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ALGORITHM OF IDENTIFICATION OF NONLINEAR TECHNICAL SYSTEMS ACCORDING TO MEASURED DATA

Application nonlinear smoothing of a procedure, evaluation of a gradient of criterion quality gradient of quasinewton procedure, enables reaching a minimum of functional of quality. For the solution in a problem of accessible smoothing effect, the nonlinear return information filter has been used.

The obtained algorithm of identification of nonlinear technical systems can be utilized for construction of mathematical models of steady and nonsteady technological procedure.

***Keywords**: procedure, quality criteria, functional quality, identification algorithm, information filter, stationary process, non-stationary process, nonlinear system.*

Introduction. The problem of solving the problems of identification in various scientific fields occupies an important place. Despite the diversity of the developed methods [1], is comparable to a large class of theoretical and applied problems that require solutions. This is especially true of nonlinear systems in some areas of technology [2].

Purpose. Develop an algorithm for parameter identification of nonlinear systems by a number of measured data.

Materials and result obtained. Algorithms for parameter identification of nonlinear systems are based on minimizing the error between measurements of the output signal of the object and output the mathematical model if applied to the input signal of some type. In most cases, this pseudo signal as white noise, or a binary sequence or harmonic signals of different frequencies (including one that changes periodically) [2, 3].

The dynamics of nonlinear systems in the general case:

$$\dot{x}(t) = f_c\left[x(t),\ u(t),\ w(t),\ t, \theta\right], \tag{1}$$

$$z(t_i) = S\,h_m\left[x(t_i),\ u(t_i),\ t_i, \theta\right] + b + v(t_i),\quad x(t_0) = x_0,\quad t_0 \le t \le t_f. \tag{2}$$

Where:

θ – vector of unknown parameters;

S – vector scaling factors for the measurements;

b – displacement vector measurements relative to the average value;

x(t), u(t), w(t), v(t) – state vectors, controls, states and noise disturbance measurements.

Vector of unknown parameters, we find from the minimum of the functional:

$$J_0 = 1/2\left(\theta - \theta_0\right)^T \Theta^{-1}\left(\theta - \theta_0\right) + 1/2\left(x(t_0) - x_0\right)^T P_0^{-1}\left(x(t_0) - x_0\right) + 1/2\int_{t_0}^{t_f} w^T(t) Q^{-1} w(t)\,dt + 1/2\sum_{i=1}^{N} v^T(t_i) R^{-1} v(t_i) \tag{3}$$

where *Θ, P, Q, R* – weighting matrix. The first term in equation (3) account a priori information on the parameters of the object, for example the results of estimating the parameters of the regression model on the basis of previous experimental data. If the model equation (1), (2) as a limitation, then the solution of the optimization problem can be written Lagrangian:

$$J = J_o + \int_{t_0}^{t_f} \lambda^T(t)\left\{f_c\left(x(t), u(t), w(t), t, \theta\right) - \dot{x}(t)\right\} dt, \tag{4}$$

where $\lambda(t)$ - vector of Lagrange multipliers. Minimizing the criterion J equivalent to minimizing the criterion J_0 with constraints (1), (2).

For the solution of the optimization problem we use the variational method. The first variation J with respect to small changes of unknown is defined as:

$$\delta J = \int_{t_0}^{t_f}\left[\frac{\partial J}{\partial x(t)}\delta x(t) + \frac{\partial J}{\partial w(t)}\delta w(t)\right] dt + \sum_{i=0}^{N}\frac{\partial J}{\partial x(t_i)}\delta x(t_i) + \frac{\partial J}{\partial \theta}\delta\theta. \tag{5}$$

In the stationary point of the functional J its first variation δJ must match the zero for arbitrary variations $\delta x(t_0), \delta w(t)$ and $\delta\theta$. For a given set of parameters of equation (4), (5) are nonlinear smoothing.

The task of smoothing on a fixed time interval for the discrete nonlinear system put as follows. For nonlinear systems described by a system of equations:

$$x(k+1) = f_d\left[x(k), u(k), w(k), k, \theta\right] \tag{6}$$

$$z(k) = S\,h_m\left[x(k), u(k), k, \theta\right] + b + v(k) \tag{7}$$

and experimental data $\{z_m(k)\},\ k = \overline{1,N}$ to find the value $x(0)$ and the sequence $\{w(k)\},\ k = \overline{0, N-1}$, minimizing the criterion:

$$J = 1/2\left[x(0) - x_0\right]^T P_0^{-1}\left[x(0) - x_0\right] + \\ + \frac{1}{2}\sum_{i=0}^{N-1}\left[w^T(k)Q^{-1}w(k) + v^T(k+1)R^{-1}v(k+1)\right], \quad (8)$$

