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GYRO-ACCELEROMETRIC METHOD OF DETERMINATION OF ANGULAR ORIENTATION PARAMETERS

Algorithms of gyro-accelerometric method are given to measure the parameters of angular orientation in the inertial-satellite systems constructed by the method of compensation or by Kalman filtering. Measurement errors of angular orientation parameters are investigated by the proposed method with aircraft performing various maneuvers and with changing the acceleration and attitude.

Keywords: *gyro-accelerometer, acceleration, angular velocity sensor, angular orientation, compensation scheme.*

Introduction

With constructing the inertial-satellite navigation systems (ISNS) the data fusion of inertial navigation system (INS) and satellite navigation system (SNS) is usually based on optimal Kalman filtering. However, the practical implementation of Kalman filtering on board the aircraft is complicated by several factors. The main one is the phenomenon of divergence, that is typical for strapdown INS (SINS). To overcome these complications a number of modifications of Kalman filter is developed, in particular adaptive and robust filtering algorithms, Yazvynskyi algorithms, and others.

In modern airborne complexes besides the optimal estimation algorithms, there are also other methods of data fusion that are well proved in practice. In particular, it is a method of mutual compensation. The appropriateness of using the method of compensation in SINS and SNS integration is explained be the fact that errors of these systems are in different frequency ranges.

Comparative analysis of data fusion algorithms in SINS and SNS given in [1] shows that the accuracy of navigation parameter estimation for compensation method with the latest dynamic filters [2] is not worse than accuracy of Kalman filtering algorithms, and the quality of noise filtering of the proposed algorithm in SNS is higher. But this method does not estimate the nonobservable components of state vector, in particular, roll and pitch angles.

For small unmanned aerial vehicles (UAV), the precision vertical gyroverticals are not used because of their unsatisfactory proportions. And low-cost micromechanical SINS, which is the basis for constructing ISNS of such UAV is not able to provide the necessary accuracy of autonomous measurement of roll and pitch angles.

Therefore, the development of alternative methods for determining the parameters of the angular orientation is the urgent problem.

Problem statement

To measure the parameters of angular orientation a number of alternative methods is proposed, including pyrometric and magnetometric methods.

Pyrometric method to measure roll and pitch is based on measuring the difference in the intensity of heat radiation of the ground surface and of the firmament using pyrometers. They provide contactless measurements of thermal radiation intensity in the infrared range. To measure roll and pitch angles, four pyrometers are installed in pairs, for example, along the axes of body-fixed coordinate system (CS). The natural thermal radiation of the ground surface has higher intensity than the intensity of firmament thermal radiation.

Therefore, if UAV is flying horizontally, one half of sensitivity area of each installed sensors P_1, P_2 (see Fig. 1) is occupied by firmament and the other half is taken by ground.

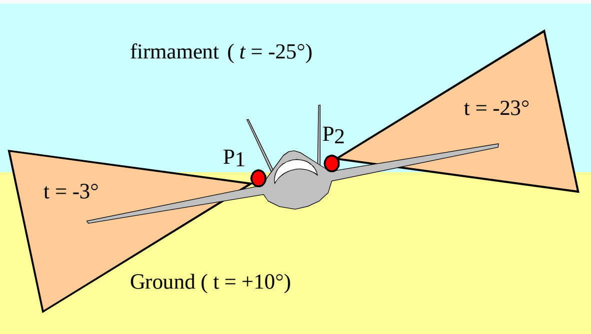
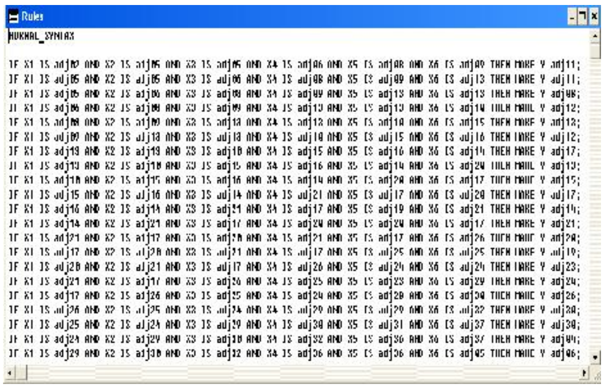


Fig. 1



Method of magnetometric measurement has areas of uncertainty, in particular with the magnetometer rotation around the axis close or coinciding with the direction of magnetic field vector and also with headings close to 90 .. 270° the uncertainty in measurements of roll and pitch angles appears.

