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A.I. Mikhalyov, A.A. Stenin, V.N. Ignatenko, M.A. Soldatova, A.S. Stenin

SYNTHESIS OF OPTIMAL CONSUMPTION FUELS ONE CLASS OF LINEAR NONSTATIONARY SYSTEMS (the method of predicted control)

Annotation. For the class of linear nonstationary systems under consideration, uniqueness is proved and the structure of controls optimal for fuel consumption is established. The problem of finding the optimal switching times is reduced to the solution of algebraic equations with an accuracy determined by the number of terms in the Walsh series expansion. As a result of the application of this approach, a program strategy of the optimal control in the sense of fuel consumption of a linear non-stationary second-order system with a constant and monotonous parameters was synthesized on the base of method predicted control.

Keywords: linear non-stationary systems, fuel consumption, optimal control structure, Walsh functions, method of predicted control, optimal program control algorithm.

Introduction

When solving the problem of synthesis of an optimal fuel control in the class of stationary systems, whose dimension n is not higher than the third order, an approach based on the combination of the maximum principle of L.S. Pontryagin and the phase space method [1-3]. In connection with the fact that optimal control in such systems is a relay type, the solution of the problem of synthesis is reduced to the construction in the phase space of the investigated system of lines ($n = 2$) or switching hypersurfaces ($n > 2$). That divide the phase space into regions, formed by the optimal trajectories of the motion of the representative point of the system at the corresponding values of the optimal control.

However, the use of a similar approach in solving the problem of synthesis of fuel-efficient controls in linear nonstationary systems in connection with the non-stationary nature of their parameters leads to a system of transcendental equations that, as a rule, do not have analytical solutions. Below we propose a procedure for synthesizing software optimal for fuel consumption controls for one class of linear non-stationary systems using the mathematical apparatus of Walsh functions [4].

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Formulation of the problem

Let the dynamics of a linear non-stationary system is described by an equation of the form

$$\dot{\bar{x}}(t) = A(t)\bar{x}(t) + B(t)\bar{u}(t), t \in [t_0, T_f], \bar{x}(t_0) = \bar{x}^{(0)}, \quad (1)$$

where $A(t) = \{a_{ij}(t)\}$, $B(t) = \{b_{ik}(t)\}$ - matrices of dimension $n \times n$ and $n \times m$ respectively, the elements of which are sign-constant

$$sign[a_{ij}(t)] = const, sign[b_{ik}(t)] = const, \quad (2)$$

monotone

$$sign[da_{ij}(t)/dt] = const, sign[db_{ik}(t)/dt] = const \quad (3)$$

functions, which have continuous first derivatives and bounded domains of definition on the control's interval.

We will also assume that the considered system (1) has a structure representing a consecutive connection of nonstationary links of the first order

$$A(t) = \begin{bmatrix} a_{11}(t) & a_{12}(t) & 0 & \cdots & \cdots & \cdots & \cdots & 0 \\ 0 & a_{22}(t) & a_{23}(t) & 0 & \cdots & \cdots & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & \cdots & \cdots & a_{ii}(t) & a_{i,i+1}(t) & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{nn}(t) \end{bmatrix}, B(t) = \begin{bmatrix} 0 \\ \vdots \\ b_n(t) \end{bmatrix} \quad (4)$$

The functional characterizing the fuel consumption has the form

$$\mathrm{I} = \int_{t_0}^{T_f} | u(t) | dt, \quad (5)$$

where T_f - fixed time, $T_f - t_0 \ge T^{\min}$, $T^{\min}$ - he minimum time required for the transition of the corresponding system from the initial state $\bar{x}(t_0)$ to the final one $\bar{x}(T_f)$. Control's action is limited

$$| u(t) | \le 1 \quad (6)$$

It is required to define a control's law $u^*(t)$ that satisfies the constraint (6) and takes the system (1) from the given initial state $\bar{x}(t_0)$ to the origin at fixed time interval and minimizes the functional (5).

