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TALKING THE DEVELOPMENT OF DEFORMATION INTO DETERMINING NORMAL AND TANGENTIAL CONTACT STRESSES AT THE FORGE

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Summari*: the methods of determination of contact normal and tangential stresses is described with allowance for deformation development against time. The obtained formulae characterizing the dynamics of deformation center change against time and these formulae are used for determination of contact normal and tangential stresses. The discrepancy between the theoretical design results and the experimental data is 1.7 – 4 % that confirms the possibility of its use for determination of contact normal and tangential stresses.*
Keywords: *stress, deformation, rolling.*

The actuality of the development and implementation of low-waste technological processes of forgings making from aluminum, titan, magnesium alloys, high-alloy corrosion-resistant, heat-resistant, high-temperature and other steels with forge-rolling process using at enterprises of aircraft industry is stipulated by the considerable application of these alloys in this branch products, increased use of metal (Metal Utilization Factor is 0.15 – 0.3), high labor intensity, long cycle of high quality roll forgings (as a rule 2 – 3 roll forgings with intermediate heating operations, flash cutting, pickling, fettling) and tasks for the improvement of metal-saving technologies. The wide use of the alloys listed above is determined by their technical, physical and mechanical features. They must have high static strength characteristics (ultimate strength, yield strength, shearing strength), satisfactory plasticity and thermomechanical characteristics that is necessary to take into consideration at the development of the technological processes of their hot deformation.

The existing methods (1, 2, 3 and others) for determination of contact normal and tangential stresses come to consideration of their distribution along deformation center. Such approach to the solution of the problem for their determination does not permit to take into consideration irregularity of deformation in transverse direction and influence of side out-contact zones adjacent to the actual deformation center. It is especially important to take these factors into consideration at determination of normal and tangential stresses at forge rolling (rolling) in calibers with various correlation of geometrical form of caliber and blanks under forge rolling. Besides, the dynamics of deformation center change is not taken into consideration.

This work purpose is the development of methods for determination of normal and tangential stresses in deformation center with allowance for the abovementioned factors.

Let's divide deformation center volume for a series of layers of dz thickness, Fig. 1.

Let's consider stresses in inclined sites of each layer, as it is shown in Fig. 2.

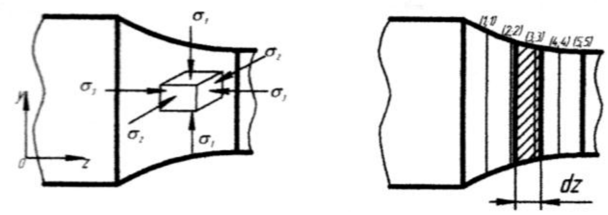


Fig. 1. Scheme of stressed state in transient zone of blank under forge rolling

Assuming that stressed and deformed state of each point in this layer will be determined by the following tensors:

At derivation of equations determining frictional force and pressure in deformation center, let's consider that the term "inclined" site means that the normal to the site coincides with no axis of the coordinate system selected by us; stresses on sides of elementary volume are distributed uniformly, and inclined site is considered as inclined cross-section in elementary parallelepiped.

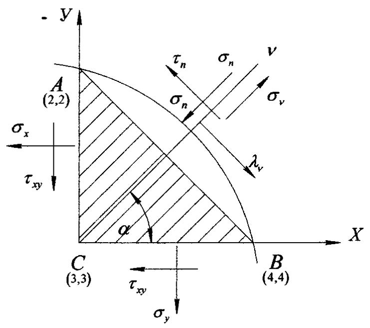


Fig. 2. Distribution of stresses in inclined site at flat stressed state

Let's consider stresses in inclined sites perpendicular to deformation center plane (Fig. 2). In ABC site which normal ν makes angle α with OX axis, normal σ_ν and tangential τ_ν stresses act. The stresses are distributed uniformly in d_z layer, face sides of ABC element are not loaded, because according to condition $\tau_{xz}= \tau_{yz} = 0$; $\sigma_z= 0$.

