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MATHEMATIC MODEL OF AND METHOD FOR SOLVING THE DIRICHLET HEAT-EXCHANGE PROBLEM FOR ONE-SHEET ROTARY HYPERBOLOID

Abstract. It is the first generalized 3D mathematical model developed for calculating temperature fields in the thin-wall one-sheet rotary hyperboloid, which rotates with constant angular velocity around the axis OZ, ; the model was created with the help of known equations of generating lines in cylindrical coordinate system with taking into account finite velocity of heat conductivity and in the form of the Dirichlet boundary problem for the hyperbolic equation of heat conduction under condition that heat-conduction properties of the body were constant, and no internal sources of the heat were available. At initial moment of time, the body temperature was constant; values of temperature on outside surfaces of the body were known and presented continuous function of coordinate.

The hyperbolic heat-conductivity equation was derived from the generalized energy transfer equation for the moving element of continuous medium with taking into account finiteness of the heat conductivity velocity.

In order to solve the boundary problem, the desired temperature field was represented as a complex Fourier series. The obtained boundary problems for the Fourier coefficients were found with the help of Laplace integral transformations and the new integral transformation for two-dimensional finite space. Intrinsic values and intrinsic functions for the integral transformation kernel were found by method of finite element and the Galerkin methods. Besides, the domain was divided into simplex element.

As a result, the temperature field in the thin-wall one-sheet rotary hyperboloid was found in the form of convergent series in Fourier functions.

Keywords: boundary problem, curvilinear integral, relaxation time.

Introduction. The use of hyperbolic surfaces in the construction industry has become a great technical breakthrough. In [1], their most important properties are summarized. First, they are pleasing to the human eye. The lace-like skeleton of the tower on the basis of one-sheet rotary hyperboloid, unusual configuration of the shell having the shape of hyperbolic rotary paraboloid looks very nice and harmoniously fits into the surrounding landscape. Secondly, geometrical properties of these surfaces justify their structural qualities. The possibility of creating a skeleton by straight beams is the most important feature of structures on the basis of hyperbolic surfaces. Thirdly, almost all surfaces, which are formed as a result of hyperbolic surfaces intersecting with other surfaces, retain their properties: rigidity and strength. For this reason, it is possible to combine different types of surfaces in one structure.

Cooling towers of the similar shape made of metal or prefabricated structural concrete reinforced in the direction of meridians and parallels are widely used in nuclear power engineering, chemical and metallurgical industry. Cooling towers made of fiberglass plastic are used in sugar mills, factories for processing meat, fish, fruits and vegetables, milk plants, breweries and other enterprises. Fiberglass plastic demonstrates good resistance to high temperatures, ultraviolet irradiation and abrasion allowing using the cooling towers even in the hardest operating conditions.

Practice of designing structures based on the hyperbolic surfaces requires analysis of their thermal strength and thermal buckling

Overview of the latest researches and publications. Thus, it is stated in [2] that conditions for reliability of calculations by finite element method and finite difference method, which are used for calculating nonstationary non-axis-symmetrical temperature fields of the rotating cylinders, are described by the same characteristics and can be expressed in the following way:

$$1-\frac{\Delta F_0}{\Delta\varphi^2}\geq 0 \quad \text{and} \quad \frac{1}{\Delta\varphi}-\frac{Pd}{2}\geq 0 .$$

If $Pd=10^5$ and corresponds to the angular velocity $\omega=1{,}671\,сек^{-1}$ of rotation of metal cylinder with radius 100 mm, then changes of $\Delta\varphi$ and ΔF_o should comply with the following conditions:

$$\Delta\varphi\leq 2\cdot 10^{-5} \quad \text{and} \quad \Delta F_o\leq 2\cdot 10^{-10}.$$

In case of uniformly cooled cylinder when Bi = 5, time period needed for temperature to reach 90% of stationary state is equal to $Fo\approx 0.025$ [2]. It means that, within this period of time, at least $1.3\cdot 10^8$ operations should be fulfilled in order to reach the stationary temperature distribution.

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Moreover, it should be mentioned that it would be necessary to make $3.14 \cdot 10^5$ calculations within one cycle of computation as the inside state of the ring should be characterized by $3.14 \cdot 10^5$ points. It is obvious that this number of calculations needed for getting a numerical result is unrealistic.

