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ASSESSMENT OF DISTRIBUTED DELAY TIME OF INPUT SIGNAL BY STATIONARY OBJECTS

Annotation. *The article proposes a new method for estimating the distributed delay time of a signal by a stationary linear object whose transfer function is normalized and does not contain zeros. It is shown that the required time is determined by the coordinate of the intersection graph of forced component of the reaction of the object on a linearly increasing input signal from the x-axis. Analytical studies are confirmed by the results of mathematical modeling of the work of the proposed algorithm on a computer.*

Key words: *density function of random values, time constant, distributed time of signal delay by the object.*

Articulation of issue

Every dynamic object is inertial in its substance. This feature belongs to it because any transfer of energetic or material resources can`t immediately be carried out. Time delay of signal transfer from input to output of the object may be transportation (pure) or distributed (capacitive) (in the econometrics it is known as «distributed lag») [1-4].

If transport delay of signal for time τ in continuous object is described by unit with transfer function $\exp(-s\tau)$, where s - Laplace operator, distributed delay is a result of the influence on input signal of transfer function without pure delay element.

Review of recent investigations and publications

The following algorithm of time assessment of distributed delay is given in the study [3]:

a) transfer function $W(s)=\int_0^\infty \exp(-st)k(t)dt$ by representationof $\exp(-s\tau)$ in the form of range $1-s+s^2/2!-s^3/3!+\ldots$ is written down as follows

$$W(s)=H_0-H_1s+H_2s^2/2!-H_3s^3/3!+\ldots$$

Where $H_i=\int_0^\infty t^i k(t)dt$ - moment i – in the order of impulsive transition function $k(t)$;

b) time of distributed delay is determined with using a formula

$$T = H_1 / H_0 . \quad (1)$$

The last expression shows that it is necessary to find an impulsive transition function and calculate its momentsfor determine of distributed time of signal delay. It is a very complex procedure.

Therefore an actual task is finding of simple and descriptive algorithm of temporal value assessment of distributed delay, which linear continuous object makes.

Objective of the study

The objective of the study is a development and analysis of time assessment signal delay algorithm in the continuous object without using of impulsive transition function and its moments.

Presentation of basic material of the research

Transfer function written above $W(s)$ can be presented in the form:

$$W(s) = (W(s) / W(0)) \cdot W(0) . \quad (2)$$

In the expression (2) a multiplier $W(0) = H_0$ is an amplification coefficient. It doesn`t have inertial properties and doesn`t delay a signal, which passes through it.

According to normalized transfer function $W(s) / W(0)$, it has an amplification coefficient, equal to one. In other words

$$\int_{-\infty}^{+\infty} k(t)dt = 1 . \quad (3)$$

If $k(t)$ corresponds also to condition

$$k(t) \geq 0 , \quad (4)$$

it can be considered as probability density function of distributed delay time T. Using impulsive transition function with such properties, we can calculate distributed delay time

$$T = \int_{-\infty}^{+\infty} tk(t)dt . \quad (5)$$

If transfer function of aperiodic unit takes the form

$$W(s) = \frac{1}{Ts + 1} ,$$

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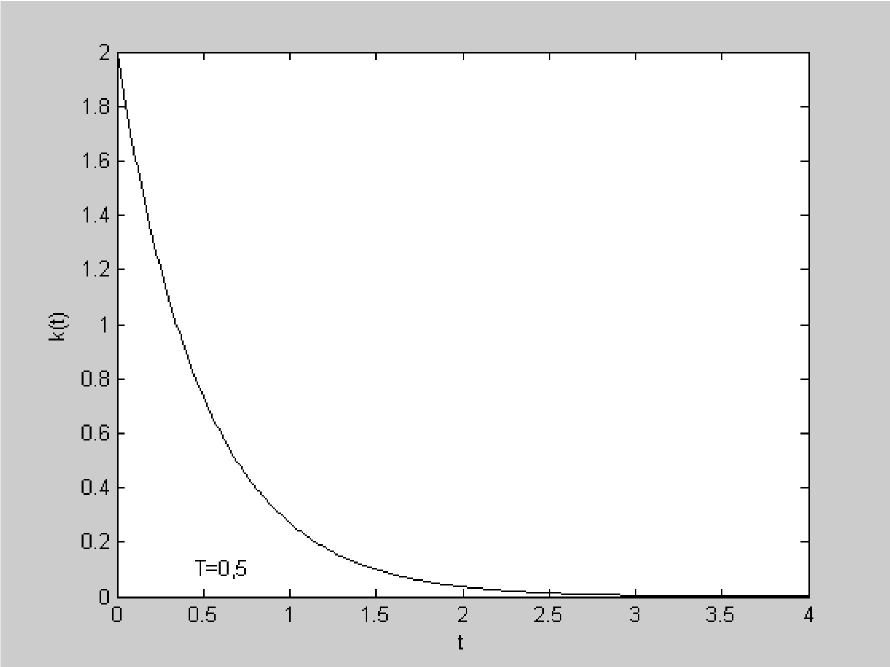
its impulsive transition function is described by the expression

