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**USING OF RASTER IMAGES FOR MEASUREMENT  
OF TIME CONSTANT  $\tau$  OF EXPONENTIAL PULSES**

*Abstract. Method for measurement of time constant  $\tau$  of exponential pulses was developed. Method uses the scan data of raster images of analog oscillograms, obtained by means of a digital photcamera. Programme for method realization was created by means of the languages HTML, CSS, JavaScript and jQuery library. Any Internet browser is the software environment for use of this programme. The measurement error of method is no more than  $\pm 1\%$ .*

**Introduction.** The merit of analog oscilloscopes in comparison with digital oscilloscopes is the direct recording without the intermediate conversions. However, a low accuracy of measurement of voltages and time intervals is a grave disadvantage of such oscilloscopes. The relative error usually has values of  $\pm (5 - 10) \%$  [1]. As was shown in the work [2], measurement error can be decreased, if to use for measurement of instantaneous voltages and time intervals the raster images of analog oscillograms, obtained by means of a digital photcamera. If to perform the scan of such images in graphics editors and then to process scan data, the error can be decreased up to the values no more than  $\pm 1,5\%$  [2].

The means of Web 2.0 technology give significant possibilities for a creation of programmes intended for the processing of raster images of analog oscillograms [3]. These means are the languages HTML, CSS, JavaScript and jQuery library. HTML and CSS allow create comfortable interface in a form of the web page. JavaScript and jQuery library give means for load of raster images, their scanning, processing of the scan data and display of final results. Any Internet browser can be used as the software environment for programmes created by means of the software mentioned above. We have used the means of Web 2.0 technology for software development in the present work.

**Problem definition.** Development of software, allowing with a high precision to define the time constant of exponential pulses from the scan

data of raster images of oscillograms, obtained by analog oscillographs is the aim of present work.

**Major part.** Raster images of oscillograms for calibration voltage  $U_C$  and single exponential pulse are shown in the Fig. 1. These images were obtained from a screen of the storage oscilloscope C8-11 by means of digital photcamera OLYMPUS. Both oscillograms were registered at the same amplification. In Fig. 1b horizontal line in a bottom of image corresponds to zero voltage. In Fig. 1a distance between the straight lines corresponds to the calibration voltage  $U_C = 100V$ .

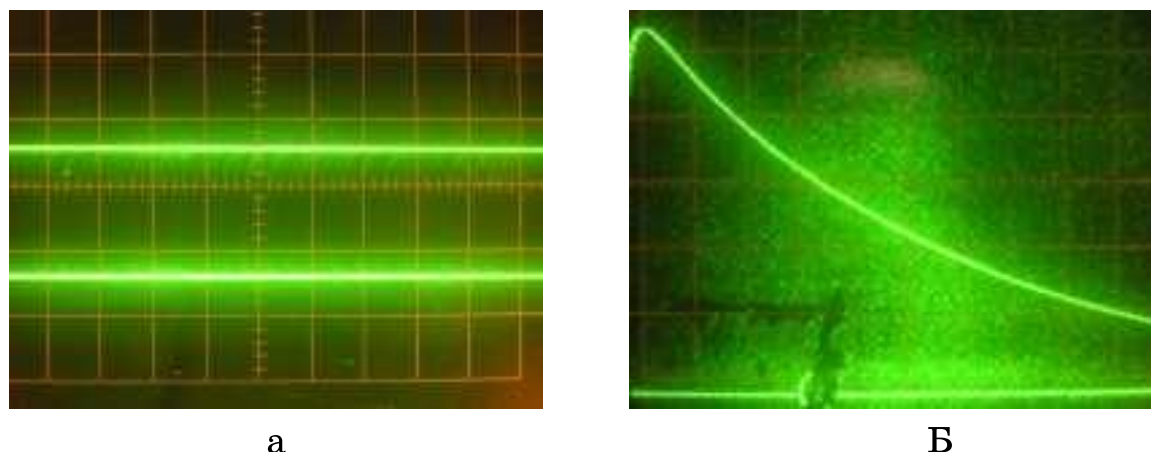


Figure 1 - Raster images of oscillograms for calibration voltage  $U_C = 100$  (a) and for exponential pulse of voltage (b).