For determination of σ_ν and τ_ν values, let's consider ABC element balance condition. Let's project all stresses to ν normal direction:

$$\sigma_\nu \cdot \sigma_x \cdot \sigma_z = \sigma_x dy \cdot \cos\alpha \cdot dz + \sigma_y dx \cdot \sin\alpha \cdot dz + \tau_{xy}(dy \cdot \sin\alpha + dx \cdot \cos\alpha)dz \qquad (1)$$

Mass forces making influence on ABC element

$$p_x \cdot \frac{1}{2} dx \cdot dy \cdot dz, \qquad p_\nu \cdot \frac{1}{2} dx \cdot dy \cdot dz$$

are stresses of 2-order infinitesimal and are absent in the equation. Taking into consideration that $\sin\alpha = \frac{dx}{ds}$, $\cos\alpha = \frac{dy}{ds}$ from relationship (1)

$$\sigma_\nu = \sigma_x \cos^2\alpha + \sigma_y \sin^2 y + \tau_{xy} \sin 2\alpha$$

Analogously, projecting all stresses to τ_ν vector direction, we can find

$$\tau_\nu \cdot ds \cdot dz = \sigma_y dx \cos\alpha \cdot dz - \sigma_x dy \sin\alpha \cdot dz + \tau_{xy}(dy\cos\alpha - dx\sin\alpha)dz \text{ or}$$

$$\tau_\nu = 1/2(\sigma_y - \sigma_x)\sin 2\alpha + \tau_{xy}\cos 2\alpha .$$

As final normal and tangential stresses σ_ν and τ_ν in deformation center are directed to sides opposite to pressure and friction forces σ_n, τ_n, they will be determined by the following expressions:

$$\sigma_n = -(\sigma_x \cos^2\alpha + \sigma_y \sin^2\alpha + \tau_{xy}\sin 2\alpha); \qquad (2)$$

$$\tau_n = \frac{1}{2}[(\sigma_x - \sigma_y)\sin 2\alpha - \tau_{xy}\cos 2\alpha]. \qquad (3)$$

Replacing σ_x, σ_y, τ_{xz}, in expressions (2) and (3) σ_x, σ_y, τ_{xz}, with deformations, using generalized Hooke law (we will consider material as uncompressed, i.e. $\varepsilon_x, + \varepsilon_y + \varepsilon_y = 0$), we obtain:

$$T_\sigma = \begin{pmatrix} \sigma_x & \tau_{xy} & 0 \\ \tau_{xy} & \sigma_y & 0 \\ 0 & 0 & 0 \end{pmatrix}; \quad T_\varepsilon = \begin{pmatrix} \varepsilon_x & \frac{1}{2}\gamma_{xy} & 0 \\ \frac{1}{2}\gamma_{xy} & \varepsilon_y & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

$$\sigma_n = -(2G^*\varepsilon_x \cos^2\alpha + 2G^*\varepsilon_y \sin^2\alpha + G^*\gamma_{xy}\sin 2\alpha); \qquad (4)$$

$$\tau_n = G^*[(\varepsilon_x - \varepsilon_y)\sin 2\alpha - G^*\gamma_{xy}\cos 2\alpha], \qquad (5)$$

where G^* is shear modulus in plastic area.

Then using Cauchy equation: $\varepsilon_x = \frac{\partial u}{\partial x}$; $\varepsilon_y = \frac{\partial \upsilon}{\partial y}$; $\gamma_{xy} = \frac{\partial u}{\partial x} - \frac{\partial \upsilon}{\partial y}$,

after substitution of them to formulae (4) , (5) we obtain:

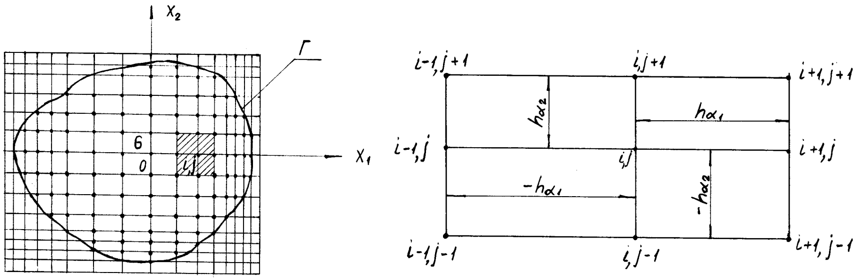
$$\sigma_n = -2G^*[\frac{\partial u}{\partial x}\cos^2\alpha + \frac{\partial \upsilon}{\partial y}\sin^2\alpha + \frac{1}{2}\left(\frac{\partial u}{\partial x} + \frac{\partial \upsilon}{\partial y}\right)\sin 2\alpha] \quad (6)$$

$$\tau_n = G^*[(\frac{\partial u}{\partial x} - \frac{\partial \upsilon}{\partial y})\sin 2\alpha - G^*(\frac{\partial u}{\partial x} - \frac{\partial \upsilon}{\partial y})\cos 2\alpha]\cdot \quad (7)$$

The formulae (6), (7) describe normal and tangential stresses on d_z elementary layer contour. For their digital realization, we use finite difference method with previous substitution of deformed Γ contour of G area with grid one (Fig. 3). Then contact normal and tangential stresses can be represented by the formulae:

$$\sigma_n = -2G^*\left(\frac{u_{i,j+1}-u_{i,j}}{2\,{}^{+}h_{\alpha_2}^{(i,j)}} - \frac{\upsilon_{i+1,j}-\upsilon_{i,j}}{2\,{}^{+}h_{\alpha_1}^{(i,j)}}\right)\sin 2\alpha + \frac{u_{i+1,j}-u_{i,j}}{{}^{+}h_{\alpha_1}^{(i,j)}}\cos^2\alpha + \frac{\upsilon_{i,j+1}-\upsilon_{i,j}}{{}^{+}h_{\alpha_2}^{(i,j)}}\sin^2\alpha; \quad (8)$$

$$\tau_n = G^*\left[\left(\frac{u_{i+1,j}-u_{i,j}}{{}^{+}h_{\alpha_1}^{(i,j)}} - \frac{\upsilon_{i,j+1}-\upsilon_{i,j}}{{}^{+}h_{\alpha_2}^{(i,j)}}\right)\sin 2\alpha + \left(\frac{u_{i,j+1}-u_{i,j}}{{}^{+}h_{\alpha_2}^{(i,j)}} - \frac{\upsilon_{i+1,j}-\upsilon_{i,j}}{{}^{+}h_{\alpha_1}^{(i,j)}}\right)\cos 2\alpha\right]\cdot \quad (9)$$



a *b*

Fig. 3. Curvilinear simple connected finite area:
a – general view; b – elementary cell (i, j) of investigated area

For arbitrary contour of deformed layer ΔZ, the expressions (8), (9) assume the form:

$$\sigma_n^* = (\sum_{n=1}^{k}\sigma_n^k)l_i\Delta Z_i; \quad (10)$$

$$\tau^* = (\sum_{n=1}^{k}\tau_n^k)l_i\Delta Z_i, \quad (11)$$

where l_i – caliber ark length; ΔZ_i- deformed layer thickness.

As an example of contact normal and tangential stresses, let's consider forge rolling of blanks of AK6 alloy of Ø25x150 mm size in oval caliber of sizes: height 13 mm, width 29 mm. The forge rolling temperature is 450 ºC; rolls rotation rate is 0.2 m/s; rolls working radius R_p = 66.5 mm.

The formulae for determination of current values of contact normal and tangential stresses are shown in Table 1.