Therefore, we will employ integral transformations for solving boundary problems, which occur during mathematic modeling of 3D non-stationary heat-exchange processes in the thin-wall one-sheet rotating hyperboloid.

Objective of the work. As review of scientific literature shows, heat exchange in the thin-walled one-sheet hyperboloids has not been studied fully yet. The purpose of the work was to construct a new generalized 3D mathematical model for calculating temperature fields in a thin-walled one-sheet hyperboloid of rotation, which rotates at a constant angular velocity, with the help of known equations of generating lines in a cylindrical coordinate system with taking into account final rate of heat conductivity and in the form of the Dirichlet boundary problem of mathematical physics for the hyperbolic equation of heat conduction, and to find solutions for the obtained boundary problem.

Presentation of the main research material. Let's consider calculation of temperature field in the thin-wall one-sheet rotary hyperboloid (fig. 1), which is restricted by two end faces (at z=-c and z=c). Equations for the generating lines in the cylindrical coordinate system $(\rho, \varphi, z)$for the inside and outside lateral surfaces are, correspondingly:

$$r = b\left(1 + \frac{z^2}{c^2}\right)^{\frac{1}{2}}, \quad r = b_1\left(1 + \frac{z^2}{c^2}\right)^{\frac{1}{2}}, \quad b_1 < b.$$

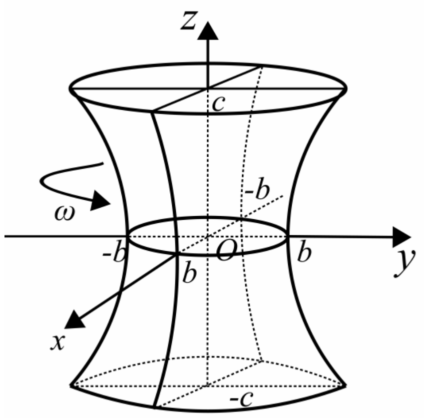

Figure 1 - The thin-wall one-sheet rotary hyperboloid with generating lines $r = \xi(z), r = \xi_1(z)$

The hyperboloid rotates with constant angular velocity ω around the axis OZ, and heat-conduction velocity is known. The heat-conduction properties of the body do not depend on temperature, and no internal sources of the heat are available. At initial moment of time, the body temperature is constant G_0, and values of temperature on outside and inside lateral surfaces of the body $V(\varphi, z)$ and $V_1(\varphi, z)$, correspondingly, are known. On the end faces, values of temperature $G_1(r,\varphi)$ and $G_2(r,\varphi)$ at z=-c and z=c, correspondingly, are known.

In the [2], a generalized heat-transfer equation is presented for the moving element of solid medium with taking into account finiteness of the heat-conduction velocity value. According to [2], a generalized equation for the energy balance of a solid body, which rotates with constant angular velocity ω around axis OZ, and whose heat-transfer properties do not depend on temperature, and no internal sources of the heat are available, can be written in the following way:

$$\gamma\, c_t \left\{ \frac{\partial \mathrm{T}}{\partial \mathrm{t}} + \omega \frac{\partial T}{\partial \varphi} + \tau_r \left[\frac{\partial^2 T}{\partial t^2} + \omega \frac{\partial^2 T}{\partial \varphi \partial t} \right] \right\} = \lambda \left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \varphi^2} + \frac{\partial^2 T}{\partial z^2} \right]. \quad (1)$$

where γ – is density of the medium; c_t is specific heat capacity; λ is heat conductivity coefficient; $T(r,\varphi,z,t)$ is temperature of the medium; t is time; τ_r is relaxation time.

Mathematically, the problem of defining cylinder temperature field consists of integration of differential equation of heat conduction (1) into

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domains $D=\{(r,\varphi,z,t)|r\in(\varsigma_1(z),\varsigma(z)),\varphi\in(0,2\pi),\mathrm{z}\in(0,h),\mathrm{t}\in(0,\infty)\}$, which, with taking into consideration the accepted assumptions, can be written as:

$$\frac{\partial\theta}{\partial t}+\omega\frac{\partial\theta}{\partial\varphi}+\tau_r\frac{\partial^2\theta}{\partial t^2}+\tau_r\omega\frac{\partial^2\theta}{\partial\varphi\partial t}=a\left[\frac{\partial^2\theta}{\partial r^2}+\frac{1}{r}\frac{\partial\theta}{\partial r}+\frac{1}{r^2}\frac{\partial^2\theta}{\partial\varphi^2}+\frac{\partial^2\theta}{\partial z^2}\right] \quad (2)$$

