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PACKING HOMOTHETIC SPHEROIDS INTO A LARGER SPHEROID WITH THE JUMP ALGORITHM

The article considers a mathematical model of the optimization problem of packing homotethic spheroids (spheres - in particular case) into a larger spheroid (sphere - in particular case). Sphere radii are supposed to be variable. A new algorithm to derive starting points belonging to the feasible region of the problem is offered. According to the jump algorithm solving the problem is reduced to solving a sequence of mathematical programming problems yielding objective improvements. A solution strategy consisting of four stages is proposed. The first stage involves formation of starting points and computation of local minima. The second stage fulfills continuous transition from one local minimum to another. The third stage realizes reduction of the solution space dimension. The fourth stage rearranges sphere pairs to refine the objective. We provide a number of numerical results both for spheres and spheroids.

Key words: *packing, sphere, spheroid, optimization, jump algorithm.*

Introduction

Problems of packing unequal spheres and spheroids have applications in medicine and biology.

Medical applications of the unequal sphere packing problem in radiosurgery are studied in [1-3]. The gamma-rays are focused on a common center, creating a spherical volume of high radiation dose. A key geometric problem in gamma knife treatment planning is to fit balls into a 3*D* irregular-shaped tumor. In this situation, overlapping balls may cause overdose, and a low packing density may result in underdose and a non-uniform dose distribution.

In biological sciences, the study of chromosome arrangements and their functional implications is an area of great current interest [4]. The territory occupied by each chromosome can be modeled as an spheroid, different chromosomes giving rise to spheroids of different size. The enclosing spheroid represents a cell nucleus, the size and shape of which differs across cell types. Overlap between chromosome territories has biological significance: it allows for interaction and co-regulation of different genes.

Sutou and Day [3] propose a global optimization approach to unequal sphere packing problems. The optimization problem is formulated as a nonconvex optimization problem with quadratic constraints and a linear objective function. Paper [5] offers a mathematical optimization method for packing unequal spheres into a cuboid based on the decremental neighborhood method and a local optimization method. An algorithm to pack unequal spheres in a larger sphere using tabu search, the quasi-human basin-hopping strategy and the Broyden–Fletcher–Goldfarb–Shanno method is developed in [6]. A set of examples are calculated.

In this paper, we adopt the jump algorithm (JA) developed for unequal circle packing [7] to pack unequal sphere in a sphere of minimal radius. JA allows to transit from one local minimum point to another one so that the larger sphere radius decreases.

First we consider the packing problem of spheres.

All given spheres

$S_i = \{v_i \in \mathbf{R}^3 : (x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2 - \hat{r}_i^2 \le 0\}$

where $v_i = (x_i, y_i, z_i)$ are center coordinates of S_i, $i \in I = \{1, 2, ..., n\}$, have to be packed into a sphere

$$S = \{(x, y, z) \in \mathbf{R}^3 : x^2 + y^2 + z^2 - R^2 \le 0\},$$

We asume that the radius R ($R \gg \hat{r}_i$, $i \in I$) of S is variable.

A sphere S_i translated by a vector v_i and a sphere S with variable size R are denoted by $S_i(v_i)$ and $S(R)$ respectively. A vector $v = (v_1, v_2, ..., v_n) \in \mathbf{R}^{3n}$ defines an arrangament of S_i, $i \in I$, in the Euclidean 3-D space $\mathbf{R}^3$.

Without loss of generality, we suppose that

$$\hat{r}_1 \le \hat{r}_2 \le ... \le \hat{r}_n, \quad \hat{r}_1 < \hat{r}_n. \qquad (1)$$

Then we consider the packing problem of spheroids.

Each homothetic spheroid $E_i(v_i)$, $i \in I$, is generated by rotation of an ellipse of semi-axes a_i and b_i, $a_i > b_i$, along the axis of revolution *OX*, therefore we assume that third semi-axe is defined as $c_i = b_i$ and $v_i = (x_i, y_i, z_i)$ is a translation vector. All spheroids have to be packed into an spheroid

$$E = \{(x, y, z) \in \boldsymbol{R}^3 : \frac{x^2}{\lambda A} + \frac{y^2}{\lambda B} + \frac{z^2}{\lambda C} - 1 \le 0\},$$

where *A*, *B*, *C* is semi-axes of container and λ is a coefficient of homothety.

