UDK 530.1

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LARGE CITIES ROAD TRAFFIC SELF-SIMILARITY RESEARCH

Annotation. The problem of estimating and analyzing the density of road traffic on the signs of self-similarity (fractality) is considered on the example of several regions of London. Experimental data includes quantitative data of road traffic for the period of 10-15 years of occupancy of different cities' areas and optimized binary image representations of geographical plans of city traffic development. Quantitative traffic data was analyzed for heterogeneity. The images of the corresponding areas are analyzed using Box Count and LCFD methods.

Keywords: fractal, road traffic, self-similarity.

Problem definition

The task of finding the best ways in transport networks is always relevant and is applicable in many areas of human activity. The vast majority of search algorithms do not take into account the fractal nature of the investigated objects.

Analysis of recent research and publications

Many studies devoted the question of analysis of fractal images. The distribution of fractal dimensions is possible to display on the graph by using the LCFD method [3]. Originally, this methodology was developed to automatically detect and objectively characterize the local anomalies of the retina vessels. Cities' road plans have also road veins branching so this method is quite applicable.

Purpose

Analyze quantitative experimental data and geographic images of the analyzed sites for fractal signs.

Main part

The density of the transport flow *k* defined as the number of vehicles per unit length of the roadway. In traffic flow, the critical density k_c and the jam density k_j are the two most important densities are. The maximum density achievable under free flow is k_c, and k_j - the maximum density achieved during overload. The inverse density value is spacing s, which is the distance between the two vehicles' centers.

$$k = \frac{1}{s}. \quad (1)$$

The density k within a length of roadway L at a given time t_1 is equal to the inverse of the average spacing of the n vehicles, where tt is the total time of travel in A area.

$$k(L, t_1) = \frac{n}{L} = \frac{n\partial t}{L\partial t} = \frac{tt}{|A|}. \quad (2)$$

Traffic flow ρ - the number of vehicles passing through the reference point per unit of time, flow speed v. The inverse of flow is headway h, which is the time that passes between the i-th car passing through a reference point in space and the $i+1$ vehicle. In congestion, h remains constant. As a traffic jam occurs, h approaches infinity.

$$\rho = kv = \frac{1}{h}. \quad (3)$$

The flow ρ passing through a fixed point x_1 over an interval T is equal to the inverse of the average headway of the m vehicles.

$$\rho(T, x_1) = \frac{m}{T} = \frac{1}{\bar{h}(x_1)}. \quad (4)$$

In the time-space diagram, the flow can be estimated in the area B, where td is the total distance traveled in area B.

$$\rho(B) = \frac{m}{T} = \frac{m\partial x}{T\partial x} = \frac{td}{|B|}. \quad (5)$$

The transport flow can be considered as a flow of one-dimensional compressive fluid assuming that the flow is maintained and there is an interdependence between the velocity and density of the traffic flow [4]. There are several main macroscopic models of traffic flows [5], described below.

Model of Tanaka (6), where $\partial(v)$ - medium (safe) distance between vehicles, L - average length of the vehicle, c_1 - time that characterizes the driver's reaction, c_2 - coefficient of proportionality of the brake path ($c_1 = 0.504$, $c_2 = 0.0285$ under normal conditions):

$$\rho(v) = \frac{1}{\partial(v)}, \partial(v) = L + c_1 v + c_2 v^2. \quad (6)$$

Greenshield's Model (7), where $\rho_{\max}$ - maximum flow density (in the absence of movement), $v_{\max}$ - maximum speed of the vehicle at an empty road:

$$\rho = \rho_{\max}(1 - \frac{v}{v_{\max}}). \quad (7)$$

Greenberg's Logarithmic Model (8), where c equals to a constant for speed-density relation in traffic flow and k_j equals to the jam density.

$$\rho = c \ln \frac{k_j}{k}. \tag{8}$$

The problem of estimating and analyzing the density of road traffic on an example of London with the help of experimental data – binary images of cities' districts and quantitative traffic data is considered. UK Department for Transport has its own road traffic monitoring system [1]. It allows to research quite important indicators for the period of 10-15 years, such as: annual average daily flow AADF (annual average daily flow) - average number of vehicles per the whole year passing through the point of the road network every day and the volume of transport (the distance passing through the vehicles of the given types on the measured segment of the road).

To analyze the distribution of traffic indicators, neighboring 5 areas of London (fig.1) were selected for which the most data are available. These are Hammersmith & Fulham, Kensington & Chelsea, City of Westminster – placed by top side of the river, Wandsworth and Lambeth – placed by the bottom one.