$$k(t) = \begin{cases} 0 & npu\ t < 0 \\ \dfrac{1}{T} e^{-\frac{t}{T}} & npu\ t \geq 0 \end{cases}.$$

Due to the fact, that $k(t)$ meet conditions (3) and (4), in this case the distributed delay time T coincides with the constant time of the aperiodic object. (Pic. 1).



Pic. 1. Impulsive transition function of aperiodic unit under *T=0,5*

Now send a signal to the input of this aperiodic unit $u(t) = t$ and \we`ll find its reaction force through Duhamel integral (Pic. 2):

$$y(t) = \int_0^t u(\vartheta)k(t-\vartheta)d\vartheta = t - T + Te^{-\frac{t}{T}}. \qquad (6)$$

During analyzing the expression (6) of output signal $y(t)$ we can see, that its forced component $t - T$ falls behind time from input signal $u(t)$ also on T value.

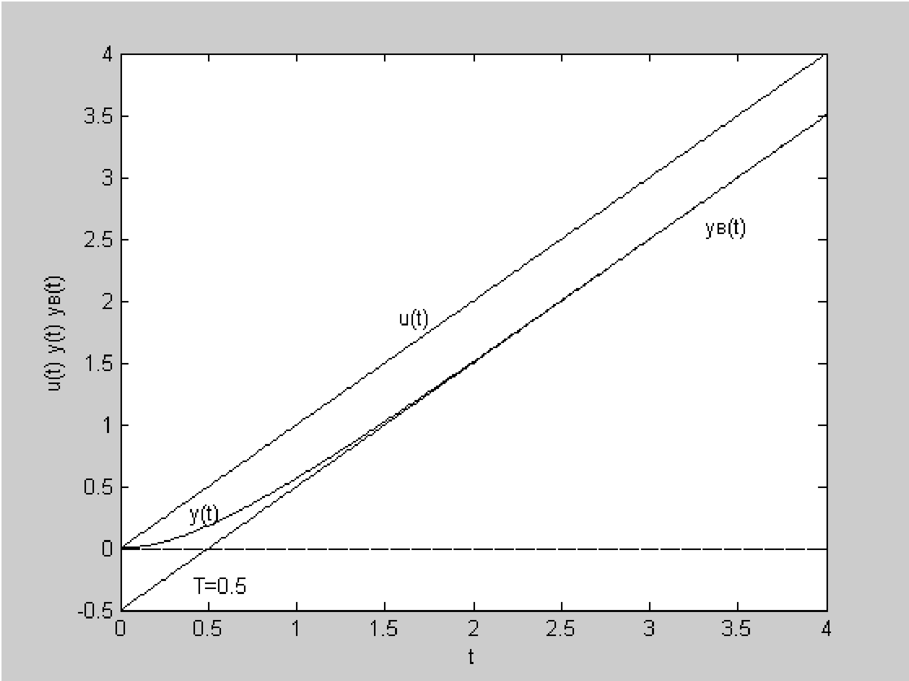
In case, when the object has a transfer function

$$W(s) = \frac{1}{T_1 T_2 s^2 + (T_1 + T_2)s + 1},$$

its impulsive transition function

$$k(t) = \frac{1}{T_1 - T_2}(e^{-\frac{t}{T_1}} - e^{-\frac{t}{T_2}})$$

meets conditions (3) and (4).