Table 1

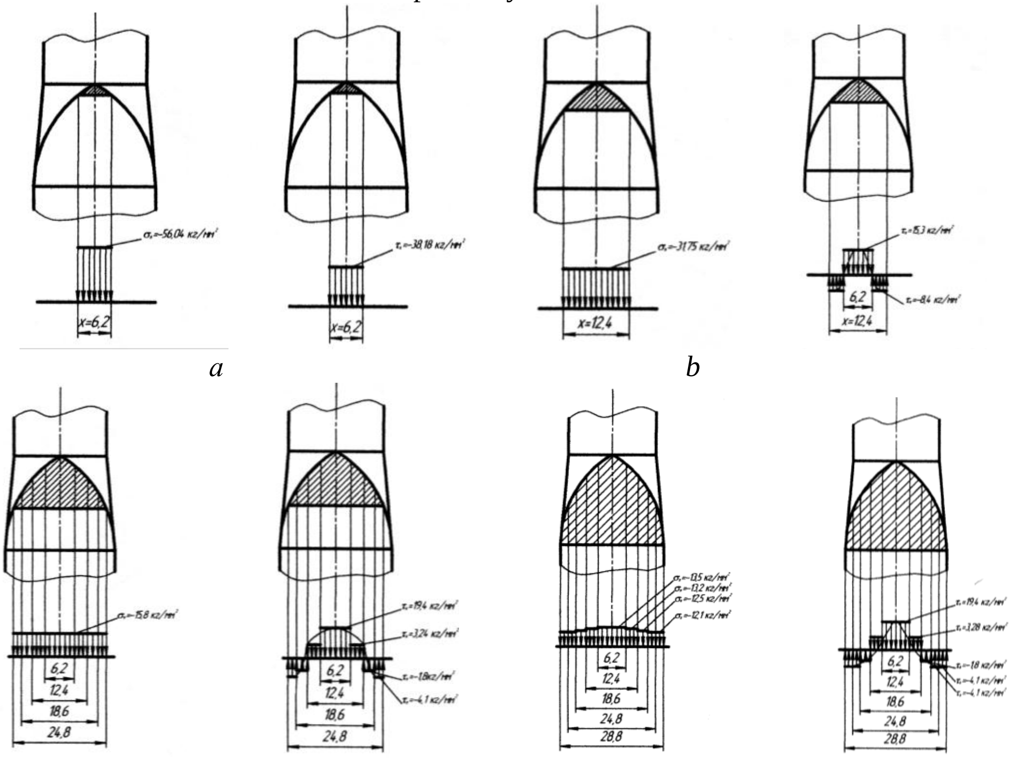
Formulae for determination of contact normal and tangential stresses

Parameter	Symbol	Design formula
1	2	3
Contact angle, rad	α	$\alpha = \arccos\left(1 - \frac{2R_з - \Delta h}{2R_p}\right) = \arccos(0,9022) = 0,4458$
Deformation time, s	t	$t = 10^{-3}\left(\frac{R_p \cdot \alpha}{\upsilon_в}\right) = 10^{-3}\left(\frac{66,5 \cdot 0,4458}{0,2}\right) = 0,145$
Caliber radius, mm	$R_к$	$R_к = \frac{\Delta h^2 + b_k^2}{4\Delta h} = \frac{12^2 + 29^2}{4 \cdot 12} = 20,5$
Oval width, mm	$b_{ов}$	$b_{ов} = b_к + \frac{2\delta(2R_к - \Delta h)}{b_к} = 29 + \frac{2 \cdot 15(41-12)}{29} = 31$
Oval height, mm	$h_{ов}$	$h_{ов} = 2\Delta h + \delta = 2 \cdot 6 + 1,5 = 13,5$
Deformation interval, c	t_i	$t_i = \frac{0,145 \cdot d_i}{\alpha_i}$
Contact arc angle, rad	$\alpha_д$	is set

