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THE OPTIMIZATION OF POWER CONSUMPTION OF MOBILE ROBOTS

Annotation. *In this article the analysis of factors affecting on battery life consumption of mobile robot for phytomonitoring of plants in the greenhouse and usage of variation calculus method for optimization of energy consumption have been conducted.*

Keywords: *energy consumption, optimization, linear velocity, calculus of variations.*

Modern robotic systems have many advantages over traditional: high efficiency, reduced costs for human resources; the possibility of upgrading. Their main drawback is limited energy resource, because of the relatively small capacity of the battery, especially of mobile robots. The following problem can be solved by increasing the battery life, or reduce energy consumption. Our research aimed at reducing power consumption during the process of robot moving [1].

Energy saving is achieved if the robot moves with optimal speed (the speed with constant acceleration and speed has infrequent changes in the conditions of the robot straight line moving).

To formulate the optimization problem we accept that the mobile robotic system for electrical phytomonitoring is nonholonomic system with symmetrical structure, as set in motion by two identical DC motors [2].

Let the work of the position (coordinates and angle) as the $P(t) = [x(t)\ y(t)\ и(t)]^T$, linear velocity - v, angular velocity - w. Then kinematic equations to describe will be shown in expression:

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{и} \end{pmatrix} = T_p \begin{pmatrix} v \\ w \end{pmatrix},\ T_p = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}^{\mathrm{T}}. \quad (1)$$

To simplify the calculations, we assume that both engines have identical: anchor resistance - R_a, anti-EMF - K_b, torque - K_t and gear ratio - n. If we will mark the battery voltage as V_s, then engine balance equation will look like:

$$R_a i = V_s u - K_b n w, \quad (2)$$

where $i = [i^R i^L]^T$ - anchor current vector, $w = [w^R w^L]^T$ - vector of angular speed of wheels and $u = [u^R u^L]^T$ - normalized input vector of control. Indices R, L correspond to the left and right engine respectively.

Dynamic dependence between the angular speed and current of the engine, considering inertia and friction for the engine it is possible to write down as:

$$J\frac{dw}{dt} + F_v w = K_t n i, \tag{3}$$

where F_v - coefficient of friction, J – matrix moments of engine inertia.

From the expressions (2) and (3) we can get next differential equation:

$$\dot{w} + Aw = Bu, \tag{4}$$

where $A = \begin{pmatrix} a_1 & a_2 \\ a_2 & a_1 \end{pmatrix} = J^{-1}\left(F_v + \frac{K_t K_b n^2}{R_a}\right)$, $B = \begin{pmatrix} b_1 & b_2 \\ b_2 & b_1 \end{pmatrix} = J^{-1}\frac{V_s K_t n}{R_a}$.

We define a state vector as $z = [v \; w]^T$, associate w and v *with* w^R and w^L in expression:

$$z = \begin{pmatrix} v \\ w \end{pmatrix} = T_q \begin{pmatrix} w^R \\ w^L \end{pmatrix} = T_q w, \quad T_q = \begin{pmatrix} \frac{r}{2} & \frac{r}{2} \\ \frac{r}{2b} & -\frac{r}{2b} \end{pmatrix}. \tag{5}$$

Using similarity transformations, from expressions (4) and (5) we receive:

$$\dot{z} + \bar{A}z = \bar{B}u, \tag{6}$$

$$\dot{z} + \bar{A}z = \bar{B}u, \tag{6}$$

where $\bar{A} = T_q A T_q^{-1} = \begin{pmatrix} \mathrm{p}_v & 0 \\ 0 & \mathrm{p}_w \end{pmatrix} = \begin{pmatrix} a_1 + a_2 & 0 \\ 0 & a_1 - a_2 \end{pmatrix}$.

$$\bar{B} = T_q B = \begin{pmatrix} \mathrm{B}_1 & \mathrm{B}_1 \\ \mathrm{B}_2 & -\mathrm{B}_2 \end{pmatrix} = \begin{pmatrix} \frac{r(b_1 + b_2)}{2} & \frac{r(b_1 + b_2)}{2} \\ \frac{r(b_1 - b_2)}{2} & -\frac{r(b_1 + b_2)}{2} \end{pmatrix}.$$

The energy that comes from the battery power converts into mechanical energy of motion and heat loss. Heat loss causes the internal resistance of the battery, resistances of the controlling device (driver) of engine, the anchor resistance of engine and viscous friction during movement.