for initial conditions

$$\theta(r,\varphi,z,0)=0,\quad \frac{\partial\theta(r,\varphi,z,0)}{\partial t}=0 \quad (3)$$

and for boundary conditions

$$\theta(\varsigma_1(z),\varphi,z,t)=\Psi(\varphi,z),\qquad \theta(\varsigma(z),\varphi,z,t)=G(\varphi,z), \quad (4)$$

$$\theta(r,\varphi,0,t)=\Theta(r,\varphi),\quad \theta(r,\varphi,h,t)=\Lambda(r,\varphi), \quad (5)$$

where $\theta=\dfrac{T(\rho,\varphi,z,t)-G_0}{T_{\max}-G_0}$ –is relative temperature of the body; $a=\dfrac{\lambda}{c_t\,\gamma}$ - is heat conductivity coefficient; $T_{\max}=\max\limits_{r,\varphi,z}\{V(\varphi,z),\ V_1(\varphi,z),\ G_1(r,\varphi),\ G_2(r,\varphi)\}$;

$h=2c$;

$$\varsigma(z)=b\left(1+\frac{(\mathrm{z}+\mathrm{c})^2}{c^2}\right)^{\frac{1}{2}};\ \varsigma_1(z)=b_1\left(1+\frac{(\mathrm{z}+\mathrm{c})^2}{c^2}\right)^{\frac{1}{2}};$$

$$G(\varphi,z),\Theta(\rho,\varphi),\Lambda(\rho,\varphi)\in\mathrm{C}(0,2\pi).$$

In this case, solution of the boundary problem (2)-(5) $\theta(r,\varphi,z,t)$ is twice continuously differentiated by r and φ, z, once - by t in the domain D and continuous on the $\overline{D}$ [3], i.e. $\theta(r,\varphi,z,t)\in C^{2,1}(D)\cap C(\overline{D})$, and functions $G(\varphi,z),\Psi(\varphi,z),\Theta(r,\varphi),\ \Lambda(r,\varphi),\ \theta(r,\varphi,z,t)$ can be decomposed into the Fourier complex series [3]:

$$\left\{\begin{array}{c}\theta(r,\varphi,z,t)\\ G(\varphi,z)\\ \Psi(\varphi,z)\\ \Theta(r,\varphi)\\ \Lambda(r,\varphi)\end{array}\right\}=\sum_{n=-\infty}^{+\infty}\left\{\begin{array}{c}\theta_n(r,z,t)\\ G_n(z)\\ \Psi_n(z)\\ \Theta_n(r)\\ \Lambda_n(r)\end{array}\right\}\cdot\exp(in\varphi), \quad (6)$$

where

$$\begin{Bmatrix} \theta_n(r,z,t) \\ G_n(z) \\ \Psi_n(z) \\ \Theta_n(r) \\ \Lambda_n(r) \end{Bmatrix} = \frac{1}{2\pi}\int_0^{2\pi} \begin{Bmatrix} \theta(r,\varphi,z,t) \\ G(\varphi,z) \\ \Psi(\varphi,z) \\ \Theta(r,\varphi) \\ \Lambda(r,\varphi) \end{Bmatrix} \cdot \exp(-in\varphi)d\varphi;$$

$$\theta_n(r,z,t) = \theta_n^{(1)}(r,z,t) + i\theta_n^{(2)}(\rho,z,t);\ \ \mathrm{G}_n(z) = G_n^{(1)}(z) + iG_n^{(2)}(z);$$

$$\Psi_n(z) = \Psi_n^{(1)}(z) + i\Psi_n^{(2)}(z); \Theta_n(r) = \Theta_n^{(1)}(r) + i\Theta_n^{(2)}(r); \Lambda_n(r) = \Lambda_n^{(1)}(r) + i\Lambda_n^{(2)}(r).$$

In view of the fact that $\theta(r,\varphi,z,t)$ is a real-valued function, let's confine ourselves by considering only $\theta_n(r,z,t)$ for n=0,1,2,..., because $\theta_n(r,z,t)$ and $\theta_{-n}(r,z,t)$ $\theta_{-n}(r,z,t)$ are complexly conjugate [3].