Problem. Find a vector v ensuring a packing of spheres $S_i(v_i)$, $i \in I$, (spheroides $E_i(v_i)$) without their mutual overlappings within the sphere S of the minimal radius R^* (spheroid E with the minimal coefficient of homothety λ^*).

Mathematical model

A mathematical model of the problem can be stated as

$$\mu^* = \min \mu, \text{ s.t. } Y = (\mu, v) \in W \subset \mathbf{R}^{3n+1} \quad (2)$$

where

$$W = \{Y \in \mathbf{R}^{3n+1} : \Phi_{ij}(v_i, v_j) \geq 0,\ 0 < i < j \in I, \ \Phi_i(v_i, R) \geq 0,\ i \in I\}, \quad (3)$$

$$\mu = \begin{cases} R \text{ in the case of sphere packing,} \\ \lambda \text{ in the case of ellipsoids packing.} \end{cases}$$

Here $\Phi_{ij}(v_i, v_j) = (x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2 - (\hat{r}_i + \hat{r}_j)^2 \geq 0$ ensures non-overlapping spheres S_i and S_j, $\Phi_i(v_i, R) \geq 0$ provides a placement of $S_i(v_i)$ within $S(R)$ and $\Phi_i(v_i, R) = -x_i^2 - y_i^2 - z_i^2 + (R - \hat{r}_i)^2$. The condition of non-overlapping homotetic spheroids is

$$\Phi_{ij}(v_i, v_j) = \frac{(x_j - x_i)^2}{(a_i + a_j)^2} + \frac{(y_j - y_i)^2}{(b_i + b_j)^2} + \frac{(z_j - z_i)^2}{(c_i + c_j)^2} - 1$$

and the condition of placement spheroid $E_i(v_i)$, $i \in I$, in container $E(\lambda)$ is

$$\Phi_i(v_i, \lambda) = 1 - \frac{(x_i)^2}{(\lambda A - a_i)^2} - \frac{(y_i)^2}{(\lambda B - b_i)^2} - \frac{(z_i)^2}{(\lambda C - c_i)^2}$$

if spheroids E_i, $i \in I$, and E are homothetic.

The mathematical model (2)–(3) possesses the same characteristics as that of the mathematical models considered in [8], i.e. local minima are reached at extreme points of W, the matrix of the inequality system in (3) is strongly sparse, the problem stated is NP-hard. Thus, in general, a global minimum of the problem can be reached but only in a theoretical manner.

Consequently, for successfully solving problem (2)–(3) it needs to be able to construct starting points belonging to the feasible region W, to compute local minima and to derive an effective non-exhaustive search for local minima.

In what follows we consider the sphere packing. One may consider homothetic spheroids instead of spheres by replacing spheres S_i by spheroids E_i, $i \in I$, and radius R by coefficient of homothety λ.

Generating starting points and searching for local minima

Primarily, we suppose that radii r_i of spheres S_i, $i \in I$, are variables and form a vector $r = (r_1, r_2, ..., r_n) \in \mathbf{R}^n$. In the case, the inequalities in system (3) take the form

$$\Phi_{ij}(v_i, v_j, r_i, r_j) \geq 0,\ 0 < i < j \in I,\ \Phi_i(v_i, r_i, R) \geq 0,\ i \in I.$$

Thus, $X = (v, r) \in \mathbf{R}^{4n}$ is the vector of all variables.

Let $R = R^0 > 0$. We form a point $X^0 = (v^0, 0)$ so that $v_i^0 \in P(R^0)$, $i \in I$, i.e. points $v_i^0 \in \mathbf{R}^3$, $i \in I$, are randomly thrown in the sphere $P(R^0)$.