On fig.1 is shown the simplified diagram of the electrical system of a mobile robot. To reduce heat losses in the engine control device used controller PWM, because it has less power consumption and generates less heat than linear voltage regulator. Therefore, we can determine the resistance of the amplifier R_{amp} and PWM u^R, u^L. To simplify the calculations, we assume that the heat loss through the internal resistance of the battery and the resistance of the amplifier control device of engine are small, and therefore they are not included.

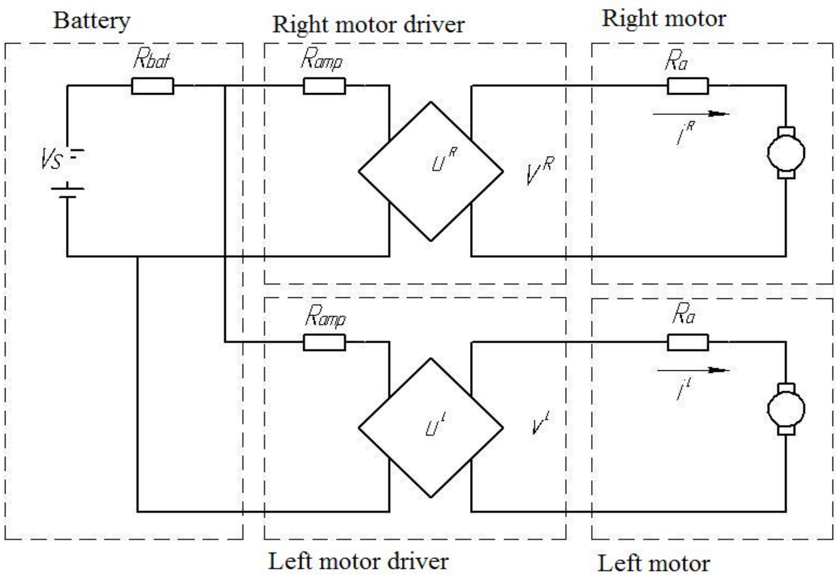

Fig. 1. Simplified electrical scheme of a mobile robot.

Thus the energy which is supplied from the battery to drive circuit of the mobile robot is a function for minimizing and it can be shown as [4]:

$$E_w = \int_{t_0}^{t_f} i^T V dt = V_s \int_{t_0}^{t_f} i^T u dt, \quad (7)$$

where $V = [V^R V^L]^T$ – input voltage, which comes to the from the accumulator battery (V_s – voltage of battery), $u = \frac{V}{V_s} = [u^R u^L]^T$.

As the battery capacity is limited, the voltage will be limited as well:

$$-u^{max} \leq u^R, u^L \leq u^{max}. \quad (8)$$

According to the phrases (2),(5) we can write down the function of optimization E_w through the velocity as:

$$E_w = \int_{t_0}^{t_f} (k_1 u^T u - k_2 z^T T^{-T} u) dt, \quad (9)$$

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де $k_1 = \frac{V_s^2}{R_a}$, $k_2 = \frac{K_b n V_s}{R_a}$, $z = [v \ \ w]^T$.

Thus, taking into consideration expressions (2), (3) overall optimization function can be represented as follows:

$$E_w = R_a \int_{t_0}^{t_f} i^T i dt + F_v \frac{K_b}{K_t} \int_{t0}^{t_f} z^T T_q^{-T} T_q^{-1} z dt + \frac{\text{稔}_b}{K_t} \int_{t0}^{t_f} \dot{z}^T T_q^{-T} J^T T_q^{-1} z dt. \quad (10)$$

In the expression (10) first term ($E_r = R_a \int_{t_0}^{t_f} i^T i dt$)– is the energy, dissipates by the resistance in the engine anchor [5]; second term ($E_F = F_v \frac{K_b}{K_t} \int_{t0}^{t_f} z^T T_q^{-T} T_q^{-1} z dt$) corresponds to the loss of energy to overcome friction; the last term ($E_K = \frac{K_b}{K_t} \int_{t0}^{t_f} \dot{z}^T T_q^{-T} J^T T_q^{-1} z dt$) - is the kinetic energy of the mobile robot that zero average value when speed the constant or final speed is equal to the initial. This means that the contribution of the last term in power consumption is zero.

