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ESTIMATION OF CONFORMITY OF MODEL AND THE ORIGINAL

The comparative analysis is lead and recommendations on using of various nonparametric criteria with reference to a problem of checking uniformity of two samples as versions of a problem of estimation of conformity of mathematical model and the original are resulted.

Keywords: modeling, comparing, nonparametric criteria, estimation, conformity

Introduction. Conformity (the term adequacy is sometimes used) - the main property of mathematical model. The level of conformity is formed at creation of model and supported at use of model. In both cases it is necessary to estimate the level of conformity.

The problem of an estimation of conformity of mathematical model and the original can be considered [1] as a version of a known problem [2-5] about checking of uniformity of two samples, formed by target variables of mathematical model and the original. This problem usually in mathematical statistics [2-5] is formulated in a following kind. Let elements of two samples $\xi 1, \xi 2, \ldots, \xi n$ and $\xi 1', \xi 2', \ldots, \xi m'$ (which are realizations of target variables of model $y_м$ and the original y_o) are mutually independent and submit to continuous distributions. The basic checking hypothesis Ho, consists in the assumption, that both samples are taken from the same sets and, so means that functions of distribution of random variables ξ and ξ' are identical. This hypothesis can be expressed identity

$$H_o : \mathrm{M}F_n(x) \equiv \mathrm{M}G_m(x), \quad (1)$$

where Fn(x) и Gm(x) - functions of the empirical distributions constructed on $\xi 1, \xi 2, \ldots, \xi n$ and $\xi 1', \xi 2', \ldots, \xi m'$ samples. Possible(Probable) competing hypotheses can be written down in the form of inequalities:

$$H_1^+ : \sup \mathrm{M}\left(G_m(x) - F_n(x)\right) \rangle 0, \quad (2)$$

$$H_1^- : \inf \mathrm{M}\left(G_m(x) - F_n(x)\right)\langle 0, \quad (3)$$

$$H_1 : \sup\left(\mathrm{M}\left(G_m(x) - F_n(x)\right)\right)\rangle 0 \quad (4)$$

Considering, that in real conditions of modeling it is often inconvenient to receive the necessary information on laws of distribution of the random variables forming samples, we use for this purpose nonparametric criteria. In this case various criteria which are subdivided on two groups can be applied.

The first group includes the criteria based on differences of functions of empirical and theoretical distributions (criteria Smirnov, Kolmogorov, analogue of criterion ω2) Thus, "simplicity" is usually reached(achieved) due to some decrease (reduction) in capacity(power) of criteria in comparison with capacity(power) of criteria of the first group.

Material and methods. Let's compare the specified criteria for a typical applied problem(task) of an estimation of conformity of mathematical model and the original in metallurgy which prominent feature is the limited volume of sample (small samples) that is caused by difficulty, and sometimes and practical impossibility, gathering of great volumes of the information.

Check of the basic hypothesis (1) At unilateral competing hypotheses (2), (3) it is recommended[2-4[to spend by means of рангового criterion Wilcoxon which though is not in regular intervals the most powerful criterion, is not displaced, well effective and simple at the practical use. For example [2], асимптотическая efficiency of criterion Wilcoxon in relation to t-criterion Student "always ≥0.864, But can exceed 1 and even to be infinite".

Unilateral competing hypotheses (alternative) of a kind (2), (3), consisting that the target variable of model stochastically is less (2 or more (3) target variables of the original have private (individual) character. More the general (common) is bilaterial alternative (4) or most the general (common) variant of alternative in the form of a condition F(x) ≠ G(x) (here F(x) and G(x) functions of corresponding (meeting) theoretical distributions). Often to a considered (an examined) problem(task) in the general(common) statement criterion Smirnov [5] which statistics is set by expression is applied:

$$D_{m,n} = \max_{|x|\langle\infty} \left| G_m(x) - F_n(x) \right|. \tag{5}$$

Practically values of statistics (5) are recommended [5] to be calculated by means of formulas:

$$D^{+}_{m,n} = \max_{1\le r\le m}\left(\frac{r}{m} - F_n\left(\eta^{©}_r\right)\right) = \max_{1\le s\le n}\left(G_m\left(\eta_s\right) - \frac{s-1}{n}\right), \tag{6}$$

$$D^{-}_{m,n} = \max_{1\le r\le m}\left(F_n\left(\eta^{©}_r\right) - \frac{r-1}{m}\right) = \max_{1\le s\le n}\left(\frac{s}{n} - G\left(\eta_s\right)\right), \tag{7}$$

$$D_{m,n} = \max\left(D^{+}_{m,n}, D^{-}_{m,n}\right). \tag{8}$$

To check of the basic hypothesis (1) at alternative (4) it is often applied [2-5] bilateral criterion Wilcoxon which concerns to group serial (ранговых) статистик. The basic hypothesis Ho it is possible to express identity:

$$H_o: \qquad \mathrm{P}\left\{\xi\langle x\right\} \equiv \mathrm{P}\left\{\xi'\langle x\right\} \qquad \left(|x|\langle\infty\right) \tag{9}$$

The statistics of criterion Wilcoxon is set by the formula:

$$W = r_1 + r_2 + \ldots + r_m, \tag{10}$$

where r_i - ranks (serial numbers) of elements of smaller sample on volume in the general(common) variational number(line).