To measure the roll and pitch angles it is also possible to use so called gyro-accelerometric method, when readings of SINS accelerometers are processed by special algorithms.

Problem statement can be formulated as following: it is necessary to develop the scheme and algorithms of gyro-accelerometric measurement of angular orientation parameters and to provide their comprehensive investigation.

Problem solution

The accelerometer can be used to measure the projections of absolute linear acceleration and also to measure indirectly the projections of gravitational acceleration. The former measurements is used to create SINS where the readings obtained from accelerometers are converted to the axes of navigation CS, then processed and integrated. As a result, the navigation parameters of object motion are obtained: velocity and coordinate. Thus, the accelerometers, together with gyros, are integral parts of the navigation and control systems of moving objects.

If the only force acting on an object is the gravity force, then the accelerometer measures the projection of gravity vectors on sensitivity axis $Ax = g\sin(\alpha)$, and therefore can be used as inclinometer to determine the static tilt angle of accelerometer sensitivity axis.

$$\alpha = \arcsin(Ax/g)$$

Such properties of accelerometer are used in the algorithms of SINS initial alignment to determine the ramp roll and pitch angles.

In practice, the moving object is experienced other forces besides the gravitational force. They can be caused by acceleration, rotation, vibrations, etc. In particular, during flight there will be accelerations that characterize the change in the relative velocity vector **Vr**, plus Coriolis acceleration $(\boldsymbol{\omega}+2\boldsymbol{\Omega})\times\mathbf{V_r}$, transferring acceleration and acceleration of gravity force. And then the acceleration **A** measured by accelerometer is the following:

$$\mathbf{A}=\left(\dot{\mathbf{V}}_{\mathbf{r}}\right)_{Earth}+(\boldsymbol{\omega}+2\boldsymbol{\Omega})\times\mathbf{V_r}+\boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\mathbf{R})-\mathbf{g}_0(\mathbf{R}).$$

The transferring acceleration caused by the Earth rotation $[\boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\mathbf{R})]$ is summed with acceleration of gravity force $\mathbf{g}_0(\mathbf{R})$ as vectors and creates the gravity acceleration

$$\mathbf{g}=\mathbf{g}_0(\mathbf{R})-\boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\mathbf{R}).$$

Then the equation of apparent accelerations of object center-of-mass measured by accelerometer takes the form:

$$\mathbf{A}=\left(\dot{\mathbf{V}}_{\mathbf{r}}\right)_{Earth}+(\boldsymbol{\omega}+2\boldsymbol{\Omega})\times\mathbf{V_r}-\mathbf{g}. \quad (1)$$

Here **Vr** is vector of UAV relative velocity; **R** is vector that characterizes the current location of object in the selected CS; μ is the angular velocity, which occurs with motion relatively the spherical surface of the Earth, which, in turn, rotates with angular velocity t.

In algorithms of SINS operating modes it is necessary to get the value of the Earth velocity vector by integrating the accelerometer signals projected on the axes of navigation CS, for example, rectangular geographical CS *OLHB*. Thus, it is necessary to subtract from these projections(a_L, a_H, a_B) the components of Coriolis acceleration projections and components of gravity acceleration projections. Then the projection of the ground velocity vector on the axes of CS *OLHB* can be obtained by integrating the equations:

$$\dot{V}_L=a_L-(V_B\omega_{H_\Sigma}-V_H\omega_{B_\Sigma})+g_L;$$

$$\dot{V}_H=a_H-(V_L\omega_{B_\Sigma}-V_B\omega_{L_\Sigma})+g_H;$$

$$\dot{V}_B=a_B-(V_H\omega_{L_\Sigma}-V_L\omega_{H_\Sigma})+g_B;$$

where $\omega_{B_\Sigma}=\omega_{B_V}+2\Omega_B$, $\omega_{H_\Sigma}=\omega_{H_V}+2\Omega_H$, $\omega_{L_\Sigma}=\omega_{L_V}+2\Omega_L$ are components of projections $(V_B\omega_{H_\Sigma}-V_H\omega_{B_\Sigma})$, $(V_L\omega_{B_\Sigma}-V_B\omega_{L_\Sigma})$, $(V_H\omega_{L_\Sigma}-V_L\omega_{H_\Sigma})$ are projections of Coriolis accelerations; ω_{B_V}, ω_{H_V}, ω_{L_V} are projections of angular rotation velocity of navigation CS *OLHB* which appears with motion relative the Earth spherical surface; $\Omega_B=\Omega_{Earth}\cos B, \Omega_H=\Omega_{Earth}\sin B, \Omega_L=0$ are projections of Earth angular velocity on the axes of navigation CS *OLHB;* V_B, V_L, V_H are projections of velocity vector on the axes of CS *OLHB*; g_L, g_B, g_H are projections of gravity force acceleration which with small altitudes ($H \le 100$ km) can be calculated up to terms зберігаючи 10^{-2}:

$$g_L=0,\ g_B=0,$$

$$g_H=-g\left(1+5.2884\cdot10^{-3}\sin^2 B\right)\left[1-\frac{2H}{a}\left(1-e\ \sin^2 B\right)\right];$$

B is geographic latitude; H is the flight altitude; g is the acceleration of gravity force at the equator; a is the major semi-axis of Earth geoid; e is the eccentricity.

With using the accelerometric method of angular orientation measurement the similar approach is used. It is proposed to subtract the projections of components of the relative acceleration $\left(\dot{\mathbf{V}}_{\mathbf{r}}\right)_{Earth}$ and Coriolis acceleration $(\boldsymbol{\omega}+\mathbf{2\Omega})\times\mathbf{V_r}$ from readings of accelerometers (1), leaving only the components of the gravity acceleration.

$$\mathbf{A}=-\mathbf{g}\,.$$

The roll and pitch angles are determined by inclinometer algorithms with measured values of undisturbed by other forces projections of gravity accelerations.

The components of Coriolis and relative accelerations in ISNS are calculated by either estimated navigation parameters (by compensation method) or directly by information from SNS. Obtained parameters of angular orientation are further processed using information from angular velocity sensors (AVS) which are parts of SINS .

The block diagram of scheme to measure the angular orientation by gyro-accelerometric method is shown in Fig. 2.

In this scheme the ISNS data about components of ground velocity vector and about components of Coriolis and relative accelerations is used to calculate their projections on the axes of body-fixed CS. The obtained components are subtracted from readings of horizontal accelerometers. Then information about undisturbed components of gravity acceleration is taken to determine the direction cosine matrix by SINS algorithms using the current values of gravity acceleration.

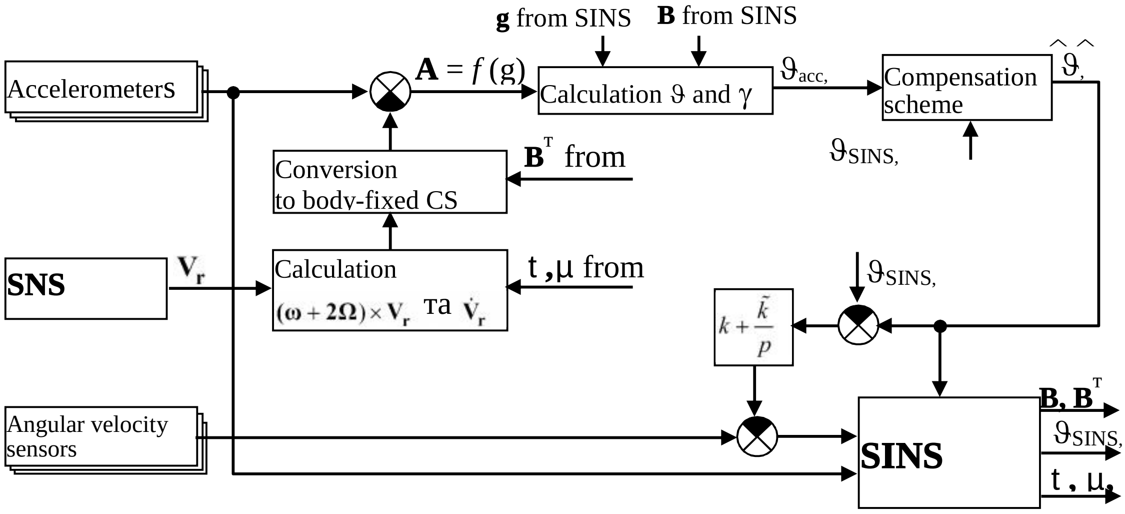


Fig. 2

And finally the current values of roll and pitch angles are calculated. Calculated roll and pitch angles by the proposed accelerometric method have the high-frequency error (sensor noise), that is why this information is further processed to get estimate from compensation scheme. Here the readings of SINS with slowly varied (low-frequency) error are fused with readings of gyro-accelerometric sensor with corresponding high-frequency noise. And the compensation scheme in this case is an ideal variant. The obtained estimates are used to correct the parameters of angular orientation calculated by SINS algorithms.