As the robot moves mostly straight, we consider only this option. Thus rotation velocity of robot - w in this period is zero, but $P(t) = [x(t)\ 0\ 0]^T$– is the position, $z(t) = [v(t)\ 0]^T$ - speed at the moment of t. Then the problem of minimizing energy consumption can be formulated as follows: find the value of linear velocity $v(t)$ and controlling figure $u(t)$, that will minimize the function:

$$E_w = \int_{t_0}^{t_f} \left(k_1 u^T u - k_2 z^T T_q^{-T} u\right) dt \quad (11)$$

under the following conditions:

1) initial and final position: $P(t_0) = [x_0\ 0\ 0]^T$ and $P(t_f) = [x_f\ \ 0\ 0]^T$;

2) initial and final velocity: $z(t_0) = [v_0\ 0]^T$ and $z(t_f) = [v_f\ \ 0]^T$;

3) satisfying value of battery, where t_0 and t_f start and end time of the robot moving.

We assume that the initial and final velocity of robot are zero and its motion starts with initial position. Then the energy minimization task can be written as [6]:

$$min\, E_w = \int_{t_0}^{t_f} (k_1 u^T u - k_2 z^T T_q^{-T} u) dt, \quad (12)$$

$$\dot{z} = -\bar{A} z + \bar{B} u, \quad (13)$$

$$z(0) = z(t_f) = [0\ 0]^T, \quad (14)$$

$$P_f = \int_0^{tf} T_p z dt = \left[x_f\ 0\ 0\right]^T, \qquad (15)$$

$$-\binom{u^{max}}{u^{max}} \leq u = \binom{u^R}{u^L} \leq \binom{u^{max}}{u^{max}}. \qquad (16)$$

Taking into account the cost of energy, which minimizes the equation (12) satisfying the constraints (14) - (16) for the system of equations (13), search for optimal speed we will do using the method of variations. Based on studies that are presented in [1], Lagrange multiplier for expression (15) will be $a = \left[a_x a_y a_и\right]^T$. Defined functions for function expression (13) $\lambda = [\lambda_v \lambda_w]^T$, where the Hamiltonian function would be:

$$H = k_1 u^T u - k_2 z^T T_q^{-T} - a^T T_p z + \frac{a^T P_f}{t_f} + \lambda^T(-\bar{A}z + \bar{B}u). \qquad (17)$$

Necessary conditions for optimal velocity $z*$ and out coming signal $u*$:

$$\frac{\partial H}{\partial u} = 2k_1 u - k_2 T_q^{-1} z + \bar{B}^T \lambda = 0, \qquad (18)$$

$$\frac{\partial H}{\bar{a} z} = -k_2 T_q^{-T} u - T_p^T a + \bar{A}^T \lambda = -\lambda, \qquad (19)$$

$$z = -\dot{\bar{A}} z + \bar{B} u. \qquad (20)$$

From the expressions (18) - (20) we obtain the following differential equation:

$$\ddot{z} - \left(\bar{B}\bar{B}^T \bar{A}^T \bar{B}^{-T} \bar{B}^{-1} \bar{A} - \frac{k_2}{k_1} \bar{B}\bar{B}^T T_q^{-T} \bar{B}^{-1}\right) z + \frac{1}{2 \cdot k_1} \bar{B}\bar{B}^T T_p^T a = 0 \qquad (21)$$

Because of $\bar{B}\bar{B}^T$ and $\bar{A}$ - are diagonal matrixes, equation (21) can be reduced to:

$$\ddot{z} - Q^T Q z + R T_p^T a = 0, \qquad (22)$$

where $Q^T Q = \bar{A}^T \bar{A} - \frac{k_2}{k_1} \bar{B}\bar{B}^T T_q^{-T} \bar{B}^{-1} \bar{A}$, $Q = \begin{pmatrix} \frac{1}{ф_v} & 0 \\ 0 & \frac{1}{ф_w} \end{pmatrix}$,

and $R = \frac{\bar{B}\bar{B}^T}{2k_1} = \begin{pmatrix} n_v & 0 \\ 0 & n_w \end{pmatrix}$.