Uniqueness and structure of optimal control

For a linear non-stationary system (1) for fixed $\bar{x}(t_0)$, t_0, T_f the value of the functional (5) is a function only $\bar{u}(t)$. We show that the functional (5) on a set U_t is a convex downward function, that is, the relation

$$I(\overline{u}^{(3)}) = I(\lambda \overline{u}^{(1)} + (1-\lambda)\overline{u}^{(2)}) \le \lambda I(\overline{u}^{(1)}) + (1-\lambda) I(\overline{u}^{(2)}) \quad (7)$$

In this case, taking into account the convexity U_t, we have

$$I(\overline{u}^{(3)}) = \int_{t_0}^{T_f} \sum_{k=1}^{m} \mu_k \, | u_k^{(3)}(t) | \, dt = \int_{t_0}^{T_f} \sum_{k=1}^{m} \mu_k \, | \lambda u_k^{(1)}(t) + (1-\lambda) u_k^{(2)}(t) | \, dt \le$$
$$\le \lambda \int_{t_0}^{T_f} \sum_{k=1}^{m} \mu_k \, | u_k^{(1)}(t) | \, dt + (1-\lambda) \int_{t_0}^{T_f} \sum_{k=1}^{m} \mu_k \, | u_k^{(2)}(t) | \, dt = \lambda I(\overline{u}^{(1)}) + (1-\lambda) I(\overline{u}^{(2)}) \quad (8)$$

Thus, the convexity properties of the reachability set and the quality functional (5) allows us to conclude that the control problem posed is convex. It's known, that the solution of a convex problem exists. This in turn means that there is a control satisfying the constraint $\overline{u}(t) \in U_t, t \in [t_0, T_f]$ under the condition $T_f - t_0 \ge T^{\min}$. Control's law translates the linear nonstationary system (1) from an arbitrary initial state to the origin, and minimizing the functional (5).

Taking into account that in [5] the uniqueness of optimal and extremal controls for nondegenerate problems is proved, this allows us to conclude that the relay control law, defined by expression

$$u^*(t) = -dez\{b_n(t) p_n^*(t)\} \quad (9)$$

absolutely minimizes the Hamiltonian.

For normal problems of optimal control in the sense of fuel consumption of the controls of a linear nonstationary system (1) of the above structure and parameters, the maximum number of control switching does not exceed 2n-1, where n is the size The state space of the system under investigation [5]. In this case, the most general sequence of values of the optimal control can be written in accordance with [6] as follows:

$$\begin{aligned} &0 \ldots u_0 \ldots 0 \ldots -u_0 \ldots 0, \ u_0 \quad (n - \text{четное}), \\ &0 \ldots -u_0 \ldots 0 \ldots u_0 \ldots 0, \ -u_0 \quad (n - \text{нечетное}), \end{aligned} \quad (10)$$

where $u_0 \pm 1$. Optimal can also be short sequences, occurring in general sequences of the form (10).

The solution of the problem

For clarity, we will seek the solution of the problem posed bang-bang control (9) for the system (1) for n = 2. In addition, it should be noted that the dynamics of most real control objects can be approximated of the dynamics of second-order systems.

Let the dynamics of the nonstationary object in the interval $[t_0, T_f]$ be described by differential equations of the form

$$\dot{x}_1(t) = a_{11}(t)x_1(t) + a_{12}(t)x_2(t),$$
$$\dot{x}_2(t) = b_2(t)u(t) \tag{11}$$

respectively, with initial and boundary conditions

$$\bar{x}^T(t_0) = \{x_1^{\ 0},\ x_2^{\ 0}\}, \bar{x}(T_f) = \{0,0\} \tag{12}$$

According to [6], in this case the most complete optimal sequence of controls has the form

$$u_0, 0, -u_0, \tag{13}$$

where $u_0 = \pm 1$, and, therefore, in the system (11) it is necessary to determine the switching times t_1 and t_2.

Suppose that we have found the approximation of the coefficients of equations (11) in the form of series in terms of the Walsh function system

$$a_{11}(t) \approx \sum_{r=0}^{R-1} a_r^{(11)}\varphi_r(t) = \bar{a}^{(11)T}\bar{\varphi}_R(t),$$
$$a_{12}(t) \approx \sum_{r=0}^{R-1} a_r^{(12)}\varphi_r(t) = \bar{a}^{(12)T}\bar{\varphi}_R(t), \tag{14}$$
$$b_2(t) \approx \sum_{r=0}^{R-1} b_r\varphi_r(t) = \bar{b}^T\bar{\varphi}_R(t),$$

where $\bar{\varphi}_R^T(t) = \{\varphi_0(t), ..., \varphi_r(t), ..., \varphi_{R-1}(t)\}$; R-dimensional vector of Walsh functions defined on the interval $[t_0, T_f]$; R is the number of terms in the Walsh series expansion; $\bar{a}^{(11)T} = \{a_0^{\ 11}, ..., a_r^{\ 11}, ..., a_{R-1}^{\ 11}\}, \bar{a}^{(12)T} = \{a_0^{\ 12}, ..., a_r^{\ 12}, ..., a_{R-1}^{\ 12}\}, \bar{b}^T = \{b_0, ..., b_r, ..., b_{R-1}\}$ -R-dimensional vectors of constant coefficients of the R-dimensional vector of Walsh functions for functions $a_{11}(t), a_{12}(t), b_2(t)$, respectively.