In order to construct a point $(v, R^0) \in W$ on the ground of the point (v^0, R^0) we solve the problem

$$\Psi(\hat{r}) = \max \sum_{i=1}^{n} r_i, \text{ s.t. } X = (v, r) \in D \subset \mathbf{R}^{4n}, \quad (4)$$

where

$$D = \{X \in \mathbf{R}^{4n} : \Phi_{ij}(v_i, v_j, r_i, r_j) \geq 0,\ 0 < i < j \in I, \ \Phi_i(v_i, r_i, R^0) \geq 0, \phi_i(r_i) = \hat{r}_i - r_i \geq 0,\ r_i \geq 0,\ i \in I\}. \quad (5)$$

It follows from the construction of X^0 that $X^0 \in D$. So taking starting point X^0 we tackle problem (4)–(5) and obtain a local maximal point $\hat{X} = (\hat{v}, \hat{r})$. Note that in addition to the characteristics of problem (2)–(3), problem (4)–(5) possesses the properties:

(i) Inequalities $\phi_i(r_i) \geq 0$, $i \in I$, in (5) imply that if

$$\Psi(\hat{r}) = \sum_{i=1}^{n} \hat{r}_i = \sum_{i=1}^{n} \hat{r}_i = b,$$

then $\hat{r} = \hat{r}$ and spheres S_i, $i \in I$, are packed into $S(R^0)$. This means the point $\hat{X}$ is a global maximal point of problem (4)–(5).

(ii) If $\Psi(\hat{r}) < b$, and $\hat{X}$ is a global maximal point of problem (4)–(5), then spheres S_i, $i \in I$, can not be packed into $P(R^0)$.

Depending on R^0 two cases can be found:
i. $\Psi(\hat{r}) = b$ and ii. $\Psi(\hat{r}) < b$.

It follows from item (i) that $(\hat{v}, R^0) \in W$ if $\Psi(\hat{r}) = b$. The point $(\hat{v}, R^0)$ is not in the general case a local minimal point of problem (2)–(3). So, taking starting point $(\hat{v}, R^0)$ we calculate a local minimal point $(\breve{v}, \breve{R})$ of problem (2)–(3) regarding the interaction of the placed spheres.

Let $\Psi(\hat{r}) < b$. Then we either choose $X^0 = (v^0, 0)$ in a random way again and solve sequentially problems (4)–(5) and (2)–(3) or try to execute a transition from $\hat{X}$ to $\hat{\hat{X}}$ so that $\Psi(\hat{\hat{r}}) > \Psi(\hat{r})$.

Solution strategy

To compute a local minimum of problem (2)–(3) we derive a step by step procedure that includes tackling problems (2)–(3) and (4)–(5).

Primarily, we choose $R = R^0$ guaranteeing an arrangement of spheres S_i of radii $\hat{r}_i$, $i \in I$, into the sphere $S(R^0)$. Then we take a point $X^0 = (v^0, 0)$ in a random way so that $v_i^0 \in S(R^0)$, $i \in I$, and, using starting point X^0, solve problem (4)–(5). As a result, a local maximal point $\hat{X} = (\hat{v}, \hat{r})$ is found. Because of the choice $R = R^0$ we always have $\Psi(\hat{r}) = b$, i.e. $\hat{r} = \hat{r} = (\hat{r}_1, \hat{r}_2, ..., \hat{r}_n)$. This means that $(\hat{v}, R^0) \in W$. So, taking a starting point $(\hat{v}, \mu^0)$, we solve problem (2)–(3) and calculate a local minimal point $(\breve{v}^0, \breve{R}^0)$.

The jump algorithm (JA) permits to execute a continuous transition from a local maximal point of problem (4)–(5) to another one ensuring an increase of $\Psi(r)$. Let $\hat{X} = (\hat{v}, \hat{r})$ be a local maximal point of the problem (4)–(5) and $\Psi(\hat{r}) = \sum_{i=1}^{n} \hat{r}_i < b$, i.e. at least one of the inequalities $\hat{r}_i - \hat{r}_i \geq 0$, $i \in I$, is not active. We formulate the auxiliary problem

$$\max V(r) = \sum_{i=1}^{n} r_i^3, \text{ s.t. } X \in M \subset \mathbf{R}^{4n}, \qquad (6)$$

$$M = \{X \in \mathbf{R}^{4n} : \Phi_{ij}(v_i, v_j, r_i, r_j) \geq 0,\ 0 < i < j \in I,$$
$$\Phi_i(v_i, r_i, R^0) \geq 0,\quad \psi_{1i}(r_i) = r_{max} - r_i \geq 0, \qquad (7)$$
$$\psi_{2i}(r_i) = -r_{min} + r_i \geq 0,\ i \in I\},$$

where $r_{max} = \max\{\hat{r}_i,\ i \in I\}$ and $r_{min} = \min\{\hat{r}_i,\ i \in I\}$.