By putting values of functions from (6) into (2)-(5) we can receive the following system of differential equations:

$$\frac{\partial\theta_n^{(i)}}{\partial t} + \vartheta_n^{(i)}\theta_n^{(m_i)} + \tau_r\frac{\partial^2\theta_n^{(i)}}{\partial t^2} + \tau_r\vartheta_n^{(i)}\frac{\partial\theta_n^{(m_i)}}{\partial t} = a\left[\frac{\partial^2\theta_n^{(i)}}{\partial r^2} + \frac{1}{r}\frac{\partial\theta_n^{(i)}}{\partial r} - \frac{n^2}{r^2}\theta_n^{(i)} + \frac{\partial^2\theta_n^{(i)}}{\partial z^2}\right] \tag{7}$$

for initial conditions

$$\theta_n^{(i)}(r,z,0) = 0, \qquad \frac{\partial\theta_n^{(i)}(r,z,0)}{\partial t} = 0 \tag{8}$$

and for boundary conditions

$$\theta_n^{(i)}(\varsigma_1(z),z,t) = \Psi_n^{(i)}(z) \qquad \theta_n^{(i)}(\varsigma(z),z,t) = G_n^{(i)}(z) \tag{9}$$

$$\theta_n^{(i)}(r,0,t) = \Theta_n^{(i)}(r), \qquad \theta_n^{(i)}(r,1,t) = \Lambda_n^{(i)}(r), \tag{10}$$

where $\vartheta_n^{(1)} = -\omega n;\ \ \vartheta_n^2 = \omega n;\ \ \mathrm{m}_1 = 2,\ \ \mathrm{m}_2 = 1;\ \ $ i=1,2.

In order to solve the boundary problems (7)-(10), let's apply the following integral transformation

$$\bar{f}(\mu_{n,k}) = \iint_{\mathrm{D}}\phi(r,z,\mu_{n,k})\cdot r\cdot \mathrm{f}(r,z)d\sigma \tag{11}$$

where $\phi(x,y,\mu_{n,k})$, $\mu_{n,k}$ – are intrinsic functions and intrinsic values.

Classical problem of the intrinsic functions and intrinsic values are formulated as a problem of defining values for numerical parameters (intrinsic values) $\mu_{n,k}$ and functions (intrinsic functions) $\phi(x,y,\mu_{n,k})$, which are

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identically not equal to zero in the domain $\Xi = \{(x, y) | \, y \in (0, h), x \in (\varsigma_1(z), \varsigma(z))\}$ and satisfy the equation:

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{1}{x}\frac{\partial \phi}{\partial x} - \frac{n^2}{x^2}\phi + \mu_{n,k} \cdot \phi + \frac{\partial^2 \phi}{\partial y^2} = 0 \tag{12}$$

and additional conditions

$$\phi(\mathrm{x}, 0, \mu_{n,k}) = 0, \qquad \phi(x, h, \mu_{n,k}) = 0, \tag{13}$$

$$\phi(\varsigma_1(z), y, \mu_{n,k}) = 0, \quad \phi(\varsigma(z), y, \mu_{n,k}) = 0, \tag{14}$$

where $\phi(x, y, \mu_n) \subset C^2(\Xi) = \{u(x, y) \in C(\Xi) : \partial_\alpha u(x, y) \in C(\Xi), \ \forall \alpha, \ |\, \alpha \,| \leq 2\}$;

$\partial_\alpha u(x, y) = \dfrac{\partial^{|\alpha|} u(x, y)}{\partial x^{\alpha_1} \, \partial x_2^{\alpha_2}}$; $|\, \alpha \,| = \alpha_1 + \alpha_2$ – is multiindex, components of which are whole integral numbers.

Let's find intrinsic values $\mu_{n,k}$ and intrinsic functions $\phi(x, y, \mu_{n,k})$ by solving problems (12)-(14) with the help of finite element method and the Galerkin method. To this end, let's divide the domains into simplex elements (fig. 2):

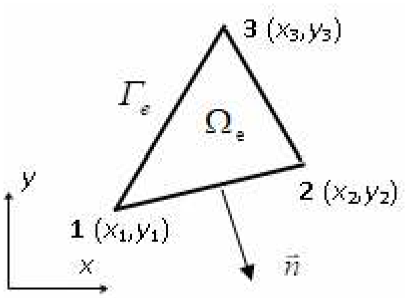

Figure 2 - Triangular element of the first order

Then, function $\phi(x, y)$ inside the simplex element is expressed through the shape functions of the N_1, N_2, N_3 with the known values of the ϕ_1, ϕ_2, ϕ_3 in the vertex of the triangle:

$$\phi_e(x, y) = N_1\phi_1 + N_2\phi_2 + N_3\phi_3 = [N_e]^T \{\phi_e\} \tag{15}$$

Where $[N_e] = [N_1, N_2, N_3]^T$; $\{\phi_e\} = \{\phi_1, \phi_2, \phi_3\}^T$ inferior index (e) means free simplex element.