To simplify the analyses of gyro-accelerometric method of angular orientation measurement let us consider the two-component SINS and longitudinal motion of UAV along the equator.

Kinematics of UAV rotational motion by pitch angle can be described in SINS algorithms as following:

$$\omega_z=\omega_{z_{\text{avs}}}+(\dot{L}+\Omega_{\text{Earth}});\;\; \vartheta=\vartheta_0+\int_0^t\omega_z dt,$$

and kinematics of translational motion of center-of-mass (in this case - motion in vertical plane) can be described as following:

$$\dot{V}_H=a_H-a_H^{\text{cor}}-g;\quad \dot{V}_L=a_L-a_L^{\text{cor}};\quad \dot{L}=\dot{V}_L/R_{\text{Earth}}\,,$$

where $\Omega_{\text{Earth}}=7,27\cdot10^{-5}\,1/\text{sec}$; $R_{\text{Earth}}=6378388$ m;

$$a_H=a_x\sin\vartheta+a_y\cos\vartheta;\quad a_L=a_x\cos\vartheta-a_y\sin\vartheta;$$

$$a_L^{\text{cor}}=-V_H(\dot{L}+2\Omega_{\text{Earth}});\; a_H^{\text{cor}}=V_L(\dot{L}+2\Omega_{\text{Earth}}).\quad (2)$$

Input data for SINS is signals of inertial sensors $\omega_{z_{\text{avs}}}$, a_x, a_y, which have errors ($\Delta\omega_z$, Δa_x, Δa_y)

containing the deterministic and random white noise components. Signals of inertial sensors can be described by the following equations:

$$\omega_{z_{avs}} = \omega_{z_{man}} - (\dot{L}_{true} + \Omega_{Earth\ true}) + \Delta\omega_z, \quad (3)$$

$$a_x = a_{x_{man}} + g_{true}\sin\vartheta_{true} + \\ +(a_{Htrue}^{cor}\sin\vartheta_{true} - a_{Ltrue}^{cor}\cos\vartheta_{true}) + \Delta a_x \quad (4)$$

$$a_y = a_{y_{man}} + g_{true}\cos\vartheta_{true} + \\ +(a_{Htrue}^{cor}\cos\vartheta_{true} - a_{Ltrue}^{cor}\sin\vartheta_{true}) + \Delta a_y \quad (5)$$

Here the angular velocity sensor measures not only the angular velocity of maneuver $\omega_{z_{man}}$, but also the angular velocity of the Earth rotation $\Omega_{Earth\ true}$ and angular velocity $\dot{L}_{true}$, caused by UAV motion relative the spherical ground surface.

Similarly it is necessary to take into account that accelerometers measure the maneuver accelerations $a_{x_{man}}$, $a_{y_{man}}$ together with components of Coriolis and gravity accelerations.

To implement the gyro-accelerometric method of pitch measurement it is necessary to subtract the calculated components of Coriolis acceleration $(a_H^{cor}\sin\vartheta - a_L^{cor}\cos\vartheta)$ and longitudinal acceleration of center-of-mass $a_{x_{man}}$ from readings of longitudinal accelerometer a_x. Then the resulting signal of longitudinal accelerometer contains the projection of gravity vector on the sensitivity axis together with deterministic and random white noise components of errors $Ax = g_{true}\sin(\vartheta) + \Delta ax$.

Components of Coriolis acceleration are calculated by formulas (2), using the estimates of ground velocity components (V_H, V_L) and rate in longitude change $\dot{L}$, and also the known value of angular velocity of the Earth rotation Ω_{Earth}. To convert these components to axes of body-fixed CS the corrected (gyro-accelerometric) value of pitch angle ϑ is used.

The component $a_{x_{man}}$ is calculated by formula:

$$a_{x_{man}} = \frac{a_L^f - a_L^{cor} + a_y\sin\vartheta}{\cos\vartheta} - g\sin\vartheta - (a_H^{cor}\sin\vartheta - a_L^{cor}\cos\vartheta).$$

Here the components of Coriolis acceleration are calculated using the estimates and corrected value of pitch angle. To project the horizontal component of UAV acceleration on the longitudinal axis of body-fixed CS it is necessary to subtract the component of normal acceleration $a_y\sin(\vartheta)$ and to reconstruct the component of Coriolis acceleration a_L^{cor}. The acceleration of gravity force g at the equator is set to be equal to 9.8 m/sec^2.