Now, let $(\breve{v}^0, \breve{R}^0)$ be a local minimal point of problem (2)–(3). We compute

$$r_i^\lambda = \hat{r}_i - \left(\frac{1}{2}\right)^{\lambda+2} \hat{r}_i = \hat{r}_i (1 - (0.5)^{\lambda+2}),\ i \in I, \lambda = 0,1,....$$

and assume that sphere radii are equal to r_i^λ, $i \in I$. Then problem (2)–(3) takes the form

$$\breve{R} = \min R \text{ s.t. } Y = (v, R) \in W^\lambda \subset \mathbf{R}^{3n+1}, \qquad (8)$$

where
$$W^\lambda = \{Y \in \mathbf{R}^{3n+1} : \Phi_{ij}^\lambda(v_i, v_j) \geq 0,$$
$$0 < i < j \in I,\ \Phi_i^\lambda(v_i, R) \geq 0,\ i \in I\},$$
$$\Phi_{ij}^\lambda(v_i, v_j) = \left\| v_i - v_j \right\|^2 - (r_i^\lambda + r_j^\lambda)^2,$$
$$\Phi_i^\lambda(v_i, \mu) = -x_i^2 - y_i^2 - z_i^2 + (R - r_i^\lambda)^2.$$

Since $r_i^\lambda < \hat{r}_i$, $i \in I$, then the point $(\breve{v}^0, \breve{R}^0) \in W^\lambda$ and $(\breve{v}^0, \breve{R}^0)$ is not a local minimal point of problem (8). So, taking starting point $(\breve{v}^0, \breve{R}^0)$, we solve problem (12) and define a local minimal point $(\breve{\breve{v}}^0, \breve{\breve{R}}^0)$. Since $\sum_{i=1}^{n} r_i^\lambda < b$, then, tackling problem (4)–(5) for starting point $X^0 = (\breve{\breve{v}}^0, r^\lambda) \in D$, we compute a local maximal point $\hat{X}^\lambda = (\hat{v}^\lambda, \hat{r}^\lambda)$. Two cases are possible: $\Psi(\hat{r}^\lambda) = b$ and $\Psi(\hat{r}^\lambda) < b$. If $\Psi(\hat{r}^\lambda) = b$, then $\hat{r}_i^\lambda = \hat{r}_i$, $i \in I$, and hence $(\hat{v}^\lambda, \breve{\breve{R}}^0) \in W$. Since the solution spaces of problems (2)–(3) and (4)–(5) are different, then $(\hat{v}^\lambda, \breve{\breve{R}}^0)$ in general is not a local minimal point of problem (2)–(3). So, taking starting point $(\hat{v}^\lambda, \breve{\breve{R}}^0)$, we solve problem (2)–(3). As a result, a new local minimum point $(\breve{v}^1, \breve{R}^1)$ is computed. In this case a local minimal point $(\breve{\breve{v}}^1, \breve{\breve{R}}^1)$ of problem (8) for the starting point $(\breve{v}^1, \breve{\mu}^1)$ is computed again, and so on until $\Psi(\hat{r}^\lambda) < b$ becomes, i.e. we have $\sum_{i=1}^{n} \hat{r}^\lambda < b$, $\hat{X}^\lambda = (\hat{v}^\lambda, \hat{r}^\lambda)$ and $(\hat{v}^\lambda, \breve{\mu}^\lambda) \notin W$ after λ iterations.

In this situation ($\Psi(\hat{r}^\lambda) < b$) we compute the steepest ascent vector Z^0 at the point $\hat{X}^\lambda$ for problem (6)–(7), define $\gamma = m$, construct a point $X^m = (v^m, r^m) \in D$ according to (8) and the ascending sequence (see (1))

$$r_{i_1}^m \leq r_{i_2}^m \leq .... \leq r_{i_n}^m. \qquad (9)$$

Since $V(r^m) > V(\hat{r})$ may occur, then, making use of sequence (9), we compute $r_{i_j}^{m0} = \min\{r_{i_j}^m, \hat{r}_j\}$, $j \in I$. This ensures the inequality $V(r^{m0}) \leq V(\hat{r})$ where $r^{m0} = (r_1^{m0}, r_2^{m0}, ..., r_n^{m0})$. Based on sequence (9), we construct two points: $\tilde{\tilde{X}}^m = (\tilde{v}^m, \tilde{\tilde{r}}^m)$ where $\tilde{v}_j^m = v_{i_j}^m$, $\tilde{\tilde{r}}_j^m = r_{i_j}^{m0}$, $j \in I$, and point $\tilde{X}^m = (\tilde{v}^m, \tilde{r}^m)$ where $\tilde{v}_j^m = v_{i_j}^m$, $\tilde{r}_j^m = r_{i_j}^m$, $j \in I$.