Pic. 2. Aperiodic unit reaction under *T=0,5* on the linearly increasing input signal

While sending a linearly increasing signal to the input of this object $u(t) = t$ an output signal will be the following

$$y(t) = t - (T_1 + T_2) + (T_1^2 e^{-\frac{t}{T_1}} - T_2^2 e^{-\frac{t}{T_2}})\frac{1}{T_1 - T_2}.$$

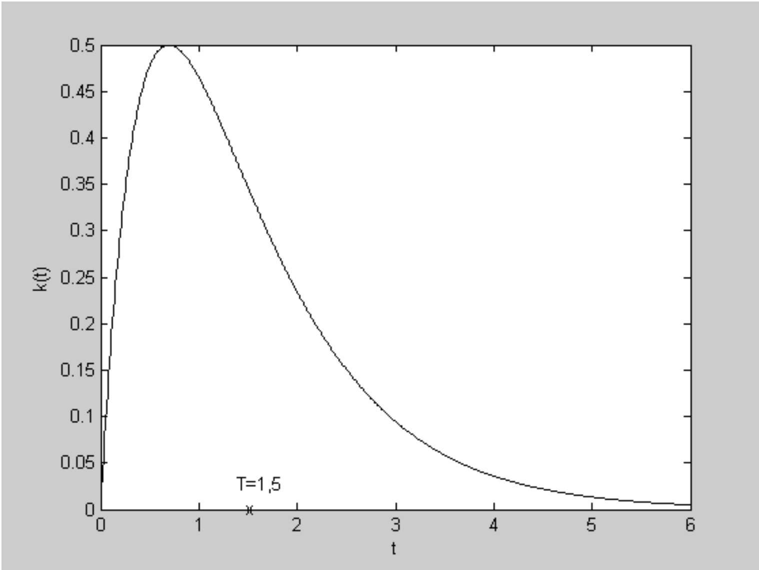
Its forced component $t - (T_1 + T_2)$ is behind from the input signal $u(t) = t$ on value $T_1 + T_2$, which corresponds with value T for $k(t)$ (Pic. 3 and 4):

Using mathematical induction method we can prove, that aperiodic units n with transfer functions, turning on consequently

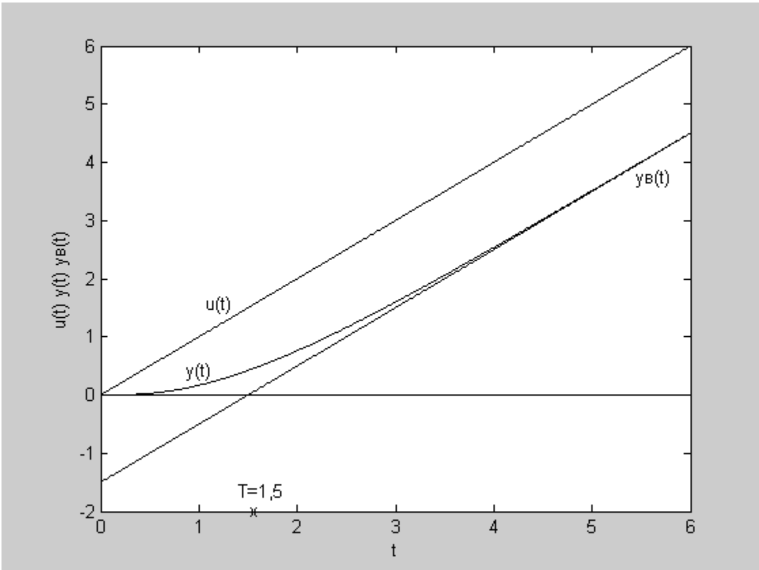
$$W_i(s) = \frac{1}{T_i s + 1}$$

$(i = \overline{1, n})$, realize a distributed delay, which value is

$$T = T_1 + T_2 + \ldots + T_n.$$



Pic. 3. Impulsive transition function of two connected in sequence aperiodic units



Pic. 4. Reaction of two connected in sequence aperiodic units on the linearly increasing input signal