For the i-node (i = 1, 2, 3), shape functions have the following forms:

$$N_i(x,y)=\frac{1}{d}(a_i+b_i x+c_i y)$$

where $d=x_2y_3-x_3y_2+x_3y_1-x_1y_3+x_1y_2-x_2y_1$; $a_i=x_jy_k-x_ky_j$; $b_i=y_j-y_k$; $c_i=x_k-x_j$; i, j, k are sequent numbering of the simplex-element nodes at counterclockwise tracing

Let's put the approximate solution (15) into equation (12) and, as a result, obtain the following equation:

$$\left(\frac{\partial^2}{\partial x^2}+\frac{1}{x}\frac{\partial}{\partial x}+\frac{\partial^2}{\partial y^2}\right)[N_e]^T\{\phi_e\}+\left(\mu_n-\frac{n^2}{x^2}\right)[N_e]^T\{\phi_e\}=0 \qquad (16)$$

Multiplying of the left side of the equation (16) by shape function $[N_e]$ and integrating by element e will give:

$$\mathrm{I}_1+\mathrm{I}_2=\{0\}$$

where

$$I_1=\iint_{\Omega_e}[N_e]\left(\frac{\partial^2}{\partial x^2}+\frac{1}{x}\frac{\partial}{\partial x}+\frac{\partial^2}{\partial y^2}\right)[N_e]^T dxdy\{\phi_e\};I_2=\iint_{\Omega_e}\left(\mu_n-\frac{n^2}{x^2}\right)[N_e][N_e]^T dxdy\{\phi_e\}$$

By integrating I_1 along the x and y, we receive

$$I_1=I_3-I_4$$

$$I_3=\left[\int_{\Gamma_e}[N_e]\frac{\partial[N_e]^T}{\partial x}dy+\int_{\Gamma_e}[N_e]\frac{\partial[N_e]^T}{\partial y}dx\right]\{\phi_e\}$$

$$I_4=\iint_{\Omega_e}\left\{\frac{\partial[N_e]}{\partial x}\frac{\partial[N_e]^T}{\partial x}+\frac{\partial[N_e]}{\partial y}\frac{\partial[N_e]^T}{\partial y}-\frac{[N_e]}{x}\frac{\partial[N_e]^T}{\partial x}\right\}dxdy\{\phi_e\}$$

By taking into account the identical relation:

$$\int_\Gamma\psi\frac{\partial\varphi}{\partial x}dy+\int_\Gamma\psi\frac{\partial\varphi}{\partial y}dx=\int_\Gamma\psi\left(\frac{\partial\varphi}{\partial x}dy+\frac{\partial\varphi}{\partial y}dx\right)=\int_\Gamma\psi\frac{\partial\varphi}{\partial n}d\Gamma$$

we receive:

$$I_3=\left[\int_{\Gamma_e}[N_e]\frac{\partial[N_e]^T}{\partial n}d\Gamma\right]\{\phi_e\}$$

where $\partial/\partial n$ is outer normal derivative; $\int_\Gamma d\Gamma$ is boundary curvilinear integral.

Summing of all elements will give us:

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$$\sum_e \left[\int_{\Gamma_e} [N_e] \frac{\partial [N_e]^T}{\partial n} d\Gamma \right] \{\phi_e\} + \sum_e \iint_{\Omega_e} \left(\mu_n - \frac{n^2}{x^2} \right) [N_e][N_e]^T dxdy \{\phi_e\} -$$

$$- \sum_e \iint_{\Omega_e} \left\{ \frac{\partial [N_e]}{\partial x} \frac{\partial [N_e]^T}{\partial x} + \frac{\partial [N_e]}{\partial y} \frac{\partial [N_e]^T}{\partial y} - \frac{[N_e]}{x} \frac{\partial [N_e]^T}{\partial x} \right\} dxdy \{\phi_e\} = \{0\} \qquad (17)$$