The time scale  $50 \mu s / \text{division}$

Upper left corner of raster image is the origin of coordinates for any its point. Relatively origin of coordinates,  $x$  coordinate is the point offset in horizontal direction (time axis of oscillogram);  $y$  coordinate is the point offset in vertical direction (voltage axis of oscillogram). Coordinates  $x, y$  in pixels for any point of the raster image of oscillograms can be obtained by means of the scan in graphics editors.

The scan at same coordinates  $x$  for lines in the Fig. 1a allows obtaining coordinates  $x, y_{C1}$  and  $x, y_{C2}$ . Then the scale of voltage  $Mu$  in V/pixel can be found as

$$Mu = U_C / |y_{C1} - y_{C2}|. \quad (1)$$

On base of scale  $Mu$  can be defined the instantaneous voltage  $U(t)$  in any points of oscillogram for raster image in the Fig. 1b. For this it is necessary to perform the coordinate scan for point of pulse line  $(x, y)$  and for zero line  $(x, y_0)$  at the same  $x$  coordinate. The value of  $x$

corresponds to chosen time moment  $t$ . From these coordinates the instantaneous voltage  $U(t)$  can be calculated by formula:

$$U(t) = Mu|y - y_0|. \quad (2)$$

As is well known, time dependence of voltage for the pulses with exponential shape is described by expression:

$$U(t) = U_0 \exp(-t/\tau), \quad (3)$$

where  $U_0$  is the pulse amplitude,  $\tau$  is the time constant.

If scan of the coordinates begin for pulse line from  $x_1$  coordinate, corresponding to time moment  $t_1$  and to voltage  $U(t_1) = Mu|y_1 - y_0|$ , the expression (3) can be presented as

$$U(t) = U(t_1) \exp\left(-\frac{t-t_1}{\tau}\right), \quad (4)$$

where the time interval  $t - t_1 = Mt(x - x_1)$ .

The scale of time  $Mt$  for raster image can be defined at known oscilloscope time scale if to use the data of scan  $(x_{C1}, y)$ ,  $(x_{C2}, y)$  obtained for time interval  $T_C$  in the graticule of oscilloscope. In this case the scale of time (time unit/pixel) is defined by formula:

$$Mt = T_C / |x_{C1} - x_{C2}|. \quad (5)$$

As from expression (4) follows, the linear dependence takes place in coordinates  $\ln(U(t)) \sim t$  for exponential pulses. It allows defining time constant  $\tau$  as

$$\tau = \Delta t / \Delta \ln(U(t)), \quad (6)$$

where  $\Delta t$  is the time increment;  $\Delta \ln(U(t))$  is the increment for natural logarithm of voltage.

It should be noted, that taking into account expression (2)  $\Delta \ln(U(t)) = \Delta \ln(|y - y_0|)$ . Consequently, the scale of voltage  $Mu$  can be excluded at determination of time constant  $\tau$ . In this case the increment for logarithm of voltage is calculated by values of coordinates in pixel obtained at scan of the raster image of oscillogram.

The array of time  $t_i = Mt|x_i - x_1|$  and the array of voltage  $U_i(t_i) = Mu|y_i - y_0|$  ( $i = 1, 2, \dots, N$ ) can be obtained by scan of pulse line and zero line in raster image of oscillogram in a chosen time interval  $t_i = Mt|x_i - x_1|$ . It is necessary the array of voltage  $U_i(t_i)$  to transform in the array of natural logarithm of voltage  $\ln(U_i(t_i))$  for determination of time constant  $\tau$  for exponential pulse. Straight line  $\ln U = b + at$  can be drawn through the points of this array by means of the least-squares

method (linear regression). Module of inverse value of  $a$  coefficient in equation of this line according to (6) is the value of  $\tau$  ( $\tau = |1/a|$ ).