Now we consider a distributed delay, which is made by oscillatory link with transfer function (a condition is not meet here (4))

$$W(s) = \frac{a^2 + b^2}{s^2 + 2as + (a^2 + b^2)}. \qquad (7)$$

The last we can write in such matter:

$$W(s) = \frac{1}{(\frac{1}{a+jb}s+1)(\frac{1}{a-jb}s+1)}.$$

Taking into attention complex values

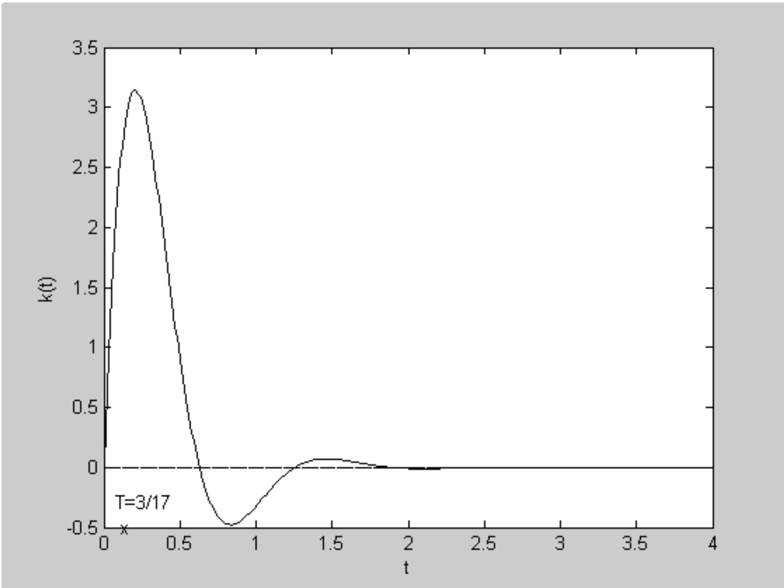
$$\frac{1}{a+jb} \text{ and } \frac{1}{a-jb}.$$

as «time constants», we`ll find their amount.

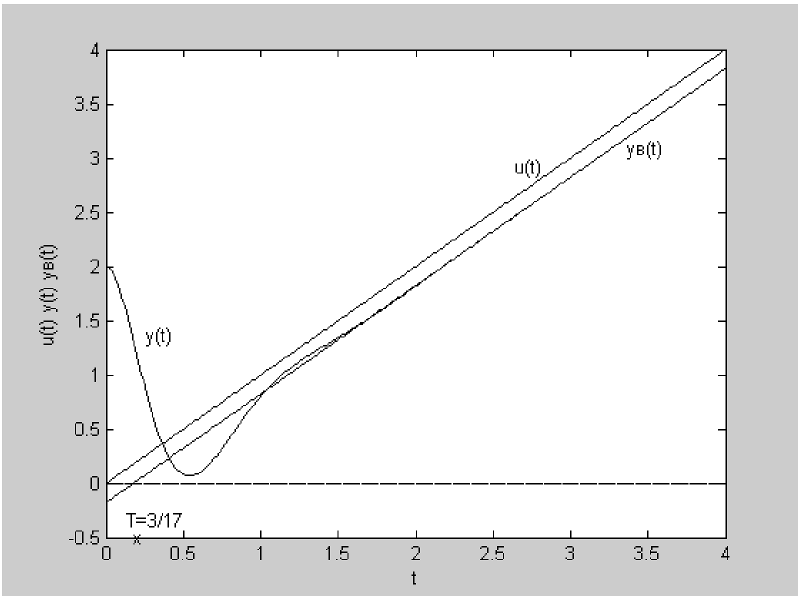
It will be equal to value

$$\frac{2a}{a^2+b^2}.$$

It will be just the time of distributed delay, which is made by oscillatory link for linearly increasing input signal.



Pic. 5. Impulsive transitiona function of the oscillatory link



Pic. 6. Reaction of the oscillatory link on the linearly increasing input signal

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On pic. 5 and 6 it is shown results of finding a input signal distributed delay value by oscillatory link with parameters $a = 3$, $b = 5$ through first (H1) and zero (H0) moments and using the input signal $u(t) = t$.

Conclusions

When performing this study the effective methodology of determination of the distributed time of signal delay has been developed by continuous stationary object with transfer function without zeros.

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