Here $ф_v = \frac{J_1 + J_2}{\sqrt{F_v(F_v + K_t K_b n^2 / R_a)}}$ – electromechanical time constant for moving,

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$\phi_w = \frac{J_1 - J_2}{\sqrt{F_v(F_v + K_t K_b n^2/R_a)}}$ electromechanical time constant for the turning of mobile robot.

As the energy loss by turning a mobile robot is not captured (considering moving on a straight line) optimal linear velocity z ^ * can be expressed as:

$$z^*(t) = \begin{pmatrix} v^*(t) \\ w^*(t) \end{pmatrix} = \begin{pmatrix} C_1 e^{t/\phi_v} + C_2 e^{-t/\phi_v} + K_v \\ 0 \end{pmatrix}, \quad (23)$$

where $C_1 = \frac{e^{-t_f/\phi_v} - 1}{e^{t_f/\phi_v} - e^{-t_f/\phi_v}} K_2$, $C_2 = \frac{1 - e^{t_f/\phi_v}}{e^{t_f/\phi_v} - e^{-t_f/\phi_v}} K_v$, and $K_v = \frac{x_f(e^{t_f/\phi_v} - e^{-t_f/\phi_v})}{2\phi_v\left(2 - e^{\frac{t_f}{\phi_v}} - e^{-\frac{t_f}{\phi_v}}\right) + t_f(e^{t_f/\phi_v} - e^{-t_f/\phi_v})}$.

Then equation ((13)gives the equation of optimal control signal of u^*:

$$(t) = \bar{B}^{-1}(\dot{z}^* + \bar{A}z^*) =$$

$$\frac{1}{2 \cdot в_1 \cdot \phi_v} \begin{pmatrix} C_1(1 + \phi_v p_v)e^{\frac{t}{\phi_v}} - C_2(1 - \phi_v p_v)e^{-\frac{t}{\phi_v}} + \phi_v p_v K_v \\ C_1(1 + \phi_v p_v)e^{\frac{t}{\phi_v}} - C_2(1 - \phi_v p_v)e^{-\frac{t}{\phi_v}} + \phi_v p_v K_v \end{pmatrix}. \quad (24)$$

For construction of dependencies of optimal velocity from relative time (from the beginning):

Parameter	Value	Parameter	Value
resistance of anchor, R_a	0,71 Ом	battery voltage, V_s	12 B
Torque, K_t	0,0230 N·м/A	Limitation of voltage, u^{max}	1
Anti-emf, K_b	0,0230 B/(rad/sec)	coefficient of friction, F_v	0,054 N·м/(rad/sec)
The radius of the wheels, r	0,15 м	Gear ratio, n	49,8
Base	0,206 м	The inertia of engines, $J = \begin{pmatrix} J_1 & J_2 \\ J_2 & J_1 \end{pmatrix}$	$\begin{pmatrix} 0.1241 & 0.0098 \\ 0.0098 & 0.1241 \end{pmatrix}$

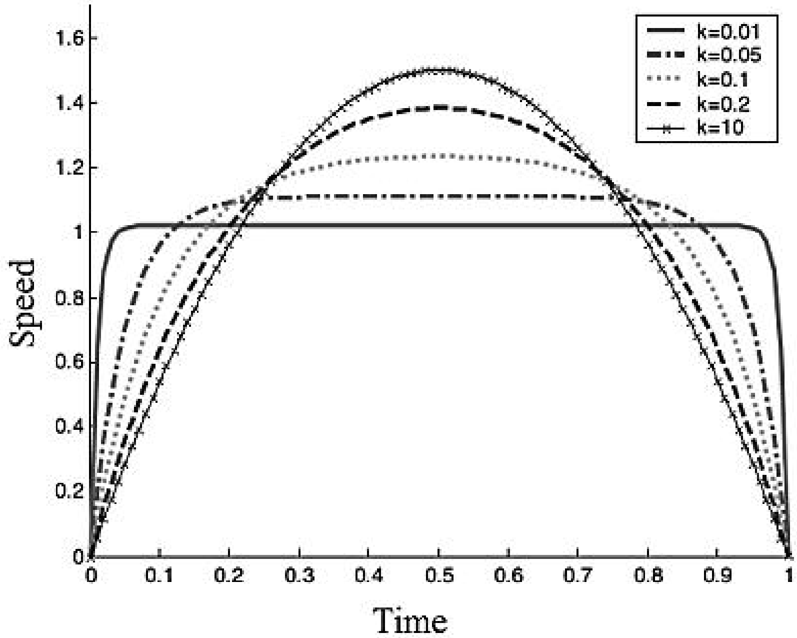

Fig. 2. Speed depends on the relative distance of moving at different values meaning of$k$ (mechanical time constant before the time of displacement $k = \tau_v/t_f$)

The analysis of materials which are shown in Fig. 2 can to make the fconclusions that the minimal power consumption rate graphic has a symmetrical shape:

○ if $k \approx 0$ ($k = \phi_v/t_f$) then graphic of optimal velocity is trapezoidal;

○ if k>0,2 then graphic takes the form of a parabola.