Continuation of Table 1

1	2	3
Contact angle in cross section of deformation center	φ_i	$\varphi_i = \dfrac{\upsilon_в \cdot t_i \left(1 - \dfrac{\Delta h}{2R_к}\right)}{R_p \cdot \alpha}$
Contact zone cross section arc length, mm	$l_{\varphi\iota}$	$l_{\varphi\iota} = R_p \varphi_i$
Oval width, mm	$b_{ов}^{(i)}$	$b_{ов}^{(i)} = \dfrac{(b_{ов} + 2R_к)}{\alpha \cdot R_p} \cdot \upsilon_в \cdot t_i + 2R_З$
Oval height, mm	$h_{ов}^{(i}$	$h_{ов}^{(i)} = 2\{R_З - R_p[\cos(\alpha - \alpha_i) - \cos\alpha]\} + m$
Deformation factors	$K_x^{(i)}; K_y^{(i)}$	$K_x = \dfrac{b_{ов}^{(i)}}{2R_З}; \quad K_y = \dfrac{h_{ов}^{(i)}}{2R_З}$
Contact area, mm^2	$F_к^{(i)}$	$F_к^{(i)} = R_к \dfrac{\upsilon_в \cdot t_i}{R_p}[(R_p + R_к)\dfrac{\varphi\upsilon_в \cdot t_i}{2R_p \cdot \alpha} - R_к \sin(\dfrac{\varphi\upsilon_в \cdot t_i}{2R_p \cdot \alpha})]$
Stresses intensity, kg/mm^2	σ_i	is set
Main deformation	$\varepsilon_1^{(i)}; \varepsilon_2^{(i)};$	$\varepsilon_1^i = \dfrac{1}{2} l_n \cdot K_x^{(i)}; \quad \varepsilon_2^i = \dfrac{1}{2} l_n \cdot K_y^{(i)}$
Deformation intensity	$\varepsilon_i^{(i)}$	$\varepsilon_i^{(i)} = \dfrac{2}{\sqrt{3}} \sqrt{\varepsilon_1^{(i)^2} + \varepsilon_2^{(i)^2} + \varepsilon_1^{(i)} \cdot \varepsilon_2^{(i)}}$
Deformation module of second kind, kg/mm^2	G^*	$G^* = \dfrac{\sigma_i^{(i)}}{3\varepsilon_i^{(i)}}$
Stress along site perpendicular to axis ОУ, kg/mm^2	$\sigma_x^{(i)}$	$\sigma_x^{(i)} = 2G^*(1 - K_x^{(i)})$
Stress along site perpendicular to axis OX, kg/mm^2	$\sigma_y^{(i)}$	$\sigma_y^{(i)} = 2G^*(1 - K_y^{(i)})$
Step in OX axis direction, mm	h_{α_1}	$h_{\alpha_1}^{(i,j)} = R_З K_x^{(i)}(\sin\beta_{i+1,j} - \sin\beta_{i,j+1})$
Step in ОУ axis direction, mm	h_{α_2}	$h_{\alpha 2}^{(i,j)} = R_З K_y^{(i)}(\cos\beta_{i,j+1} - \cos\beta_{i+1,j})$
Angles of cross section division, row	$\beta_{i+i,j}; \beta_{i,j+i}$	are set
Boundary values on blank surface, mm	$u_{i,j+1}^{(г)}$	$u_{i,j+1}^{(г)} = R_З(1 - K_x^{(i)})\sin\beta_{i,j+1}$
Boundary values on blank surface, mm	$u_{i+1,j}^{(г)}$	$u_{i+1,j}^{(г)} = R_З(1 - K_x^{(i)})\sin\beta_{i+1,j}$
Boundary values on blank surface, mm	$\upsilon_{i,j+1}^{(г)}$	$\upsilon_{i,j+1}^{(г)} = R_З(1 - K_y^{(i)})\cos\beta_{i,j+1}$
Boundary values on blank surface, mm	$\upsilon_{i+1,j}^{(г)}$	$\upsilon_{i+1,j}^{(г)} = R_З(1 - K_y^{(i)})\cos\beta_{i+1,j}$
Normal inclination angle to blank surface	$tg\alpha_{i,j}$	$tg\alpha_{i,j} = \dfrac{K_x^{(i)}(\sin\beta_{i+1,j} - \sin\beta_{i,j+1})}{K_y^{(i)}(\cos\beta_{i,j+1} - \cos\beta_{i+1,j})}$
Tangential stresses along sites parallel to coordinate axes, kg/mm^2	τ_{xy}	$\tau_{xy} = \dfrac{G^*}{2}\left(\dfrac{u_{i,j}^{(г)} - u_{i+i,j}^{(г)}}{h_{\alpha_2}^{(i,j)}} - \dfrac{\upsilon_{i+1,j}^{(г)} - \upsilon_{1+i,j}^{(г)}}{h_{\alpha_1}^{(i,j)}}\right)$
Normal pressure, kg/mm^2	$\sigma_n^{(i)}$	$\sigma_n^{(i)} = -(\sigma_x \cos^2\alpha_{i.j} + \sigma_y \sin^2\alpha_{i.j} + \tau_{xy}\sin 2\alpha_{i,j})$
Tangential stress kg/mm^2	$\tau_n^{(i)}$	$\tau_n^{(i)} = 0{,}5(\sigma_x - \sigma_y)\sin 2\alpha_{i,j} - \tau_{xy}\cos^2\alpha_{i.j}$
Theoretical averaged pressure	$\sigma_{ср}^m$	$\sigma_{ср}^m = 1{,}15(\sigma_{n.ср})$
Pressure error	$h_{пог}^{(i)}$	$h_{пог}^{(i)} = \left(\dfrac{\sigma_{ср} - \sigma_{ср}^m}{\sigma_{ср}}\right)100\%$
Average value of design error	$h_{пог}^{ср}$	$h_{пог}^{ср} = \dfrac{1}{m}\sum_{n=1}^{m} h_{пог}^{(n)}$