Value  $a$  at the linear regression is determined by relationship:

$$a = (\overline{xy} - \bar{x}\bar{y}) / (\overline{x^2} - \bar{x}^2), \tag{7}$$

where in our case

$$\bar{x} = \frac{1}{N} \sum_{i=1}^N t_i; \bar{y} = \frac{1}{N} \sum_{i=1}^N \ln(U_i(t_i)); \overline{x^2} = \frac{1}{N} \sum_{i=1}^N t_i^2; \overline{xy} = \frac{1}{N} \sum_{i=1}^N t_i \ln(U_i(t_i)) \tag{8}$$

and  $t_i = Mt|x_i - x_l|$ ,  $U_i(t_i) = Mu|y_i - y_0|$ .

Fig. 2 illustrates a scattering of the points, obtained at scan of oscillogram in raster image relatively the regression line. Thick solid line shows the regression line. In Fig. 2  $\Delta$  is the mean-square deviation of scan points relatively the regression line  $\ln U = b + at$ . This deviation can be calculated by formula:

$$\Delta = \sqrt{\frac{1}{N} \sum_i (\ln(U_i(t_i)) - b - at_i)^2} \tag{9}$$

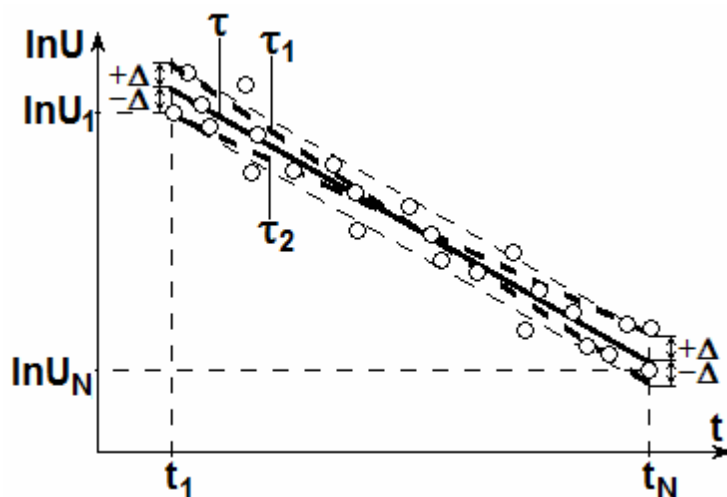


Figure 2 - Scattering of the scan points relatively the regression line

The absolute error  $\Delta\tau$  and the relative error  $\delta$  at measuring of time constant  $\tau$  can be determined at using of  $\Delta$  value. Taking into account mean-square deviation  $\Delta$ , the straight lines through scan points can be drawn by two way with the maximum deviation from the regression line, as it is shown in the Fig. 2 by the thick dotted lines. Two values of time constant  $\tau_1$  and  $\tau_2$  are defined by the slope of these lines. These values allow finding the negative  $\tau_1 - \tau$  and the positive  $\tau_2 - \tau$  abso-

lute errors. Analysis has shown that these errors are defined by expressions:

$$\tau_1 - \tau = \frac{-2\tau^2\Delta}{t_N - t_1 + 2\tau\Delta}; \quad \tau_2 - \tau = \frac{2\tau^2\Delta}{t_N - t_1 - 2\tau\Delta}. \quad (10)$$

From the Eq. (10), when  $2\tau\Delta \ll t_N - t_1$  values of  $\Delta\tau$  and  $\delta$  are defined as

$$\Delta\tau = \pm \frac{2\Delta}{a^2(t_N - t_1)} = \pm \frac{2\tau^2\Delta}{(t_N - t_1)}, \quad (11)$$

$$\delta = \pm \frac{2\Delta}{a(t_N - t_1)} 100\% = \pm \frac{2\tau\Delta}{(t_N - t_1)} 100\%. \quad (12)$$

On base of above-mentioned method the programme for measurement of the time constant of exponential pulses was developed. Programme was created by means of the languages HTML, CSS, JavaScript and jQuery library.