The estimated (filtered) value of horizontal component of center-of-mass acceleration a_L^f is obtained by data fusion of information from SINS and differentiated signal V_L from SNS. With differentiation of noisy radio signal of SNS the standard filtering procedures are used.

Fig. 3 shows the estimation results of horizontal component of center-of-mass acceleration a_L^f in comparison with differentiated signal V_{LSNS} during the UAV maneuver "Zoom".

By information Ax the current value of pitch angle is calculated – so called accelerometric pitch angle : $\vartheta_{acc} = \arcsin(Ax/g)$.

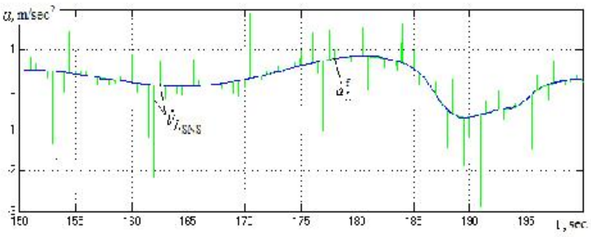


Fig. 3

By the difference between the current pitch angle calculated by SINS algorithms and accelerometric pitch angle the resulting signal is formed to correct the readings of angular velocity sensor (to eliminate the deterministic component of error).

$$\Delta\omega_{z_{cor}} = \left[K_{\omega z} + \frac{\tilde{K}_{\omega z}}{p}\right](\vartheta - \vartheta_{acc}). \quad (6)$$

The result of correction is the estimated (gyro-accelerometric) pitch angle ϑ.

The investigation of accelerometric algorithms of pitch measurement has been done by mathematical modeling in program software *Simulink MATLAB.*

For the investigation the model of two-component SINS has been created which describes the kinematics of UAV motion in vertical plane.

Simultaneously the ideal navigation system has been simulated identical to SINS but it has had the input signals as signals of ideal sensors and true values of the Earth model. Information from the ideal navigation system has been used to form the true (not calculated) values of navigation parameters and also to estimate the accuracy characteristics of SINS by comparison the calculated and true values of navigation parameters.

With SNS simulation the output information of ideal navigation system has been used and corrupted by white noise components of SNS errors.

The simulation of inertial sensors has been done by formulas (3)…(5), but the components of model errors $\Delta\omega_z$, Δa_x, Δa_y have been absent.

The above mentioned algorithms of gyro-accelerometric sensor have been also simulated.

The results of simulation are given in Fig. 4. They illustrate the change in pitch measurement errors with no maneuvering with higher zooming.

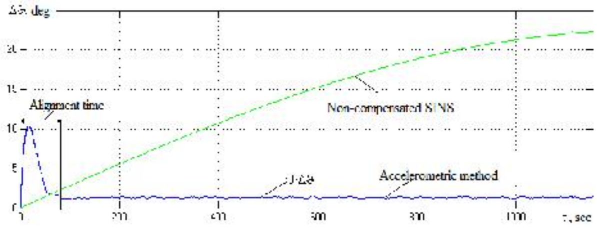


Fig. 4

The comparative analyses proves the significant improvement in accuracy of pitch measurement by gyro-accelerometric sensor even using the low-accurate SINS. The error of pitch measurement for autonomous operation of not-corrected SINS previously has raised up to 23° during 25 minutes of flight, and now it is decreased to 0.028°. At the beginning of operation the initial alignment stage is obviously observed (determination and elimination of systematic component of angular velocity sensor error)

With using very rough accelerometers in SINS structure the error of pitch measurement (Fig. 5) does not exceed 0.25° (in the model of accelerometer errors the systematic component has been increased by an order of magnitude).

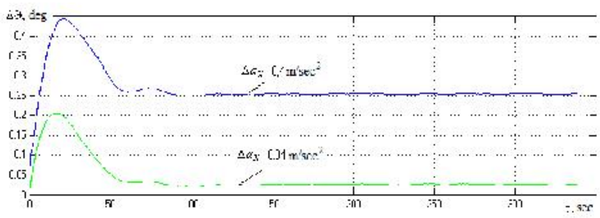


Fig. 5

The estimation of measurement errors for gyro-accelerometric method has been done for UAV maneuvering to climb and descent. The change in parameters of longitudinal motion during such maneuvering is shown in Fig. 6. In particular, the pitch angle has varied in range ±15°, normal overload has reached 5 m/sec^2, and longitudinal acceleration has been (±0,8 m/sec^2).