If $V(\hat{r}) > V(\tilde{\tilde{r}}^m) > V(\hat{r}^\lambda)$, then the new steepest ascent vector Z^0 at the point $\tilde{X}^m$ for problem (6)–(7) is calculated. Taking $\hat{X} = \tilde{X}^m$, we build a new point $X^m = \hat{X} + (1/2)^m Z^0$ and construct new points $\tilde{\tilde{X}}^m = (\tilde{v}^m, \tilde{\tilde{r}}^m)$ and $\tilde{X}^m = (\tilde{v}^m, \tilde{r}^m)$ in accordance with sequence (13), and so on. The iterative process is continued until either $V(\tilde{\tilde{r}}^m) = V(\hat{r})$ or $V(\tilde{\tilde{r}}^m) \leq V(\hat{r}^\lambda) < V(\hat{r})$ occurs.

If $V(\tilde{\tilde{r}}^m) = V(\hat{r})$, i.e. $\tilde{\tilde{r}}_i^m = \hat{r}_i$, $i \in I$, then taking starting point $(\tilde{v}_i^m, \breve{R}^\lambda)$, we tackle problem (2)–(3) and calculate a new local minimal point $(\breve{v}^0, \breve{R}^0)$. The process is repeated until $V(\tilde{\tilde{r}}^m) \leq V(\hat{r}^\lambda) < V(\hat{r})$ becomes.

Reduction of the solution space dimension is realized by means of sequencial fixing initial values of sphere radii without fixing their center coordinates.

To the aim we take the point $\tilde{X}^m$, single out $\tilde{r}_i^m > \hat{r}_i$, $i \in I^1 \subset I$, and calculate

$$\delta^1 = 4/3 \cdot \max\{(\tilde{r}_i^m)^3 - (\hat{r}_i)^3,\ i \in I^1\}.$$

Let δ^1 correspond to radius $\hat{r}_t$. Hence, δ^1 is an increment of volume of the sphere S_t when varying its radius from $\hat{r}_i$ to r_t^m. This means that if $r_t = \hat{r}_t$, then there is a volume reserve around the sphere S_t. In order to use the reserve we fix radius $r_t = \hat{r}_t$ and derive a point $\tilde{X}^{m1} = (\tilde{v}^m, \tilde{r}_1^m, \tilde{r}_2^m, ..., \tilde{r}_{t-1}^m, \tilde{r}_{t+1}^m, ..., \tilde{r}_n^m) \in \mathbf{R}^{4n-1}$, i.e. r_t is no longer a variable and, hence, the dimension of the solution spaces D and M decreases by 1. Then, taking starting point $\tilde{X}^{m1}$ we realize JA in the space $\mathbf{R}^{4n-1}$. If

$$V(\tilde{r}^{m1}) = V(\hat{r}) = \sum_{i=1}^{n} (\hat{r}_i)^3,$$

then we take starting point $(\tilde{v}_i^{m1}, \breve{R}^{\lambda})$, tackle problem (2)–(3) and calculate a new local minimal point $(\breve{v}^0, \breve{R}^0)$. If $V(\tilde{r}^{m1}) < V(\hat{r})$, we continue to reduce the solution space dimension. If $V(\tilde{r}^{m1}) < V(\hat{r})$ and all spheres S_i, $i \in I^1 \subset I$, are exhausted, we increase λ by 1 and realize JA again. The process is continued until $(1/2)^{\lambda+2}\hat{r}_i < \varepsilon$, $i \in I$.

After that we take the local minimal point $(\breve{v}^0, \breve{R}^0)$ of problem (2)–(3) and rearrange sphere pairs whose radii are slightly distinguished. This allow to improve the objective value of problem (2)–(3). An algorithm executing such rearrangements is described in [9]. In order to obtain a good approximation to a global minimum of problem (2)–(3) we repeat the step-by-step procedure consisting of the construction of a starting point and the search for a local minimum of problem (2)–(3) with JA ν times. As a result local minimum points (v^{*t}, R^{*t}), $t \in T = \{1, 2, ..., \nu \le 10\}$ are computed.