For example, it was examined [5] application of criteria Smirnov and Wilcoxon for one typical task of checking up conformity of mathematical model and original for data at a significance value of 5 %.

Application of both criteria leads to identical results in identical conditions of estimation conformity of mathematical model and original. Both of criterions allow to allocate area of modeling (value of entrance variables at which is available conformity of mathematical model and original).

Though application of criterion Smirnov assumes performance of a significant amount of calculations (cycles by quantity of elements in samples) and the choice of greatest of the calculated sizes, it cannot be recognized by essential lack (except for cases when calculations are carried out "manually" without application of the corresponding computer programs). More essential lack is that for "small" samples some variants of data sets cannot be subjected to check by means of criterion Smirnov. In particular, it concerns to all cases (for all significance values) when one of samples contains only one value of a target variable and besides variants when in one sample two values, and in another - two, three or four values.

Criterion Wilcoxon provides an opportunity of the analysis small samples, including (unlike criterion Smirnov) and at one value in one of samples (the minimum quantity of values is equal the second sample 9 at a significance value of 10 %). Considering, that criterion Wilcoxon possesses, besides with sufficient power and efficiency [2-5], allows to allocate area of modeling, criterion Wilcoxon is recommended to be used as the core at estimation conformity of mathematical model and original.

More difficult is task to estimate adequateness of model when the quantity of information is too small, for example, as it may be for the metallurgical processes. In this case it is necessary to use other criteria, which is based on small samples as for model so for original. The criteria may be built in such way. For bigger sample (indifferently model or origin) calculate statistical characteristics (11) and (12), then determine the value of criteria (13):

$$\bar{\xi} = \frac{1}{n}\sum_{i=1}^{n}\xi_i, \tag{11}$$

$$(S^*)^2 = \frac{1}{n}\sum_{i=1}^{n}\left(\xi_i - \bar{\xi}\right)^2 \tag{12}$$

$$\zeta\left(\bar{\xi}, S^*\right) = max\left|\left(\xi_r' - \bar{\xi}\right)/S^*\right|,\ r = \overline{1,m}. \tag{13}$$

where n – volume of bigger sample, ξ_r' - testing variety.

It shows what is the difference between testing variety and average value for biggest sample (for original or model) on a relation to mean quadratic deviation for this sample. According to rule of criteria main hypothesis (about the conformity of testing variety) is true if it is true such condition (14)

$$\zeta\left(\bar{\xi}, S^*\right) < \zeta(n, Q). \tag{14}$$

Practical using of criteria may be more simple if take its limit values (15), (16)

$$\xi_{max} = \bar{\xi} + \zeta(n, Q)\cdot S^*, \tag{15}$$

$$\xi_{min} = \bar{\xi} - \zeta(n, Q)\cdot S^*. \tag{16}$$

If testing variety is placed between limit values (15), (16) so it means that hypothesis about the conformity of testing variety is true with a definite level of significance value Q.

For example (table 1), only model of Полухина have the conformity with original at a significance value of 5 % (exsept last cage). Other models have the conformity for only certain gates.

Table 1

Values [6] of average specific pressure at the cages calculated by different models

Number of cage	Model of Целикова	Model of Полухина	Model of Гелеи	Model of Экелунда	ξmax	ξmin
5	20.5	21	12.5	11.5	24.19967	19.60033
6	32	26	24	19	29.14960	23.85040
7	42	34	28	26	39.59946	32.40054
8	42.5	39.5	32.5	31	44.54939	36.45061
9	56.5	43.5	37	40	45.64937	37.35063
10	39	38	39	34	47.29935	38.70065

Results of estimating conformity considerably depends from significance value, volume of bigger sample and it mean quadratic deviation. For example (table 2), for increasing sample with bigger mean quadratic deviation interval of conformity (15), (16) becomes wider.

Table 2

Values [6] of average specific pressure at the cages calculated by different models

Number of cage	Model of Целикова	Model of Полухина	Model of Гелеи	Model of Экелунда	ξmax	ξmin
5	20.5	21	12.5	11.5	27.96430	16.03570
6	32	26	24	19	33.73428	19.36572
7	42	34	28	26	45.75977	26.24023
8	42.5	39.5	32.5	31	51.52974	29.57026
9	56.5	43.5	37	40	52.80085	30.29915
10	39	38	39	34	54.65750	31.34250

Conclusions. For estimation of adequacy may be used criteria Wilcoxon or special criteria for cases when it is few information about model and original. Both criteria have equal effectiveness in similar cases and allows to allocate area of modeling.

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