By multiplying the augend by the expression $\{\phi_e\}\{\phi_e\}^T$ we receive:

$$I_3 = \sum_e \phi_e\}^T \left[\int_{\Gamma_e} [N_e] \frac{\partial [N_e]^T}{\partial n} d\Gamma \right] \{\phi_e\} = \sum_e \left[\int_{\Gamma_e} \left(\{\phi_e\}^T [N_e] \right) \cdot \frac{\partial \left([N_e]^T \{\phi_e\} \right)}{\partial n} d\Gamma \right]$$

$$= \sum_e \left[\int_{\Gamma_e} \phi_e \frac{\partial \phi_e}{\partial n} d\Gamma \right]$$

Therefore, with taking into consideration the Dirichlet boundary condition, the augend in (17) can be neglected. In this case, the (17) takes the following form:

$$[K] + \mu_{n,k} \cdot [M] = \{0\} \qquad (18)$$

Where

$$[K] = \sum_e \left\{ - \iint_e \left(\frac{\partial [N_e]}{\partial x} \frac{\partial [N_e]^T}{\partial x} + \frac{\partial [N_e]}{\partial y} \frac{\partial [N_e]^T}{\partial y} - \frac{[N_e]}{x} \frac{\partial [N_e]^T}{\partial x} \right) dxdy - \right.$$

$$\left. - \frac{n^2}{x^2} \iint_e [N_e][N_e]^T dxdy \right\} \phi_e ;$$

$$[M] = \sum_e \iint_e [N_e][N_e]^T dxdy \cdot \{\phi_e\}.$$

Therefore, intrinsic functions $\phi(x, y, \mu_{n,k})$ and intrinsic values $\mu_{n,k}$ can be found from (18), and formula of inverse transformation takes the following form:

$$\mathrm{f}(\rho, z) = \sum_{j=1}^{\infty} \frac{\phi(\rho, z, \mu_{n,j})}{\left\| \phi(\rho, z, \mu_{n,j}) \right\|^2} \bar{f}(\mu_{n,j}). \qquad (19)$$

Let’s employ the integral transformation (11) for the system of differential equations (7), and, as a result, we receive the following system of ordinary differential equations:

$$\frac{d\overline{\theta}_n^{(i)}}{d\,t}+\vartheta_n^{(i)}\left[\overline{\theta}_n^{(m_i)}+\tau_r\,\frac{d\overline{\theta}_n^{(m_i)}}{d\,t}\right]+\tau_r\,\frac{d^2\overline{\theta}_n^{(i)}}{d\,t^2}=\Omega_{\mathrm{n,k}}^{(\mathrm{i})}-\mu_{n,k}\overline{\theta}_n^{(i)} \tag{20}$$

for initial conditions

$$\overline{\theta}_n^{(i)}\left(\mu_{n,k},t\right)=0,\qquad \frac{\partial\overline{\theta}_n^{(i)}\left(\mu_{n,k},t\right)}{\partial t}=0 \tag{21}$$

where

$$\Omega_{\mathrm{n,k}}^{(\mathrm{i})}=\int_0^h\left[\varsigma_1(z)\cdot\frac{\partial Q\left(\mu_n,\varsigma_1(z),z\right)}{\partial\rho}\cdot\Psi_n^{(i)}(z)-\varsigma(z)\cdot\frac{\partial Q\left(\mu_n,\varsigma(z),z\right)}{\partial\rho}\cdot G_n^{(i)}(z)\right]\cdot dz$$

$$-\oint_L\rho\left(\theta_n^{(i)}\frac{\partial Q\left(\mu_{n,k},\rho,z\right)}{\partial z}\right)d\rho;\quad \mathrm{i}=1,2.$$

The curvilinear integral is calculated by the closed positively-oriented contour ABCD (fig. 3)

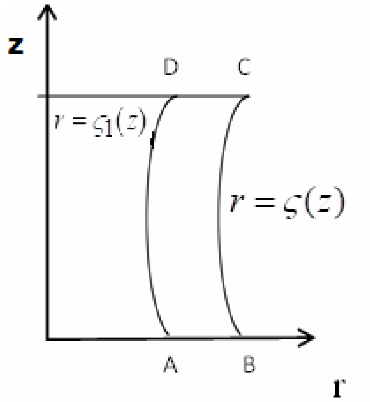

Figure 3 - Close contour with generating lines $r=\varsigma_1(z)$, $r=\varsigma(z)$