Then we single out a local minimal point (v^{*0}, R^{*0}) corresponding to $R^{*0} = \min\{R^{*t}, t \in T\}$. The point (v^{*0}, R^{*0}) is taken as an approximation to a global minimum of problem (2)–(3).

Numerical examples

In order to verify effectiveness of JA, we solve the benchmark instances for packing spheres into a sphere considered in [6]. Moreover, we solve examples for packing spheroids.

We compare results of packing spheres $r_i = i$, $i = 1, 2, ..., n$, into a larger sphere calculated by the algorithm [6] and JA. In Table 1, the first and the second column give example names and numbers of spheres to be packed. The third and the forth column summarize the best values of radii obtained in [6] (R_H) and with JA (R^{*0}). The percentage of improvement of JA against the best known results is shown in the last column. The calculation time by means of JA varied from 10 seconds to 12 hours depending on the number of spheres.

Table 1

Results of packing spheres of radii $r_i = i$ into a sphere

Example	n	R_H	R^{*0}	improve
ZHXF16	16	33.6582	**33.6572**	0
ZHXF17	17	36.2030	**36.2021**	0
ZHXF18	18	**38.8463**	38.8467	-0
ZHXF19	19	**41.5452**	41.5462	-0
ZHXF20	20	44.2737	**44.2557**	0.04
ZHXF21	21	47.0342	**47.0332**	0
ZHXF22	22	49.9068	**49.8666**	0.08
ZHXF23	23	52.8368	**52.7425**	0.18
ZHXF24	24	55.7546	**55.5782**	0.32
ZHXF25	25	58.4684	**58.4665**	0
ZHXF26	26	61.4745	**61.3883**	0.14
ZHXF27	27	64.4854	**64.4141**	0.11
ZHXF28	28	67.4837	**67.4173**	0.1
ZHXF29	29	70.5257	**70.3911**	0.19
ZHXF30	30	73.4813	**73.3704**	0.15
ZHXF31	31	76.5336	**76.5057**	0.04
ZHXF32	32	79.8018	**79.6075**	0.24
ZHXF33	33	83.1967	**82.8314**	0.44
ZHXF34	34	86.2430	**85.9206**	0.37
ZHXF35	35	89.3454	**89.1536**	0.21
ZHXF40	40	-	**105.6146**	-
ZHXF50	50	-	**140.7613**	-
ZHXF60	60	-	**178.1920**	-
ZHXF70	70	-	**217.0801**	-
ZHXF80	80	-	**258.4230**	-
ZHXF90	90	-	**300.9910**	-
ZHXF100	100	-	**345.5416**	-

Illustration for ZHXF33 is shown in fig. 1.

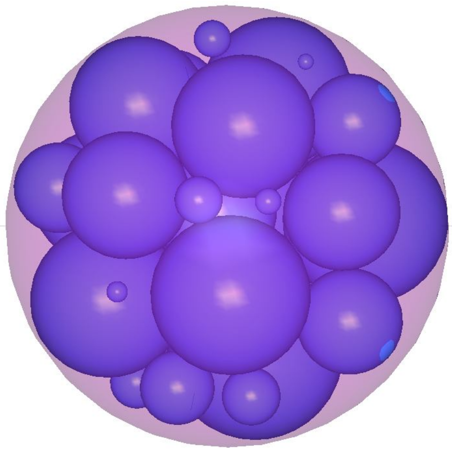

Fig. 1. Example ZHXF33

We provide results of packing homothetic spheroids (spheroids) E_i, $i \in I$, into a spheroid. For $n = 15$ spheroids $\lambda^* = 0.22659$ and sizes $(A^*, B^*, C^*) = (16.9948, 5.6649, 5.6649)$. In case $n = 20$ $\lambda^* = 0.321$ and sizes $(A^*, B^*, C^*) = (39.0005, 13.0002, 13.0002)$. For $n = 30$ ellipses $\lambda^* = 0.1421$ and sizes $(A^*, B^*, C^*) = (21.3155, 7.10516, 7.10516)$. $\lambda^* = 0.4578$ and sizes $(A^*, B^*, C^*) = (160, 55, 55)$ for $n = 50$ spheroids.