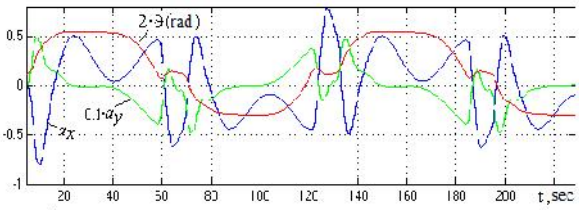


Fig. 6

Fig. 7 demonstrates the change in gyro-accelerometric errors during the maneuver "Zoom". For comparison the results of pitch measurement by inclinometer and gyro-accelerometric sensor (with complementary filter) are given.

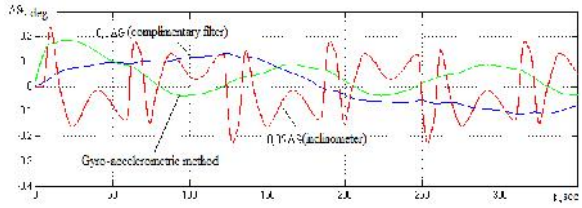


Fig. 7

The complementary filter provides data fusion of pitch measurement by inclinometer ϑ_{inc} = arcsin(a_x/g) and by SINS algorithm ϑ_{SINS}. The algorithm of complementary filter is quite simple:

ϑ_{comp} = (1J K_f) ϑ_{SINS} + $K_f \cdot \vartheta_{inc}$,

where K_f – coefficient of complementary filter.

By the difference (ϑ_{SINS} J ϑ_{comp}) the signal $\Delta\omega_{Zcor}$, is formed similar to (6) and provides correction of readings of angular velocity sensor.

The analyses of simulation results (Fig. 7) shows that with maneuvering the error of pitch measurement by gyro-accelerometric sensor does not exceed 0.1°, error by using complimentary filter is about 1°, and error of inclinometer disturbed by accelerations reaches 5°.

The gyro-accelerometric method has been also tested for UAV with highly energetic maneuvering (climbing and descending). And besides the white noise component of accelerometer errors the vibration of UAV engines has been also simulated and included to readings of accelerometers (Fig. 8).

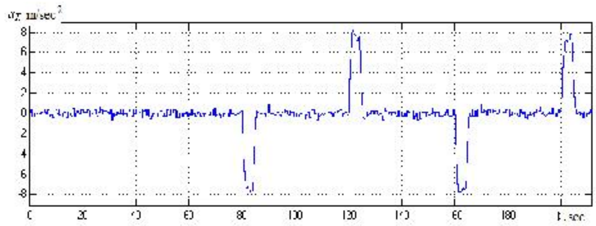


Fig. 8

Fig. 9 demonstrates the change in errors of gyro-accelerometric sensor during such energetic maneuvering. For comparison the results of pitch measurement by complimentary filter are also given. The results of simulation show that even for energetic maneuvering and severe conditions of accelerometer operation the error of pitch measurement does not exceed 0.25° for gyro-accelerometric sensor, and error of complimentary filter raises to 3°.

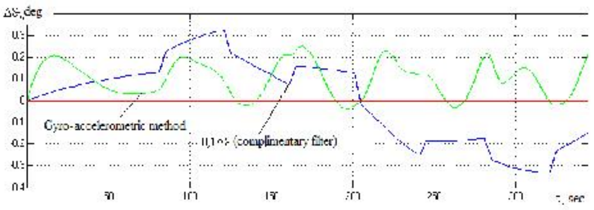


Fig. 9

If Coriolis accelerations are neglected in algorithms of gyro-accelerometric method then the significant simplification can be reached. In particular, only component $a_{x_{\text{man}}}$ is subtracted from readings of longitudinal accelerometer and it is calculated by formula:

$$a_{x_{\text{man}}} = \frac{a_L^{\text{f}} + a_y \sin\vartheta}{\cos\vartheta} - g\sin\vartheta .$$

Then

$$\vartheta_{\text{acc}} = \arcsin\left(\frac{a_x - a_{x_{\text{man}}}}{g}\right).$$

