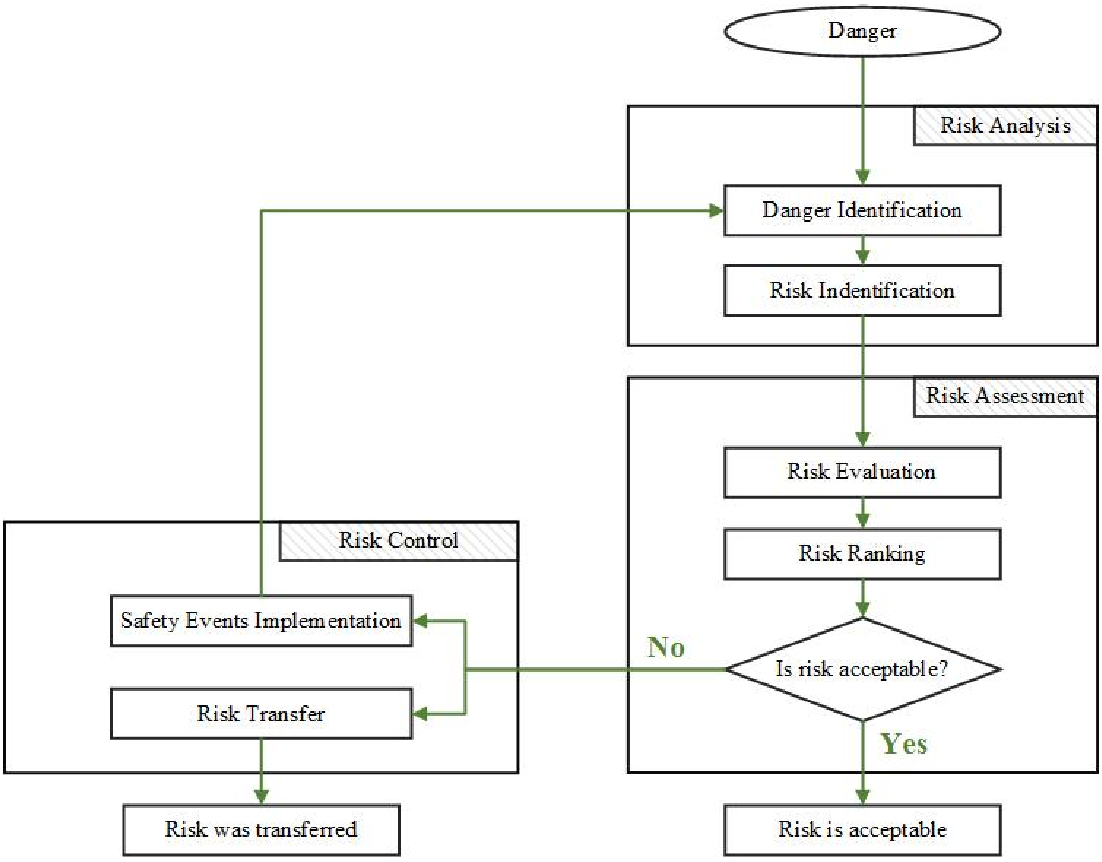

Fig. 1. General Scheme of Risk Management

On this basis, all planning decisions are divided into three groups:

–taken under certainty;

–taken under probabilistic certainty (based on risk);

–taken in complete uncertainty (unreliable).

Decision making under probabilistic certainty based on statistical decision theory. Incomplete and unreliable information are supplemented by examining random events and processes that may occur. The behavior of random objects is described by probabilistic characteristics. Themselves probabilistic characteristics are not random, and they can be used to find the optimal solution as well as with deterministic characteristics. General criteria for finding the optimal solution is a medium risk.

However, in most cases, decision-making and risk assessment take place in conditions of uncertainty and incomplete information, because the application of probability as the classic characteristics of mass processes is impossible. One of the promising areas of modern high-tech is fuzzy modeling, due to the trend of increasing complexity and formal mathematical models of physical systems and management processes related to the desire to increase their adequacy and consider set of different factors that influence the decision-making processes.

The uncertainty (lack of) information makes risk common with situation of decision making under non-determined parameters. Given the need for a quantitative risk assessment, it is reasonable to define risk through a combination of the event value (the consequences of the event) and possibility of its occurrence [2]. In practice, to get a point value of risk, the product of their numerical values is used.