This nonlinear programming problem because constraints (6), (7) and criteria as relatively unknown nonlinear parameters. In addition to the usual quadratic nonlinearities that are inherent criterion J, last term of (8) contains other types of nonlinearities. This is because the error model is defined as

$$v(k+1) = z_m(k+1) - z(k+1) = z_m(k+1) - \\ - \{E\ h_m[x(k+1),\ u(k+1),\ k+1,\ ¯] + b + v(k+1)\}. \quad (9)$$

This nonlinear problem is solved by successive computation of minimum extremist neighboring trajectories satisfying the constraint (6), (7) until you reach the minimum criteria J. For this purpose is defined by the nominal trajectory, which also satisfies the equations that form restrictions. For the solution of the problem must specify the initial conditions $x(0) = x_0$ and the sequence of values $\left[w(i)\right]$, that in many cases the solution of equations are zero. equation (3) used to calculate quality criterion that is linked to this trajectory. Using as a basis the basic trajectory, then calculate the adjacent path so that the criterion for the quality received with less importance than in the previous cycle. The procedure continues until, until you reach the minimum criteria.

To calculate the family of trajectories using variational approach. Variation second order quality criterion has the form [1, 4]:

$$¢J = [x(0) - x_0]^T P_0^{-1}¢x(0) + 1/2¢x(0)^T P_0^{-1}¢x(0) + \\ + \sum_{i=0}^{N-1}\left[w^T(i)Q^{-1}¢w(i) + 1/2¢w^T(i)Q^{-1}¢w(i)\right] + \\ + \sum_{i=1}^{N}\left[-v^T(i)R^{-1}Eh_x(i)¢x(i) + \\ + 1/2¢x^T(i)h_x^T(i)R^{-1}Eh_x(i)¢x(i)\right] \quad (10)$$

Variation constraint (6), linking variations $¢x(k)$, $¢x(0)$ and $¢x(k)$:

$$¢x(k+1) = f_x(k)¢x(k) + f_w(k)¢w(k), \quad (11)$$

$$k = \overline{0, N-1}.$$

Gradients $f_x(k),\ f_w(k)$ and $h_x(k)$ in equations (10) and (11) are defined by the expressions:

$$f_x(k) = \left.\frac{\partial f_d(x,u,w,k,¯)}{\partial x}\right|_{x(k),u(k),w(k)}, \\ f_w(k) = \left.\frac{\partial f_d(x,u,w,k,¯)}{\partial w}\right|_{x(k),u(k),w(k)}, \quad (12) \\ h_x(k) = \left.\frac{\partial h_m(x,u,k,¯)}{\partial x}\right|_{x(k),u(k)}.$$

and evaluated along the nominal trajectory defined by sequences of values $[x(k)], [u(k)], [w(k)]$.

Thus, it is necessary to calculate the trajectories of the family, given variations $¢x(0)$ and $[¢w(k)]$. This variation of the quality criterion should take a more negative value and satisfy restriction (11). This leads to the solution of the problem of so-called "minimum achievable." Based on the fact that the partial derivatives $f_x(k),\ f_w(k)$ and $h_i(k)$ remain approximately constant when moving along the face and adjacent trajectories (this is confirmed by the decomposition equations forming limit and the rejection of the second order term of the expansion and above), one could argue that the minimization problem similar to the problem of smoothing for linear systems and can be solved by using one of the known algorithms for smoothing. To prove this similarity, modify the criteria as $¢J$ by introducing additional members:

$$¢J_1 = ¢J + 1/2[x(0) - x_0]^T P_0^{-1}[x(0) - x_0] + \\ + \frac{1}{2}\sum_{i=0}^{N-1}\left[w^T(k)Q^{-1}w(k) + v^T(k+1)R^{-1}v(k+1)\right] \quad (13)$$

Additional terms in (13) will not function variables $¢x(0)$ and $[¢w(k)]$, but because they are constants in solving minimization problem. If a value criterion of equation (10) into equation (13) and perform the appropriate transformations we obtain the extended criteria:

$$¢J_1 = 1/2[¢x(0) + x(0) - x_0]^T P_0^{-1}[¢x(0) + x(0) - x_0] + \\ + \frac{1}{2}\sum_{i=0}^{N-1}\left[[w(k) + ¢w(k)]^T Q^{-1}[w(k) + ¢w(k)] + \\ + v_1^T(k+1)R^{-1}v_1(k+1)\right], \quad (14)$$

where

$$v_1(k) = z_m(k+1) - \\ - E\{h_x(k+1)¢x(k) + h_m[x(k+1), u(k+1), k+1, ¯]\} - b \quad (15)$$

Minimizing the criterion $¢J_1$ under the constraint (11) is a linear smoothing, which $¢x(0)$ – unknown initial conditions; $[¢w(k)]$ - unknown sound of an object. For the solution of the problem "achievable" smoothing, we use a nonlinear inverse filter information. This allows you to calculate the next trajectory for which the criterion value decreases nonlinear smoothing. Neighboring trajectories (family) are calculated iteratively as long as the change variations $¢x(0)$ and $[¢w(k)]$ is sufficiently small. From this it follows that $¢J$ as "small" and the result is a minimum criterion.