Figure 1 - Analyzed areas and an example of an optimized binary image of a city for fractal analysis

The study analyzed the main roads, which include highways and roads of the "A" type. These roads usually have a high level of traffic flow and are often the main arteries of the main directions.

If you place the average daily traffic of the AADF area on the histogram (fig. 2, left), it is possible to see that the graph has a stable distribution. Placing the

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value of the histogram in the opposite direction, in general, we would get an exponential curve (fig. 2, right side). The graph presented in fig. 2 has a "heavy-weight tail", which is a sign of the self-similarity (fractality) of experimental data. If we approximate the obtained values, we can obtain an analytic expression that characterizes the distribution of signs of self-similarity.

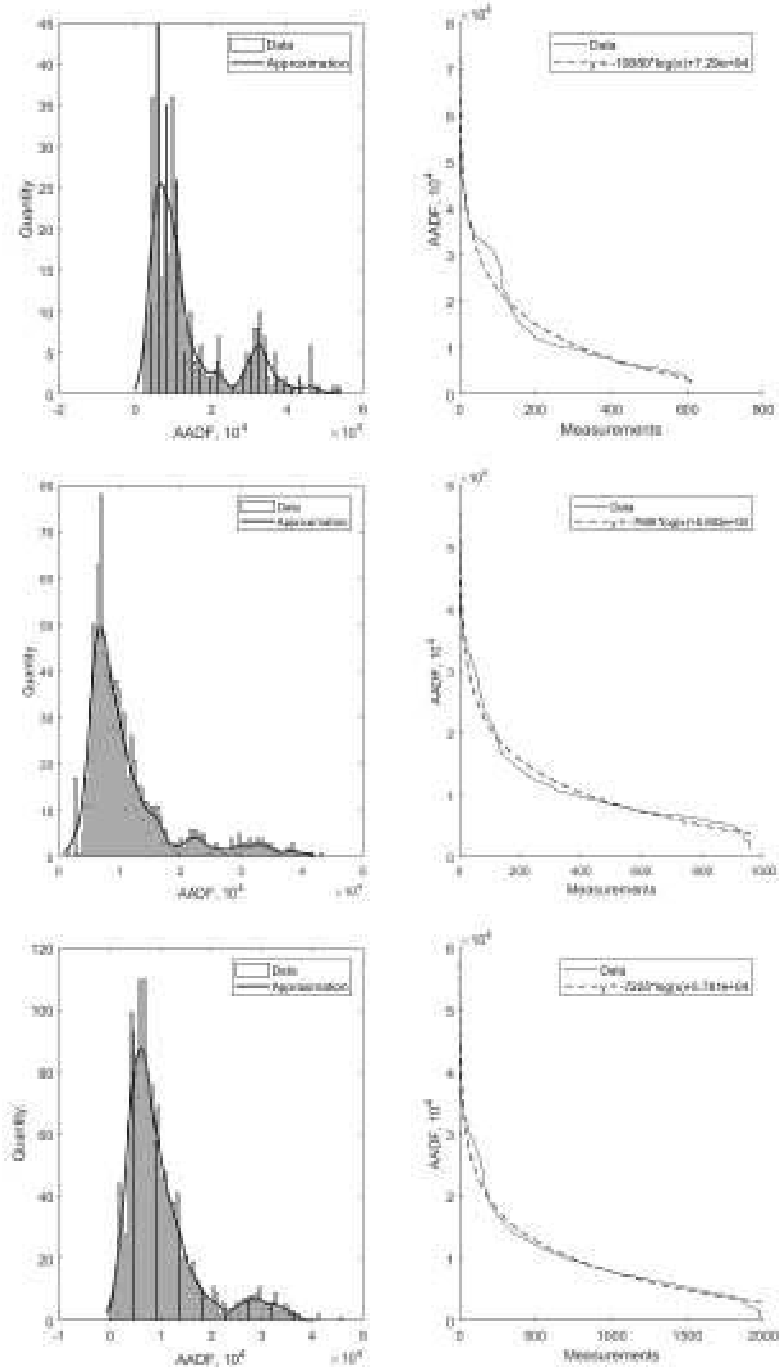

Figure 2 - Distribution of indicators of average daily traffic in areas of London (left side - AADF histogram with approximation; right - reversed by the size of the AADF chart). Areas: Hammersmith & Fulham, Kensington & Chelsea, Westminster

The greatest correspondence of AADF is observed with an approximation by the logarithmic equation of the form:

$$\widetilde{\rho}(x) = c\ln(x) + b,\ x \in |\mathrm{B}|. \quad (9)$$

The Greenberg's logarithmic model (8) is a well-established expression for macroscopic traffic, the result of observing data sets of density values for tunnels, and equation (9) fits to observed model. This model has gained very good popularity because this model can be derived analytically. However, main drawbacks of this model is that as density tends to zero, speed tends to infinity. This shows the inability of the model to predict the speeds at lower densities.