However, the measure of the possibility of an event is usually chosen as probability of its occurrence P. The consequences of unwanted event A can be assessed by various specific parameters – from the economic to the ethical or political. Hence the risk is

$$R = A \cdot P. \tag{1}$$

The biggest problems arise whenthere are two (or more) types of threats at the same time. In this case the risk can be represented in vector form with various types of threats corresponding coordinate axes:

$$R_X = A_X \cdot P_X. \tag{2}$$

This view assumes that the variables do not interact with each other. In practice, as a rule, this assumption is not satisfied. For example, the risk of health is directly related to the risk to the environment. Therefore, for proper risk assessment is necessary to develop adequate mathematical apparatus [2].

1 Fuzzy Models

In many cases, the risk is a function of statistical parameters as well as fuzzy parameters. Due to differences in the theories used to describe these two types of parameters (probability theory, for example), the problem of aggregation becomes non-trivial. To solve it, you need to convert the parameters to the same type. This involves transforming a probability density distribution P into distribution of opportunities R, or, conversely, R into P. Moreover, in such transformations the amount of information (uncertainty) must be kept in the distributions P and R.

Fuzzy models assume a wide range of options for aggregation of event values and measure of its occurrence possibility. There are possible options for the generalization of such models [2].

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Aggregation is used in fuzzy inference and recognition, multi-criteria decision-making problems. The operator of aggregation is often called a function on N variables (criteria) that has some desired properties and is defined on the unit interval. The range of values of this function is a unit interval. Fuzzy measure expresses a subjective weight or importance of each subset of criteria.

Using of T-norms for the assessment of confidence in the validity of the formula.

The formula for risk (1) can has non-strict nature because the value of the probability P and damage A may not be known for sure, but with some degree of confidence $x(P)$ and $y(A)$. The degree of confidence $z(R)$ in the validity of the formula may depend on the degrees of confidence $x(P)$ and $y(A)$ as follows [2]:

–as their minimum – $z(R)=T_M\left[x(P),y(A)\right]=x(P)\wedge y(A)$ – Zadeh's T-norm;

–as their product – $z(R)=T_P\left[x(P),y(A)\right]=x(P)\times y(A)$ – probability T-norm;

–other T-norms.

Replacement of A and P values by fuzzy numbers (linguistic variables), and products – by expanded (on basis of generalization) product of fuzzy numbers.

In this case, the formula (1) will look like [2]:

$$R = A\otimes P \Leftrightarrow \mu_R(z)=\vee\left[\mu_A(x)\wedge\mu_P(y)\right]$$

$$z = x\times y$$

Here R, A and P – fuzzy numbers; $\mu_R(z)$, $\mu_A(x)$ and $\mu_P(y)$ – membership functions characterizing the degree of belonging to the elements of fuzzy sets R, A and P respectively; $\otimes$ – operation of expanded product of fuzzy numbers; $\wedge$ – operation minimum (disjunction); $\vee$ – operation maximum (conjunction). It should be noted that there are some other approaches to expansion. Their consideration is usually associated with the problems of "interaction" and "compensation" variables.

Replacement of A and P values by fuzzy relationship, and products – the composition of these relations.

Fuzzy relation is called a fuzzy set in the Cartesian product of base sets [2].

$$\mu_R(x,y)=\mu_{A\times P}(x,y)=min\left[\mu_A(x),\mu_P(y)\right]$$

The formula (1) can be rewritten using the composition operation as follows:

$$R(x,y)=A(x,y)\circ P(x,y)$$

Here $R(x,y)$, $A(x,y)$ and $P(x,y)$ – some fuzzy relationships, and $\circ$ – the operation of the composition. Among the known composition transactions often used max-min and max-product operations, i.e.:

$$\mu_R(x,y)=\vee\left[\mu_A(x,y)\wedge\mu_P(x,y)\right]$$

and

$$\mu_R(x,y)=\vee\left[\mu_A(x,y)\cdot\mu_P(x,y)\right]$$

respectively.

We can also use a max-T composition of fuzzy relations, where T – parametric norm. The relationships $R(x,y)$, $A(x,y)$ and $P(x,y)$ can be interpreted as the ratio of modeling, that is $X=(x_1,\ldots,x_N)$ – a universe that may be normalized, $X_{norm}=[0,1], x_1,\ldots,x_N\in[0,1]$; $Y=(y_1,\ldots,y_K)$ – the name (s) of elements of term-set of linguistic variables that express fuzzy values of R, A and P. Normalization X (converting to universal scale) made special monotonic transformation F: $X\to[0,1]$, in the simplest case $F(x)=\dfrac{x}{|X|}$ [2].