Finding solution using nonlinear inverse filter information based on the following steps:

1. Based on the initial conditions $x(0)$ and sequencing $[w(k)]$, obtained at the previous iteration (or initial conditions), calculated nominal trajectory (equation (6), (7); quality criterion J ; equation (8) and gradient matrix $f_x(k), f_w(k)$ and $h_i(k)$; equation (12).

2. For nonlinear inverse filter information put zero terminal conditions $y_{N/N}=0$ and $S_{N/N}=0$.

For $i=\overline{N,1}$ find smoothed estimates in reverse time [5, 6]:

$$y_{k/k}=y_{k/k+1}+h_x^T(k)E^T R^{-1}\times \times\{z(k)-Eh_m[x(k),u(k),k,¯]-b\}, \quad (16)$$

$$S_{k/k}=S_{k/k+1}+h_x^T(k)E^T R^{-1}E\,h_x(k), \quad (17)$$

$$K_B(k)=\left[Q^{-1}+f_w^T(k)\,S_{k+1/k+1}\right], \quad (18)$$

$$w_B(k)=Q\ \ f_w^T(k)\,[I-f_w(k)\,K_B(k)]^T\,y_{k+1/k+1}, \quad (19)$$

$$y_{i/i+1}=f_x^T(k)\,[I-f_w(k)K_B(k)]^T\times \times[y_{k+1/k+1}+S_{k+1/k+1}\ \ f_w(k)w(k)], \quad (20)$$

$$S_{k/k+1}=f_x^T(k)\,[I-f_w(k)\,K_B(k)]^T\times \times S_{k+1/k+1}\,f_x(k), \quad (21)$$

Remember: $y_{k/k}, w_B, S_{k+1/k+1}$ i $K_B(k)$.

3. Calculate the initial conditions for smoothing in the forward direction:

$$¢\,x(0)=\left[S_{0/1}+P_0^{-1}\right]^{-1}\left\{y_{0/1}+P_0^{-1}[x_0-x(0)]\right\},$$

$$x_+(0)=x_-(0)+¢\,x(0), \quad (22)$$

Where $x_+(0), x_-(0)-$ previous and the new initial value.

For $i=0,...,N-1$, compute

$$w_+(k)=w(k)+¢\,x(k)= =w_B(k)-K_B(k)[f_x(k)¢\,x(k)-f_w(k)w(k)], \quad (23)$$

$$¢\,x(k+1)=f_x(k)\,¢\,x(k)+f_w(k)[w_+(k)-w(k)], \quad (24)$$

$$ª(k)=S_{k/k}¢\,x(k)-y_{k/k}. \quad (25)$$

4. Perform iterations until until the change of values $x(0)$ and $w(k)$ will not be "enough" small, ie criterion J reaches the minimum value. From equation (5), after the solution of the problem of smoothing variation δJ must be zero. But:

$$¢\,J=\frac{\partial J}{\partial ¯}¢¯\equiv J-¢¯. \quad (26)$$

Then the stationary point of the product $J-¢¯$ must also match zero. However, given the availability of methodological and computational errors, the last piece will not be exactly zero, and therefore need to find a new vector of evaluation parameters ¯, reducing the value of the criterion as J. Using the adjusted variables $\hat{\delta}(t), w(t)$ and $\hat{\lambda}(t)$, can calculate the gradient criterion J with respect to the parameter vector ¯ with the following functions:

$$\frac{\partial \bar{J}}{\partial\theta(i)}=\sum_{j=1}^{p}\frac{\theta(j)-\theta_0(j)}{\Theta_0(i,j)}-$$
$$-\sum_{k=1}^{N}\hat{v}^T(t_k)R^{-1}E\frac{\partial h_m\left[\hat{x}(t_k),u(t_k),w(t_k),t,\theta\right]}{\partial\theta(i)}+$$
$$+\int_{t_0}^{t_f}\hat{\lambda}(t)\frac{\partial f_c\left[\hat{x}(t_k),u(t_k),w(t_k),t,\theta\right]}{\partial\theta(i)}dt;\quad i=1,\ldots,p. \quad (27)$$

where E - matrix of dimension ($m\times m$),whose elements are defined as $E_i(j,k)=1$, if $j=k=i$, and $E_i(j,k)=0$ otherwise. It is assumed that the weight matrix n is block-diagonal with blocks n¯, n_E i n_b, and the matrix n_E and n_b - diagonal.