The simplified algorithm of gyro-accelerometric method has been simulated for UAV with highly energetic maneuvering - climbing and descending (see Fig. 8). In Fig. 10 for comparison the errors of pitch measurement by simplified and full algorithms are given. The difference in pitch measurement errors does not exceed 0.05°

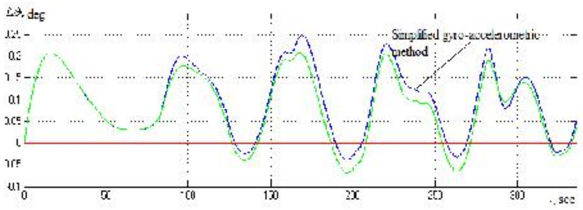


Fig. 10

The basic disadvantage of the proposed gyro-accelerometric method is its non-autonomy. That is why it can be used only in aided navigation systems, in particular, in ISNS. It must be noted, that the increasing of accuracy of angular orientation measurement significantly influences the accuracy of navigation parameters calculated by SINS algorithms.

If the satellite signal is lost, then the navigation complex of UAV passes to the aerometric mode of dead reckoning and data of a_L^{f} can be obtained by differentiating the airspeed.

In Fig. 11 the results of simulation of gyro-accelerometric method are given in case of navigation complex operation in aerometric mode during 70 seconds. UAV performs maneuvering "Zoom". During the simulations the sluggishness, errors of airspeed measurement including noise components, differentiating errors have been taken into account.

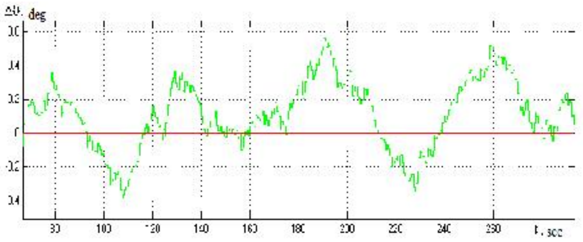


Fig. 11

Analyses of simulation results shows that even for coarse measurements of maneuvering parameters, the error of pitch measurement varies in range ±0.5°.

In case of absence the standby system in the structure of navigation complex, the pitch and roll measurement is recommended to perform with the help of gyro-accelerometric sensor using the heading method of en-route control.

Conclusions

The given research of proposed gyro-accelerometric method shows high accuracy of angular orientation measurement. In particular, the errors of roll and pitch measurement by coarse MEMS sensors do not exceed the errors of existent precise gyroverticals. The basic disadvantage of the proposed gyro-accelerometric method of angular orientation measurement is non-autonomy in comparison with the gyroverticals. Therefore, it can be used only in aided navigation systems of small UAV, in particular, in ISNS. Moreover, in this case, the algorithms of data fusion for SINS and SNS are significantly simplified, since there is no necessity to estimate the non-observed components of state vector.

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ГІРО-АКСЕЛЕРОМЕТРИЧНИЙ МЕТОД ВИЗНАЧЕННЯ ПАРАМЕТРІВ КУТОВОЇ ОРІЄНТАЦІЇ

М.К. Філяшкін, М.П. Мухіна

Приводяться алгоритми гіро-акселерометичного методу вимірювання параметрів кутової орієнтації в інерціально-супутникових системах, побудованих за методом компенсації або на основі калманівської фільтрації, досліджуються похибки вимірювання параметрів кутової орієнтації запропонованим методом при виконанні об'єктом різного роду маневрів з прискореннями і зміною кутового положення.

Ключові слова: *акселерометр; прискорення; датчик кутової швидкості; кутова орієнтація; схема компенсації.*

ГИРО-АКСЕЛЕРОМЕТРИЧЕСКИЙ МЕТОД ОПРЕДЕЛЕНИЯ ПАРАМЕТРОВ УГЛОВОЙ ОРИЕНТАЦИИ

Н.К. Филяшкин, М.П. Мухина

Приводятся алгоритмы гиро-акселерометрического метода измерения параметров угловой ориентации в инерциально-спутниковых системах, построенных по методу компенсации или на основе калмановской фильтрации, исследуются погрешности измерения параметров угловой ориентации предложенным методом при выполнении объектом различного рода маневров с ускорениями и изменением углового положения.

Ключевые слова: *акселерометр; ускорения; датчик угловой скорости; угловая ориентация; схема компенсации.*