Let's employ the Laplace integral transformation [5] for the system of differential equations (20) with conditions (21):

$$\widetilde{f}(s)=\int_0^\infty f(\tau)e^{-s\tau}\,\mathrm{d}\tau$$

As a result, we receive the following system of equations:

$$s\widetilde{\overline{\theta}}_n^{(i)}+\vartheta_n^{(i)}\left(\widetilde{\overline{\theta}}_n^{(m_i)}+\tau_r s\widetilde{\overline{\theta}}_n^{(m_i)}\right)+\tau_r s^2\widetilde{\overline{\theta}}_n^{(i)}=q_{n,k}\left(\frac{\widetilde{\Omega}_{\mathrm{n,k}}^{(\mathrm{i})}}{\mu_{n,k}}-\widetilde{\overline{\theta}}_n^{(i)}\right) \tag{22}$$

where

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$$\widetilde{\Omega}^{(\mathrm{i})}_{\mathrm{n,k}} = \int_0^1 \left[\varsigma_1(z) \cdot \frac{\partial Q(\mu_n, \varsigma_1(z), z)}{\partial \rho} \cdot \widetilde{\Psi}_n^{(i)}(z) - \varsigma(z) \cdot \frac{\partial Q(\mu_n, \varsigma(z), z)}{\partial \rho} \cdot \widetilde{G}_n^{(i)}(z) \right] \cdot dz -$$

$$\oint_L \rho \left(\widetilde{\theta}_n^{(i)} \frac{\partial Q(\mu_{n,k}, \rho, z)}{\partial z} \right) d\rho; \quad q_{n,k} = \frac{\mathrm{a}}{\mathrm{R}^2} \mu_{n,k}; \mathrm{i} = 1,2.$$

Having solved the system of equations (20) we receive:

$$\widetilde{\widetilde{\theta}}_n^{(i)} = \alpha_{n,k} \frac{\widetilde{\Omega}^{(\mathrm{i})}_{\mathrm{n,k}} \left(\tau_r s^2 + s + \mathrm{q}_{\mathrm{n,k}} \right) + (-1)^{i+1} \omega n \widetilde{\Omega}^{(m_i)}_{\mathrm{n,k}} (1 + s\tau_r)}{\left(\tau_r s^2 + s + \mathrm{q}_{\mathrm{n,k}} \right)^2 + \omega^2 n^2 (1 + s\tau_r)^2}, \tag{23}$$

where $\alpha_{n,k} = \frac{\mathrm{a}}{\mathrm{R}^2}$; $i = 1,2.$

By applying the Laplace formula of inverse transformation [5] for the expression of function (23), we receive the following original functions:

$$\overline{\theta}_n^{(1)}(\mu_{n,k}, t) = \sum_{j=1}^{2} \zeta_{n,k}(s_j) \cdot \left\{ \widetilde{\Omega}^{(1)}_{\mathrm{n,k}}(s_j) \cdot \left[(2\tau_r s_j + 1) + \tau_r \omega n i \right] + \widetilde{\Omega}^{(2)}_{\mathrm{n,k}}(s_j) \cdot \kappa_j \right\} \cdot$$

$$\left(e^{s_j t} - 1 \right) + \sum_{j=3}^{4} \zeta_{n,k}(s_j) \cdot \left\{ \widetilde{\Omega}^{(1)}_{\mathrm{n,k}}(s_j) \cdot \left[(2\tau_r s_j + 1) - \tau_r \omega n i \right] + \widetilde{\Omega}^{(2)}_{\mathrm{n,k}}(s_j) \cdot l_j \right\} \left(e^{s_j t} - 1 \right), \tag{24}$$

$$\overline{\theta}_n^{(2)}(\mu_{n,k}, t) = \sum_{j=1}^{2} \zeta_{n,k}(s_j) \cdot \left\{ \widetilde{\Omega}^{(2)}_{\mathrm{n,k}}(s_j) \cdot \left[(2\tau_r s_j + 1) + \tau_r \omega n i \right] - \widetilde{\Omega}^{(1)}_{\mathrm{n,k}}(s_j) \cdot \kappa_j \right\} \cdot$$

$$\left(e^{s_j t} - 1 \right) + \sum_{j=3}^{4} \zeta_{n,k}(s_j) \cdot \left\{ \widetilde{\Omega}^{(2)}_{\mathrm{n,k}}(s_j) \left[(2\tau_r s_j + 1) - \tau_r \omega n i \right] - \widetilde{\Omega}^{(1)}_{\mathrm{n,k}}(s_j) \cdot l_j \right\} \left(e^{s_j t} - 1 \right), \tag{25}$$

where $\kappa_j = \tau_r \omega n - (2\tau_r s_j + 1) i$; $l_j = \tau_r \omega n - (2\tau_r s_j + 1) i$;