Compile formula using fuzzy integrals.

Expanding formula (1) using parametric T-norms leads to fuzzy interpretation of the formula through convolution of T-norm from distribution of the possible consequences of unwanted event A – μ_A and probability measure P [2]:

$$R(x)=\int T\left[\mu_A(x),P(x)\right]dx$$

The concept of fuzzy measure and fuzzy integral taken from classical set theory, fuzzy set theory and measure theory. Fuzzy measure and fuzzy integrals have some important properties – they may reflect the importance of certain criteria and represent the interaction between criteria. These properties make fuzzy measure and fuzzy integrals most rational for the risk assessment.

Fuzzy integral can be represented by the expression

$$FI=\int h\circ\mu$$

where FI –fuzzyintegral; h–measurable function(membership function); $\circ$– composition (the most often is usedmax-mincomposition); μ–fuzzymeasure.

It is also possibleto useSugenoandChoquetintegrals as afuzzyinterpretationof the formula (1).

Let $X=(x_1,\ldots,x_N)$ and P – fuzzy measure on X. Then risk can be defined as Sugeno integral of the function A: $X\to[0,1]$ with respect to the measure P [2]:

$$R(x)=(S)\int A\circ P=\bigcup_{i=1}^{N}\left[A\left(x_{(i)}\right)\wedge P\left(A_{(i)}\right)\right]$$

Here (i)means that the indices are arranged, so that $0\le A\left(x_{(1)}\right)\le\ldots\le A\left(x_{(N)}\right)\le 1$ and $A_{(i)}=\left\{x_{(i)},\ldots,x_{(N)}\right\}$.

Risk can also be defined as the Choquet integral of the function A: $X\to R$with respect to the measure P:

$$R(x)=(C)\int AdP=\sum_{i=1}^{N}\left[A\left(x_{(i)}\right)-A\left(x_{(i-1)}\right)\right]\cdot P\left(A_{(i)}\right)$$

with the samesymbolsthatweredescribed above and $A\left(x_{(0)}\right)=0$.

Risk membership function μ_R fully characterizing the risk as fuzzy value. It provides defuzzified risk value, such as its most possible value, and variations of the risk values [2].

The main types of relationships between the criteria in the context of aggregation by using Choquet integral were described by Marichall in [3]. A positive (negative) correlation and interdependence (replacement) of criteria are formalized using fuzzy measure by the sign and the index of interaction $I(i,j)$. With a positive correlation and criteria substituting, the index $I(i,j)$ is taken as negative, and with the negative correlation and substituting – as positive. Preferred dependence of the criteria and its opposite – preferred independence of the criteria – are types of dependencies between criteria that well known in the utility theory [2].

Analysis of the literature showed that the fuzzy integrals (Sugeno and Choquet) have special properties that are suitable for a variety of data merge criteria [4]:

–Sugeno and Choquet integrals are idempotent (i.e. repeated action over the object does not change it), continuous monotonically nondecreasing operators;

–Choquet integral with respect to the additive measure μ coincides with the weighted arithmetic average where weights w_i are $\mu(\{x_i\})$;

–Choquet integral is stable for positive linear transformations. Sugeno integral does not have this feature, but it fulfills a similar property with min and max replacement of selection and sum. Choquet integral is suitable for the basic aggregation, while the Sugeno integral, more suitable for the progressive aggregation;

–Sugeno and Choquet integrals containing all ordinals statistical characteristics, in particular, min, max and mean;

–Weighted minimum and maximum are special cases of Sugeno integral.

The disadvantage of fuzzy integral is the complexity of definition of fuzzy measures.

There also some difficulties of practical application of fuzzy integrals. According to Grabish [5] "With the introduction of fuzzy integrals, as aggregation operators it is implied that nonadditivity of fuzzy measures should allow to model the preferred dependence of the criteria. However, an apparatus that allows doing it strictly formal, is still not developed as well as the phenomenon of dependence of the criteria was studied poorly". If fuzzy measure is additive, the criteria do not interact with each other and interaction indexes of these criteria are equal

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to zero. Therefore, if in expert's opinion the criteria are mutually preferably independent, the corresponding indexes of interaction are equal to zero. If the expert believes the preferred dependence of the criteria, it can only be formalized by a partial order on a set of criteria implementations X (training set).