The diagrams of normal and tangential stresses are shown in Fig. 4, it is evident from them how their values change with increase of contact area and respectively deformation time.



a *b* *c* *d*

Fig. 4. Diagrams of normal and tangential stresses. Deformation time t:
a - 0,029 s, b - 0,058 s, c - 0,116 s, d - 0,145 s

The discrepancy between the theoretic design results and the experimental data at the proposed methods test is 1.7 – 4 %, that confirms the possibility of its use for determination of contact normal and tangential stresses

Conclusions

The methods of determination of normal and tangential stresses at forge rolling with allowance for deformation development against time is described in the paper. The obtained formulae are presented, they characterize the dynamics of deformation center change, and they are used for determination of contact normal and tangential stresses. The results of the performed tests are described. The discrepancy between the theoretic design results and the experimental data at test of the proposed methodology is 1.7–4 % that confirms the possibility of its use for determination of contact normal and tangential stresses.

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УРАХУВАННЯ РОЗВИТКУ ДЕФОРМАЦІЇ ДЛЯ ВИЗНАЧЕННЯ НОРМАЛЬНИХ І ДОТИЧНИХ КОНТАКТНИХ НАПРУЖЕНЬ ПРИ ВАЛЬЦЮВАННІ ЗАГОТОВОК

Анотація: *у роботі описано метод визначення контактних нормальних і дотичних напружень, з урахуванням розвитку деформації в часі. Представлено отримані формули, що характеризують динаміку зміни області деформації в часі й використовуванні для визначення контактних нормальних і дотичних напружень. Розбіжність результатів теоретичного розрахунку з експериментальними даними при перевірці запропонованого методу становить 1,7-4 % , що підтверджує можливість його застосування для визначення контактних нормальних і дотичних напружень.*

Ключові слова: *напруження, деформація, вальцювання.*

УЧЕТ РАЗВИТИЯ ДЕФОРМАЦИИ ДЛЯ ОПРЕДЕЛЕНИЯ НОРМАЛЬНЫХ И КАСАТЕЛЬНЫХ КОНТАКТНЫХ НАПРЯЖЕНИЙ ПРИ ВАЛЬЦОВКЕ ЗАГОТОВОК

Аннотация: *в работе описан метод определения контактных нормальных и касательных напряжений, с учетом развития деформации во времени. Представлены полученные формулы, характеризующие динамику изменения очага деформации во времени и используемые для определения контактных нормальных и касательных напряжений. Результаты теоретического расчета с экспериментальными данными при проверке предложенного метода составляет 1,7 – 4 % , что подтверждает возможность его применения для определения контактных нормальных и касательных напряжений.*

Ключевые слова: *напряжение, деформация, вальцовка.*