Basing on (12) – (15) we can receive:

$$v^*(t) = \frac{x_f}{\phi_v} \cdot \frac{\sin h\left(\frac{t_f}{\phi_v}\right) - \sin h\left(\frac{(t_f - t)}{\phi_v}\right) - \sin h\left(\frac{t_f}{\phi_v}\right)}{2\left(1 - \cos h\left(\frac{t_f}{\phi_v}\right)\right) + \frac{t_f}{\phi_v}\sin h\left(\frac{t_f}{\phi_v}\right)}. \quad (25)$$

Energy consumption are defined by losses because of anchor resistance (the first component of expression (10)) - minimization of energy of an anchor were investigated; losses because of resistance of an anchor and losses on friction (the first and second parts of expression (10) - minimization of expenses of energy.

At the parabolic graphic of velocity of power consumption is somewhat compensated for the account of partial operation of the engine in the generator mode.

Comparing two ways of optimization of power consumptions, we draw a conclusion that minimization of power consumptions of an anchor is

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expedient in case of movement of the robot on distance to 5 meters, (the friction significantly doesn't influence this movement). Full minimization of energy is otherwise more expedient.

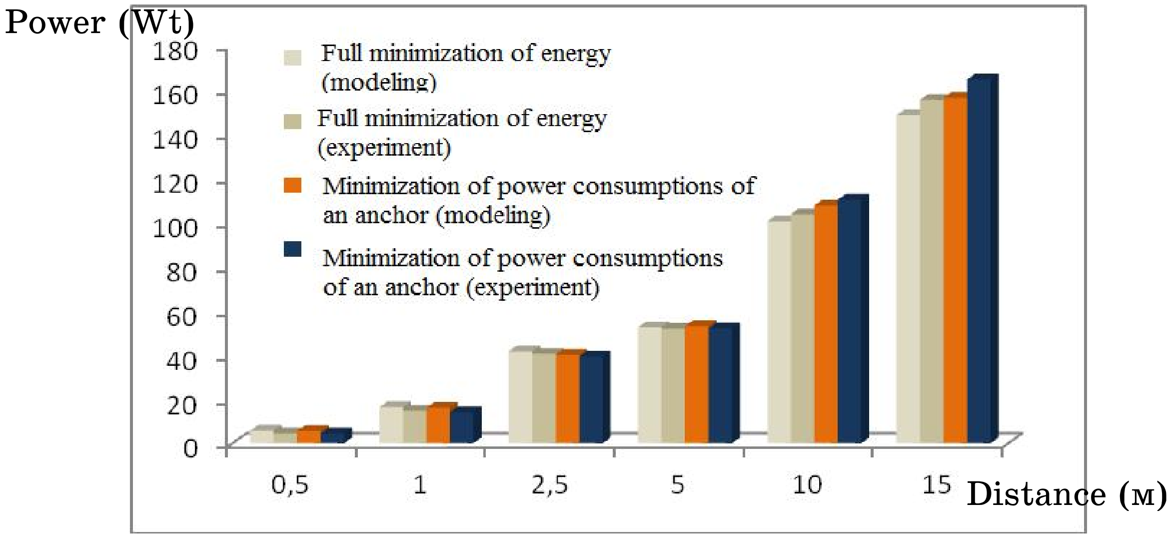

Fig. 3. Comparison of data of modeling to the data received during experiment.

CONCLUSION

1. By results of analytical designing of regulators, expression of optimum linear velocity of movement of the mobile robot that minimizes power expenses is received.

2. The choice of velocity of movement and parameters of optimization depends on distance on which the robot has to move: at distance to 5 meters the parabolic graphic of velocity is used, and at the bigger – trapezoidal.

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