Evaluation gradient matrix and updating quality criterion with respect to the parameter vector will perform with dvoranhovoyi procedure. let $¯_i$ and $\nabla_¯ J_i$ - gradient vector of parameters and quality criteria with respect to the parameter vector for $i-$й iteration. New estimates of the parameters are calculated using the following procedure kvazin'yutonivskoyi [2]:

$$¯_{i+1}=¯_i-\sim_i B_i^{-1}\nabla_¯ J_i, \quad (28)$$

where B_i - Hessian estimate for the vector $¯_i$. scalar $\sim_i$ - weighting (step size), which provides the convergence criterion as to minimum along the search direction, which is given by $-B_i^{-1}\nabla_¯ J_i$.

Gradient as a criterion for the new parameters $¯_{i+1}$ defined as $\nabla_¯ J_i$. Growth parameters and gradient vector is given by:

$$p_i=¯_{i+1}-¯_i,\quad q_i=\nabla_¯ J_{i+1}-\nabla_¯ J_i. \quad (29)$$

Evaluation Hessian B_i updated using two outer products of vectors p_i and q_i. This matrix is updated B_i accordance with the expression

$$B_{i+1}=B_i+\frac{q_i q_i^T}{q_i^T p_i}-\frac{B_i p_i\ p_i^T B_i^T}{B_i^T p_i^T p_i}. \quad (30)$$

In order to show the peer property of this procedure, we write equation (30) as:

$$B_{i+1}=B_i+\frac{q_i q_i^T}{q_i^T p_i}+\sim_i\frac{\left(\nabla_¯ J_i\right)\left(\nabla_¯ J_i\right)^T}{\left(\nabla_¯ J_i\right)^T p_i}. \quad (30)$$

Where $\sim_i$ - scalar which sets the step size for the quasi-Newton procedure (28). From equation (31) we can conclude that a peer update to occur when the update of the gradient vector q_i place simultaneously with the update vector $\nabla_¯ J_i$.

The initial value of the matrix B_0 You can specify any positive definite symmetric matrix. Very often this is done using the identity matrix, is the first update of the parameters in the direction of steepest descent. In this algorithm, double rank procedure is used as part of the quasi-Newton procedure for updating the parameters of the process.

Finally, the linear identification algorithm can be summarized as follows:

1) to set parameters ‾ , obtained in the previous iteration (or initial conditions), solve the problem of nonlinear smoothing to calculate the smoothed time series for classes and functions that perturb using the above algorithm for nonlinear smoothing. Necessary to calculate and evaluate quality criterion J_0 , defined by equation (3);

2) calculate the gradient estimate J with respect to the parameters ‾ using equations (27);

3) update the parameter vector ‾ using the quasi-Newton procedure (28), using it for reference Hessian double-rank algorithm updates the parameters;

4) repeat the algorithm as long as the quality criterion reaches a minimum value.

Conclusions. The resulting algorithm allows to construct nonlinear mathematical model for stationary and non-stationary processes. In particular, the authors see a promising application of the algorithm for creating a mathematical model of a specific production process blocks of transparent quartz glass.

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АЛГОРИТМ ИДЕНТИФИКАЦИИ НЕЛИНЕЙНЫХ СИСТЕМ ПО ИЗМЕРЕННЫМ ДАННЫМ

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Использование процедуры нелинейного сглаживания, оценивания градиента критерия качества, квазиньютоновской процедуры, дает возможность достичь минимума функционала качества. Для решения задачи достижимого сглаживания использован нелинейный обратный информационный фильтр.

Полученный алгоритм идентификации нелинейных технических систем может быть использован для построения математических моделей стационарных и нестационарных технологических процессов.

Ключевые слова: *процедура, критерий качества, функционал качества, алгоритм идентификации, информационный фильтр, стационарный процесс, нестационарный процесс, нелинейная система.*

АЛГОРИТМ ІДЕНТИФІКАЦІЇ НЕЛІНІЙНИХ СИСТЕМ ЗА ВИМІРЯНИМИ ДАНИМИ

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Використання процедури нелінійного згладжування, оцінювання градієнта критерія якості, квазіньютоновської процедури, дає можливість досягти мінімума функціонала якості. Для вирішення задачі згладжування використаний нелінійний зворотній інформаційний фільтр.

Отриманий алгоритм ідентифікації нелінійних технічних систем може бути використаний для побудови математичних моделей стаціонарних та нестаціонарних технологічних процесів.

Ключові слова: *процедура, критерій якості, функціонал якості, алгоритм ідентифікації, інформаційний фільтр, стаціонарний процес, нестаціонарний процес, нелінійна система.*