Fig. 3 shows the initial view of programme window opened in Google Chrome browser. Programme interfaces contain the buttons for control of data processing, as well as the fields of forms and the tables for load of initial data, output of scan data and results of data processing. In the top left angle of browser window are displayed the coordinates  $x$  and  $y$  for current position of cursor. At click by mouse these coordinates are transmitted to the arrays of data in programme and appear in the table columns «X», «Y» (Fig. 3). As required, user can delete data entered to the table beginning from the last row. It ensures a pressing of the button “Удалить строку”.

Programme uses the button “Выберите файл” for load raster image of oscillogram to the browser window. Sequential transition to the different stages of data processing takes place at pressing of the button “Продолжить”. Programme displays interface of stages in the block “Инструментарий”. Interface for the every stages contains necessary set of buttons and fields of forms for data input-output as well as user instruction for the given stage. The button «Сброс» assigned for cleaning of tables and fields as well as for return to the initial state of programme.

Programme sequentially performs the following stages for processing of raster images of oscillograms: 1) loading of image; 2) assignment the quantity of scan points in raster image; 3) determination of time scale; 4) scan of zero line of voltage; 5) scan of pulse line and calculation of time constant  $\tau$ . In the block “Инструментарий”, programme has the individual

interface for every processing stage. It uses the functions `show()` and `hide()` of jQuery library for display and concealment of these interfaces. The object `FileReader` is used in programme for the loading of image file to browser window. Current coordinates of cursor  $x$ ,  $y$  are recorded by means of the event “mousemove” (move of the cursor). The properties “offsetX” and “offsetY” of object “event” are used for this aim. Script with the function “text()” of jQuery library ensures the display of current coordinates.

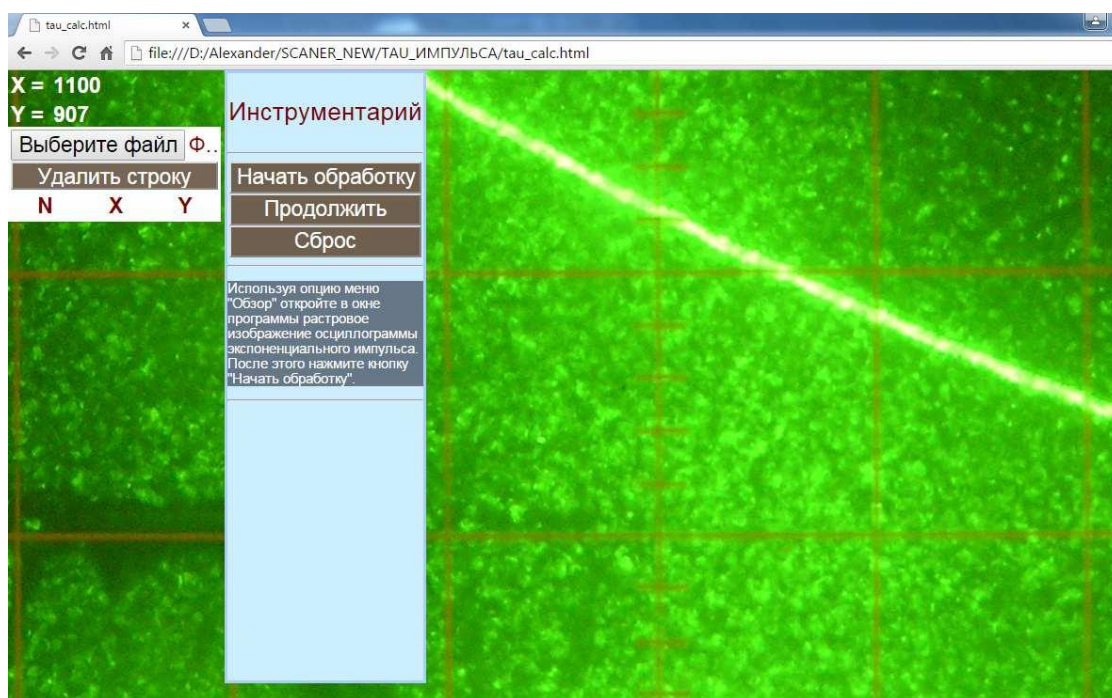


Figure 3 - Initial view of programme window with the loaded raster image of oscillogram. A part of oscillogram in Fig 1b are displayed in browser window

Image processing begins after pressing of the button “Начать обработку”. Scan of calibration time interval, zero line and pulse line takes place by means of the event “click” (press of the left mouse button). At the last stage of processing after pressing of button “Вычислить”, programme calculates the time constant of exponential pulse  $\tau$ , the absolute  $\Delta\tau$  and the relative  $\delta$  errors. It uses the formulas presented above. Results are displayed in the forms of interface (Fig. 4).