We note that if the coefficients of equation (11) are known functions, then the constant coefficients of the expansions (14) are determined by the formula

$$a_j = \int_0^1 f(x)\varphi_j(x)dx \, (j = 0,1,...) \tag{15}$$

taking into account the time scale $x = \frac{t - t_0}{T_f - t_0}, x \in [0,1]$. If the mathematical description of the object is known up to parameters, then the corresponding approximations can be obtained using the parametric identification method for linear dynamical systems proposed by the authors in [7].

We integrate the system (11) from the given initial state $\bar{x}(t_0)$ to the zero final state $\bar{x}(T_f) = \bar{0}$. Moreover, on each of the intervals of constancy of control (13), the initial conditions of the subsequent interval are determined from the condition of conjugation of the trajectory of motion. When integrating, we use the operation matrix of integration $P^{1}_{(RxR)}$ [8], which, taking into account the control interval under consideration, can be defined in the form $P^{'}_{(R\times R)} = (T_f - t_0)P_{(R\times R)}$.

If consider the interval $[t_0, t_1], u^* = u_0$ and from (11) taking account of (14) and P1(RxR) for $x_2(t)$ we get

$$x_2(t) = x_2^0 + u_0 \overline{\beta}^T \overline{\varphi}_R(t) \quad (16)$$

where $\overline{\beta}^T = \overline{b}^T P^{'}_{(R\times R)}$.

Taking into account (14) and (16) we have

$$x_1(t) = \{\int_{t_0}^{t} (\overline{a}^{(12)T} \overline{\varphi}_R(t))(x_2^0 + u_0 \overline{\beta}^T \overline{\varphi}_R(t)) \times$$
$$\times \exp(-\int_{t_0}^{t} \overline{a}^{(11)T} \overline{\varphi}_R(t)dt)dt + C_1\} \exp(\int_{t_0}^{t} \overline{a}^{(11)T} \overline{\varphi}(t)dt) \quad (17)$$

We introduce the notation

$$\overline{\alpha}^{(11)T} = \overline{a}^{(11)T} P^{'}_{(R\times R)},$$
$$\overline{d}^{(1)T} \overline{\varphi}_R(t) = x_2^0 \overline{a}^{(12)T} \overline{\varphi}_R(t) + u_0 \overline{f}^T \overline{\varphi}_R(t), \quad (18)$$

where $\overline{f}$ - R-dimensional vector of constant coefficients, each element of which, by virtue of the multiplicative property of a system of Walsh functions, is defined as

$$f_r = \sum_{r_1=0}^{R-1} a_{r_1}^{(12)} \beta_{r_1 \oplus r} \qquad (r = \overline{0, R-1}) \quad (19)$$

Taking into account the introduced notation (18) and the operational matrix of integration, we write (17) in the form

$$x_1(t) = \{\int_{t_0}^{t} \overline{d}^{(1)T} \overline{\varphi}_R(t) \exp(-\overline{\alpha}^{(11)T} \overline{\varphi}_R(t))dt + C_1\} \exp(\overline{\alpha}^{(11)T} \overline{\varphi}_R(t)) \quad (20)$$

As

$$\overline{d}^{(12)T} \overline{\varphi}_R(t) = \exp(-\overline{\alpha}^{(11)T} \overline{\varphi}_R(t)),$$
$$\overline{d}^{(3)T} \overline{\varphi}_R(t) = \exp(\overline{\alpha}^{(11)T} \overline{\varphi}_R(t)),$$

then the integrand in (20) can be written as

$$\overline{d}^{(4)T} \overline{\varphi}_R(t) = \overline{d}^{(1)T} \overline{\varphi}_R(t) \overline{d}^{(2)T} \overline{\varphi}_R(t),$$

where $\overline{d}^{(4)}$ - R-dimensional vector of constant coefficients, defined similarly to expression (19).