Illustration for example of packing 15(30) spheroids is shown in fig. 2. The Interior Point Optimizer (IPOPT) exploiting information on Jacobians and Hessians [10], and the concept of ε-active inequalities [8,11] are used when tackling problems (2)–(3), (4)–(5) and (6)–(7).

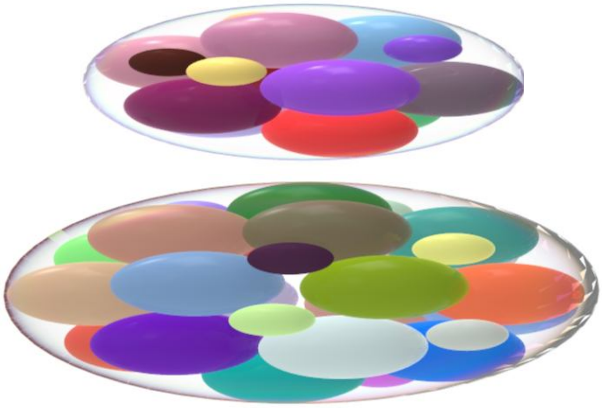

Fig. 2. Examples of packing 15 and 30 spheroids

Conclusion

Algorithm JA is effective to solve the sphere and homothetic spheroids packing problems and improves known results for benchmark.

The algorithm is especially effective if neighbor initial radii of spheres (half-axes of spheroids) in the sequence (1) are slightly distinguished.

A decrease of the problem dimension by means of sequential fixing sphere radius values sometimes permits to improve the objective values of problem (2)–(3).

The algorithm can be adopted to solve the problem of packing spheres and homothetic spheroids in containers of more compex geometric shapes.

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УПАКОВКА ГОМОТЕТИЧНИХ СФЕРОЇДІВ У БІЛЬШОМУ СФЕРОЇДІ ЗА ДОПОМОГОЮ АЛГОРИТМУ СТРИБКА (JUMP ALGORITHM)

О.М. Хлуд, Г.М. Яськов

У статті розглядається математична модель задачі оптимальної упаковки гомотетичних сфероїдів (сфер у конкретному випадку) більший сфероїд (сфера у конкретному випадку). Радіуси сфер мають бути змінними. Запропоновано новій алгоритм знаходження стартових точок, що належать області допустимих значень. З використанням алгоритмн стрибка, вирішення задачі зводиться до розв'язання послідовності задач математичного програмування, що дає об'єктивні покращення. Запропонована стратегія розв'язання складеться с чотирьох етапів. Перший етап включає формування стартових точок та обчислення локального мінімуму. Під час другого етапу виконуються безперервній перехід від одного локального мінімуму до іншого. На третьому етапі відбувається зменшення розмірності простору рішення. На четвертому етапі пари сфер перебудовуються, щоб отримати задані. Ми приводимо результати чисельних експериментів для сфер та сфероїдів.

Ключові слова: *упаковка, куля, сфероїд, оптимізація, алгоритм стрибка.*

УПАКОВКА ГОМОТЕТИЧНЫХ СФЕРОИДОВ В БОЛЬШЕМ СФЕРОИДЕ С ПОМОЩЬЮ АЛГОРИТМА ПРЫЖКА (JUMP ALGORITHM)

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В статье рассматривается математическая модель задачи оптимальной упаковки гомотетичных сфероидов (сфер в конкретном случае) больше сфероид (сфера в конкретном случае). Радиусы сфер должны быть переменными. Предложено новой алгоритм нахождения стартовых точек, принадлежащих области допустимых значений. С использованием алгоритма скачка, решение задачи сводится к решению последовательности задач математического программирования, дает объективные улучшения. Предложенная стратегия решение сложится с четырех этапов. Первый этап включает формирование стартовых точек и вычисления локального минимума. Во время второго этапа выполняются непрерывной переход от одного локального минимума в другой. На третьем этапе происходит уменьшение размерности пространства решения. На четвертом этапе пары сфер перестраиваются, чтобы получить заданные. Мы приводим результаты многочисленных экспериментов для сфер и сфероидов.

Ключевые слова: *упаковка, шар, сфероид, оптимизация, алгоритм прыжка.*