2 Applications of Fuzzy Measures and Integrals

To apply Choquet integral, fuzzy measure must first be identified based on experts' knowledge. This identification is difficult with exponentially increasing complexity in the sense that it is necessary to set the measure value for each subset of criteria. The choice of all 2^N coefficients of fuzzy measure $P\left(X_{(i)}\right)$ for relevant 2^N subsets of set of criteria indexes A is very difficult and even impossible for expert. Note that in case of even three criteria for determining the fuzzy measure we need to get $2^3 = 8$ coefficients [6]. Despite this complexity, Choquet integral can still be applied in practice. In order to do this, Grabish proposed the concept of fuzzy measure of k-order or k-additive fuzzy measures, where the order k is less than the number of aggregated criteria, or $k < |X| = N$ [12]. The essence of the concept is that in order to simplify the specifying of fuzzy measures, dependencies among more than k criteria are excluded from consideration. According to the concept of fuzzy measure of k-order in most practical cases it is possible to use Choquet integral of fuzzy measure of 2nd order, or Choquet integral of 2nd order, because it allows you to simulate the interaction between the criteria, while remaining relatively simple [6].

In addition to increasing complexity, there is also the problem of understanding the meaning of the coefficients of the expert fuzzy measure [5]. To solve this problem Grabish [7] proposed the idea of a graphical interpretation of the Choquet integral of 2nd order. The point of this interpretation is to build on the coordinate plane the restriction lines for interaction index values and Shapley indices for two criteria. This idea was developed in [8], in which the methods of fuzzy measure identification by using a hierarchical diagram of paired comparisons (in the form of "diamond"), based on the interpretation of Grabish where analyzed.

With the implementation of this approach to work with an expert in the process of formalizing its knowledge, we should choose a mathematical method for fuzzy measure identification based on formalized knowledge. These methods differ in the type of information that is required as input. Overview of methods applied to utility theory is given in [9]. The method of least squares is not well suited for solving practical problems because it involves for expert to know the desired values of aggregation result on the training sample of criteria implementations. The method based on the maximum separation is suitable for recognition problems as it involves maximizing the minimum difference between the results of the aggregation on the training set. The expert describes an instance of each class, and ranks them by using non-strict order that serves as input to the method. Method based on the minimization of the variance of fuzzy measures, or what is the same thing as maximizing the entropy of fuzzy measures, is the most appropriate to solve many practical problems [6]. It is based on the principle of maximum entropy. With regard to the construction of the aggregation operators, this principle involves the use of all available information about the aggregation criteria, but the most open-minded attitude towards the not available information.

The following practical examples briefly discussed the application of fuzzy measures and fuzzy integrals, in particular, the assessment of the properties of interfaces, technical diagnostics, and navigation.

The authors of [10] offered a solution to the problem of determining the degree of convenience of a software interface to the user. This assumes direct specifying by experts of fuzzy measures by filling special tables for multiple criteria (about four). Direct assignment of fuzzy measures is very time-consuming, and in the case of even a slight increase of the number of criteria is impossible. However, this example showed that using of fuzzy measures could improve the accuracy of estimation convenience interface.

Another practical example of the application is the analysis of the technological processes based on fuzzy expert knowledge [6]. On the first level of state analysis, the values of the membership functions of diagnostic parameters of fuzzy sets corresponding to the terms of reference of linguistic variables are determined. On the second level, by aggregating valuesof the membership function by using fuzzy measures and fuzzy integrals, the membership values of the current state of the process are obtained and corresponds to a particular class of fuzzy states such as class of the health status and the proper functioning of equipment. Identification of fuzzy measures carried out based on minimizing the variance by involving the rendering engine [6]. This example also confirms the possibility of increasing the reliability of technical diagnostics by using the fuzzy integrals.

Another example illustrates the use of the fuzzy measures in [11], which describes the navigation system for pedestrians. The inputs to this system are the subjective evaluation of various characteristics of the routes, in particular, distances, road surface quality, noise level, etc. All of these criteria are related to each other in a nontrivial way. Therefore, the aggregation of such criteria is conveniently carried out using Choquet integral [6]. As a method for identifying fuzzy measures the method of least squares is used. Despite the relatively high labor intensity of implementation, this example points to the flexibility of the Choquet integral as an aggregation operator of such subjective criteria.