As shown in fig. 2, the heavy distribution tails of the AADF preferably have a similar pattern (a peak and a quite long tail on the right), because the analyzed areas are geographically close to each other. The stable heterogeneity of distributions characterizes the fractal nature of the object. Due to the analysis of these heterogeneous areas of the city it is possible to offer recommendations on the regulation and optimization of traffic in these areas. It is important to remember that the heterogeneity of distributions is dictated by the relief nature, and therefore it is impossible to completely eliminate the heterogeneity.

With the experimental quantitive traffic data, it is interesting to compare them with the analysis and evaluation with the fractality degree of the corresponding areas' images. For easier image analysis, binary images were used. Using the capabilities of Google Maps, you can get plans for selected areas of London in black and white and then convert it to binary (fig. 1).

Table 1

Fractal dimension D of districts of the city

Area name	D
Hammersmith & Fulham	1,7633
Kensington & Chelsea	1,8009
Westminster	1,8338
Wandsworth	1,7187
Lambeth	1,7247

The study of images was performed using the Fraclac software [2] using the box count method (a method for analyzing complex models by partitioning an images into smaller and smaller partials, and analyzing bits at each smaller scale). The resulting fractal dimensions of the images D displayed in table 1. The obtained results have a quite stable range for areas at two sides of the river - [1.76; 1.83] and [1.71; 1.72] respectively. Therefore, we can assert that it is a stable sign of the fractality of the city plan.

The image of road plans was also analyzed using the method of locally connected fractal dimensions (D_{LC}, Local Connected Fractal Dimension [3]). It's a method intended for binary images analysis. Additionally it allows to get the distribution of local variations of the image and visualize them on the graph. The distribution of LCFD for first three districts is shown in fig.3.

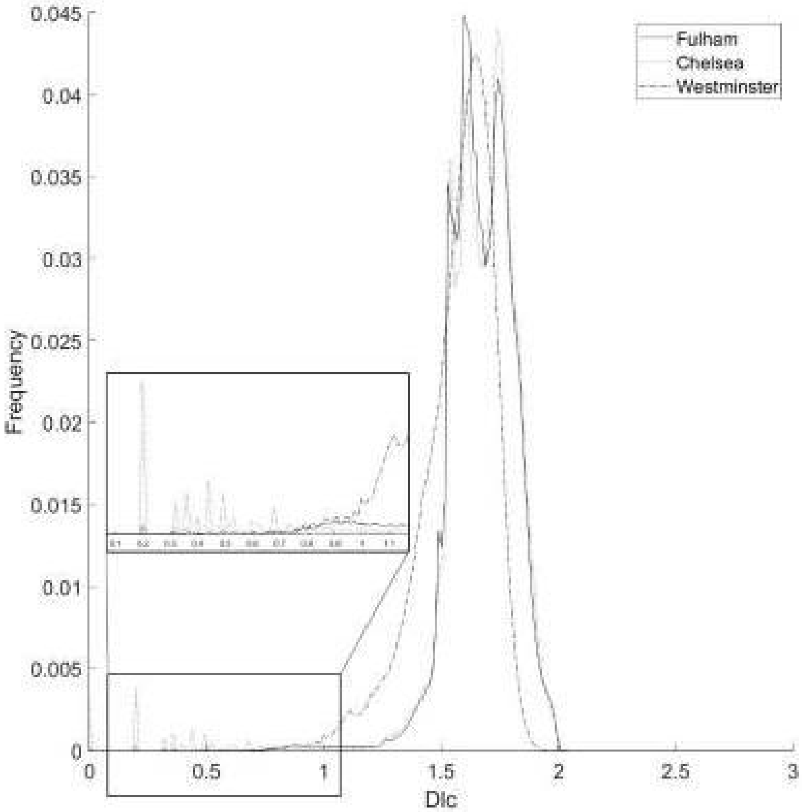

Figure 3 - Distribution of locally connected fractal dimensions of city plans

Figure 3 shows the distribution of LCFD for one group of districts. The distribution has a quite similar pattern and has a heavy tail to the left of the peak. All this is a sign of fractal characteristics of the researched object data.

Conclusions

Several models of traffic flow were observed. Available quantitative experimental data greatly corresponds to Greenberg's logarithmic model. The city plans' pictures and quantitative experimental data showed the presence of self-similarity. This characteristic can be used to find problem areas that require additional optimization. Effective road traffic management in these conditions should ensure that the transport network is loaded at the limit of its throughput and maintain continuous, even movement.

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