$\zeta_{n,k}(s_j) = \frac{0.5 s_j^{-1} \alpha_{n,k}}{(2\tau_r s_j + 1)^2 + (\tau_r \omega n)^2}$, and values of s_j for j=1,2,3,4 are determined by the following formulas:

$$s_{1,2} = \frac{(\tau_r \omega n i - 1) \pm \sqrt{(1 + \tau_r \omega n i)^2 - 4\tau_r q_{n,k}}}{2\tau_r},$$

$$s_{3,4} = \frac{(\tau_r \omega n i + 1) \pm \sqrt{(1 - \tau_r \omega n i)^2 - 4\tau_r q_{n,k}}}{2\tau_r}.$$

By this way, and by taking into account formulas of inverse transformation (6) and (19), we receive a temperature field of the thin-wall one-sheet rotary hyperboloid, which rotates with constant angular velocity ω around the axis OZ, with taking into account finite velocity of the heat conductivity:

$$\theta(r,\varphi,z,t)=\sum_{n=-\infty}^{+\infty}\left\{\sum_{k=1}^{\infty}\left[\bar{\theta}_n^{(1)}(\mu_{n,k},t)+i\bar{\theta}_n^{(2)}(\mu_{n,k},t)\right]\frac{Q(\mu_{n,k},r,z)}{\left\|Q(\mu_{n,k},r,z)\right\|^2}\right\}\exp(in\varphi)$$

where values of $\bar{\theta}_n^{(1)}(\mu_{n,k},t)$ і $\bar{\theta}_n^{(2)}(\mu_{n,k},t)$ are determined by the formulas (24),(25).

Conclusions. It is the first mathematical model developed for calculating temperature fields in the thin-wall one-sheet rotary hyperboloid, which rotates, with taking into account finite velocity of the heat conductivity; the model is created in the form of boundary problem of mathematical physics for hyperbolic equations of heat conductivity with the Dirichlet boundary conditions. An integral transformation was formulated for the two-dimensional finite space, with the help of which a temperature fields in the thin-wall one-sheet rotary hyperboloid was found by the Fourier functions in the form of convergent series by the Fourier function.

The obtained solution of the generalized boundary problem can be used for modelling temperature fields in the cooling towers of the similar shape, which are made of metal or prefabricated structural concrete and are widely used in nuclear power engineering, chemical and metallurgical industry as well as in cooling towers made of fiberglass plastic used in sugar mills, factories for processing meat, fish, fruits and vegetables, milk plants, breweries and other enterprises.

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Математична модель і метод рішення узагальненої задачі Діріхле теплообміну однопорожнинного гіперболоїда обертання

Вперше побудована математична модель розрахунку полів температури у тонкостінному однопорожнинному гіперболоїді обертання з урахуванням кінцевої швидкості поширення тепла, який обертається, у вигляді крайової задачі математичної фізики для гіперболічного рівняння теплопровідності з граничними умовами Діріхле. Побудоване інтегральне перетворення для двовимірного кінцевого простору, із застосуванням якого знайдено температурне поле у тонкостінному однопорожнинному гіперболоїді обертання у вигляді збіжних рядів по функціям Фур'є.

Математическая модель и метод решения обобщенной задачи Дирихле теплообмена однополостного гиперболоида вращения

Впервые построена математическая модель расчета полей температуры в вращающемся тонкостенном однополостном гиперболоиде вращения с учетом конечной скорости распространения тепла в виде краевой задачи математической физики для гиперболического уравнения теплопроводности с граничными условиями Дирихле. Построенно интегральное преобразование для двумерного конечного пространства, с применением которого найдено температурное поле в тонкостенной однополостном гиперболоиде вращения в виде сходящихся рядов по функциям Фурье.

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