In [12, 13] proposed fuzzy-probabilistic model to assess the risk of transportation of important cargoes in view of possible terrorist attacks. The value is a function of many factors (parameters) of "technical" nature: the number of ways, arrows, bridges etc., as well as the "human factor". In addition, the value depends on the possible risk of terrorist attacks. The possibility of these actions influenced by the proximity of the highways to the forests, human settlements; the presence and quantity of the accompanying protection, the ability to call reinforcements and time of arrival etc. Some factors have statistical nature and can be quantified from the available information using traditional statistical methods based on probability theory. Other factors have a fuzzy nature and can not be properly described as part of a probabilistic approach. Some of them can be described using linguistic variables, such as expert judgment. In this regard, more urgent task is aggregating of the information, including information of a different nature. Another important issue – correct assessment of the possible values spread of the risk with illegible source data. These problems are solving using "soft computing" – using not only the methods of probability theory, but also the fuzzy sets theory, possibility theory et al [14]. This approach yields the distribution function of risk, taking into account distributions of parameters influencing in a different nature, and therefore make an adequate risk assessment and its possible spread. Moreover, this variation corresponds to the real uncertainties (vagueness) of the original data.

As an example, in the task of creating optimal portfolio of investment project the objective function $f(x) \to max$ with restrictions $\varphi_i(x) \le 0$ where $i = 1,\ldots,m$ and $x \in X$, where X – the set of alternatives; $f : X \to R^1$ and $\varphi : X \to R^1$ are known functions. There are various integrated performance counters which used as the parameters of the objective function $f(x)$. However, despite of some advantages and disadvantages of each of the indicators, many scientists think that net present value (NPV) is the most preferred parameter of the objective function. NPV has the additivity attribute that gives an opportunity to assess the profitability of the entire investment project portfolio as the sum of profitability of investment projects, which form the present portfolio. There are different versions of the task of forming the optimal portfolio [15].

Usually, the economic meaning of the objective function $f(x)$ is the maximization of economic benefits from investment activity, and the meaning of the restrictions $\varphi_i(x) \le 0$, which imposed on the set of feasible solutions of the problem, shows the limited funds with considering the possibility of different budget constraints for each of the time intervals of the project life.

Strategic decisions, including those related to the formation of the optimal investment projects portfolio, aimed at the long term and therefore inherently connected with significant uncertainty, and have subjective component, so the using of fuzzy mathematical programming to solve the problem of forming optimal project portfolio has many benefits.

As an example, we can consider a situation in which the set of feasible alternatives (investment projects) is a collection of various ways of resource allocation that decision maker is going to invest in order to create an optimal investment portfolio. Obviously, that it is inappropriate beforehand to introduce a strict distinction for many acceptable alternatives (e.g., strict limits on the size of the investment budget of the company during the period t). In some cases, it may happen that the allocation of resources lies beyond this limit, which shows the effect of

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exceeding of small desirability (e.g., the size of the investment costs) of these distributions for decision maker. Thus, fuzzy description is more appropriate in reality than arbitrarily adopted a strict description of the problem.

Fuzzy description forms of initial information in tasks of decision-making may be different; hence, there are differences in the mathematical formulation of relevant tasks of fuzzy mathematical programming [15].

Discussion and Conclusions

Fuzzy logic has been used to handle uncertainty in human-centered systems (e.g., ergonomics, safety, occupational stress) analysis, as a way to deal with complex, imprecise, uncertain and vague data. In the models of risk management, triangular norm can extend the probabilistic models to fuzzy models and thereby make possible their use in conditions of poor statistics.

The issues of practical application of fuzzy measures and fuzzy integrals where reviewed the difficulties that arise and possible ways to overcome them were analyzed. The main obstacle to practical use of these tools is the complexity of working with an expert to formalize his knowledge in the form of coefficients of fuzzy measures. The area of research related to fuzzy measures and integrals is intensively developing.

In sum it should be noted, that fuzzy sets theory is the most promising and adapted mathematical apparatus that allows implement scientific task of developing methods for assessing individual risks in terms multifactor and uncertainties. Due to the large dynamic range of factors involved in the formation of risks, in order to develop a new method based on fuzzy sets theory for assessing individual risks, it is advisable to implement on the universe of variables that have a wide range of values, such as the likelihood of accident and risk.

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