A high resolution of raster images and a small scan error ( $\pm(1 - 2)$  pixels) ensure a high measurement accuracy of  $\tau$  by means of raster images of oscillograms. The relative error no more than  $\pm 1\%$ . Method allows defining  $\tau$  using relatively small parts of the raster image of oscillogram. In the case of rectangular pulse, this gives possibility to define the time con-



stant of a transient process which gives distortion of pulse front. At measurement of time constant  $\tau$  for such process it is necessary to perform the scan of raster image in the region of pulse front.

**Conclusion.** Method for determination of the time constant  $\tau$  of exponential pulses by data obtained at scan of raster image of oscillograms was developed. Programme for method realization was created by means of the languages HTML, CSS, JavaScript and jQuery library. Internet browsers are the software environment for this programme. Relative error of the determination of time constant  $\tau$  by scanning data of raster image of oscillograms no more than  $\pm 1\%$ .

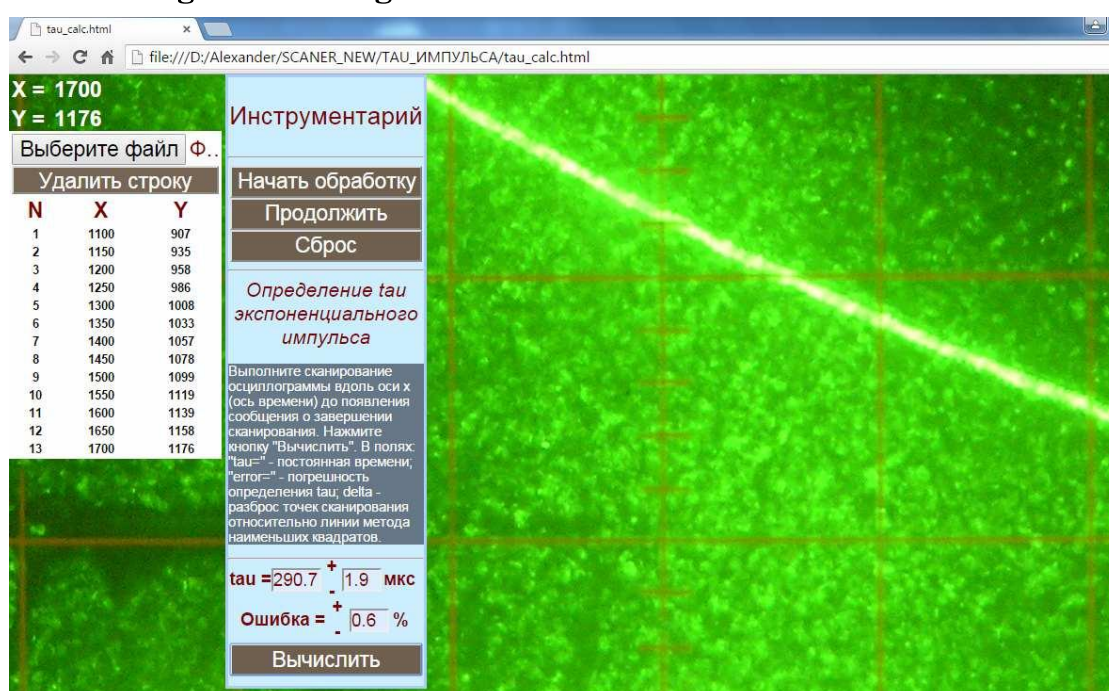


Figure 4 - Programme window at the final stage of processing of raster image

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