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Finally, we obtain expression (20) in the form

$$x_1(t) = \overline{d}^{(6)T}\overline{\varphi}_R(t) + C_1\overline{d}^{(3)T}\overline{\varphi}_R(t) \tag{21}$$

where $\overline{d}^{(6)}$ - R-dimensional vector of constant coefficients, defined as

$$d_r^{(6)} = \sum_{r_1}^{R-1} d_{r_1}^{(5)} d_{r_1 \oplus r}^{(3)} \quad (r = \overline{0, R-1});$$

$\overline{d}^{(5)}$ - R-dimensional vector of constant coefficients, defined as

$$\overline{d}^{(5)T} = \overline{d}^{(4)T} P'_{(R \times R)}$$

Taking into account the initial conditions (12), from (21) we define the constant integration

$$C_1 = (x_1^0 - \overline{d}^{(6)T}\overline{\varphi}_R(t_0)) / \overline{d}^{(3)T}\overline{\varphi}_R(t).$$

Similarly, we integrate (11) in the interval $[t_1, t_2]$ when $u^* \equiv 0$ and find

$$x_2(t) = x_2^0 + u_0\overline{\beta}^T\overline{\varphi}_R(t_1) = H,$$

$$x_1(t) = \{\int_{t_1}^{t} (\overline{a}^{(12)}\overline{\varphi}_R(t))(x_2^0 + u_0\overline{\beta}^T\overline{\varphi}_R(t_1)) \times$$

$$\times \exp(-\int_{t_1}^{t} \overline{a}^{(11)T}\overline{\varphi}_R(t)dt)dt + C_2\} \exp(\int_{t_1}^{t} \overline{a}^{(11)T}\overline{\varphi}_R(t)dt) \tag{22}$$

Arguing as in the previous case, we write, taking into account (22), $x_1(t)$ in form

$$x_1(t) = H\overline{d}^{(9)T}\overline{\varphi}_R(t) + C_2\overline{d}^{(3)T}\overline{\varphi}_R(t) \tag{23}$$

where $\overline{d}^{(9)}$ - R-dimensional vector defined as

$$d_r^{(9)} = \sum_{r_1=0}^{R-1} d_{r_1}^{(8)} d_{r_1 \oplus r}^{(3)} \quad (r = \overline{0, R-1}),$$

$\overline{d}^{(8)}$ - R-dimensional vector defined as

$$\overline{d}^{(8)T} = \overline{d}^{(7)T} P'_{(R \times R)},$$

$\overline{d}^{(7)}$ - R-dimensional vector defined as

$$d_r^{(7)} = \sum_{r_1}^{R-1} a_{r_1}^{(12)} d_{r_1 \oplus r}^{(2)} \quad (r = \overline{0, R-1}).$$

We define the integration constant in equation (23) with allowance for (21) as follows:

$$C_2 = (\overline{d}^{(6)Y}\overline{\varphi}_R(t_1) + C_1\overline{d}^{(3)T}\overline{\varphi}_R(t_1) - H\overline{d}^{(9)T}\overline{\varphi}_R(t_1)) / \overline{d}^{(3)T}\overline{\varphi}_R(t_1).$$

Integrating (11) on the last interval for, we obtain

$$x_2(t) = C_3 - u_0 \int_{t_2}^{t} \overline{b}^T \overline{\varphi}_R(t)dt = C_3 - u_0 \overline{\beta}^T \overline{\varphi}_R(t) \tag{24}$$

whence taking into account (22) has the form

$$C_3 = H + u_0 \overline{\beta}^T \overline{\varphi}_R(t_2)$$

For $x_1(t)$, taking into account (24), we obtain expression

$$x_1(t) = \{\int_{t_2}^{t} \overline{a}^{(12)T} \overline{\varphi}_R(t)(C_3 - u_0 \overline{\beta}^T \overline{\varphi}_R(t)) \exp(\overline{\alpha}^{(11)T} \overline{\varphi}_R(t))$$

$$+ C_4\} \exp(\overline{\alpha}^{(11)T} \overline{\varphi}_R(t)) = C_3 \overline{d}^{(9)T} \overline{\varphi}_R(t) - u_0 \overline{d}^{(12)T} \overline{\varphi}_R(t) + C_4 \overline{d}^{(3)T} \overline{\varphi}_R(t) \tag{25}$$

where $\overline{d}^{(12)}$ - R-dimensional vector of constant coefficients, defined as

$$\overline{d}_r^{(12)} = \sum_{r_1=0}^{R-1} d_{r_1}^{(11)} d_{r_1 \oplus r}^{(3)} \quad (r = \overline{0, R-1}),$$

$\overline{d}^{(11)}$ - R-dimensional vector of constant coefficients, defined as

$$\overline{d}^{(11)T} = \overline{d}^{(10)T} P'_{R \times R}$$

$\overline{d}^{(10)}$ - R-dimensional vector of constant coefficients, defined as

$$\overline{d}_r^{(10)} = \sum_{r_1=0}^{R-1} d_{r_1}^{(7)} \beta_{r_1} \quad (r = \overline{0, R-1}).$$

The integration constant from (25) with allowance for (23) is defined in the form

$$C_4 = (H\overline{d}^{(9)T} \overline{\varphi}_R(t_2) + C_2 \overline{d}^{(3)T} \overline{\varphi}_R(t_2) -$$

$$- C_3 \overline{d}^{(9)T} \overline{\varphi}_R(t_2) + u_0 \overline{d}^{(12)T} \overline{\varphi}(t_2)) / \overline{d}^{(3)T} \overline{\varphi}_R(t_2).$$

Since in the optimization problem under consideration the target region is the origin, then putting $x_2(T_f) = x_1(T_f) \leq \varepsilon$, where ε is the specified accuracy of reaching the final state, from (24) and (25) we obtain the following inequality for the determination of t_1 and t_2:

$$H_4 H_1 + H_2 + H_3 f(t_1, t_2) \leq \varepsilon, \tag{26}$$

where

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$$H_1 = \overline{d}^{(9)T} \overline{\varphi}_R(T_f);$$

$$H_2 = -u_0 \overline{d}^{(12)T} \overline{\varphi}_R(T_f);$$

$$H_3 = \overline{d}^{(3)T} \overline{\varphi}_R(T_f);$$

$$H_4 = u_0 \overline{\beta}^T \overline{\varphi}_R(T_f) = H + u_0 \overline{\beta}^T \overline{\varphi}_R(t_2) = C_3$$

are constant and computed in advance; $f(t_1, t_2)$ - a function of switching times t_1, t_2, equal to:

$$f(t_1, t_2) = [(H - H_4)\overline{d}^{(9)T} \overline{\varphi}_R(t_2) + u_0 \overline{d}^{(12)T} \overline{\varphi}_R(t_2) + H_5] / \overline{d}^{(3)T} \overline{\varphi}_R(t_2) +$$
$$+ [\overline{d}^{(6)T} \overline{\varphi}_R(t_1) - H\overline{d}^{(9)T} \overline{\varphi}_R(t_1)] / \overline{d}^{(3)T} \overline{\varphi}_R(t_1) \overline{d}^{(3)T} \overline{\varphi}_R(t_2) \quad (27)$$

Here $H_5 = (x_1^0 - \overline{d}^{(6)T} \overline{\varphi}_R(t_0)) / \overline{d}^{(3)T} \overline{\varphi}_R(t_0) = const$ and is also calculated in advance.

Optimal predicted control algorithm

Optimal switching times are determined of the following optimal predicted control algorithm:

Step 1. Depending on the initial state, the sign of u_0 is selected;

Step 2. If the starting point $\overline{x}(t_0)$ is in I or in III quadrant, the time of its transfer to the axis x_1 for the system (11) is determined by equation

$$\varepsilon = x_2(t_0) + u_0 \overline{\beta}^T \overline{\phi}_R(t') .$$

Step 3. By the number of functions in the Walsh system $R = 2^d, d$ - an integer $d = 1, 2, \dots$, the lower bound of the number of intervals of the partition of a given interval $[t_0, T_f] - l, l \ge R$ is determined, from which the duration of the partition step $\Delta t = (T_f - t_0) / l$ is found for the subsequent search of the required switching moments;

Step 4. Set the value; $t_1 = t' + k\Delta t, k = 1$.

Step 5. By checking the values t_2 of from $t_1 + \Delta t$ with discreteness Δt, inequality (26) is checked. If inequality (26) is satisfied at some point, then the desired values of t_1 and t_2 are found. Otherwise, the following value $k = k + 1, t_1 = t' + k\Delta t$ is selected, and the algorithm continues.

It should be noted that this algorithm takes into account the existence of non-optimal areas for which it will not have a solution, i.e. for any initial conditions it has a finite, computationally calculated procedure.

Conclusion

For ordinary optimization problems for linear non-stationary systems, the structure of which is a serial connection of typical dynamic systems of the second order with monotone and constant signs, the upper limit of the number of switching and their sequence is defined, which allows us to synthesize algorithms of optimal predicted control.

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