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**Varnaliy Zahariy**, Doctor of Economics Sciences, Professor, Department of Finance, Taras Shevchenko National University of Kyiv, Vasilkovskaya str., 60, Kyiv, Ukraine, 01033  
E-mail: [vzs1955@gmail.com](mailto:vzs1955@gmail.com)

**Onyshchenko Svitlana**, Candidate of Economic Sciences, Associate Professor, Department of Finance and Banking, Poltava National Technical Yuri Kondratyuk University, Pershotravnevyi ave., 24, Poltava, Ukraine, 36000  
E-mail: [sponomarenko@list.ru](mailto:sponomarenko@list.ru)

**Masliy Alexandra**, Senior lecturer of Finance and Banking, Department of Finance and Banking, Poltava National Technical Yuri Kondratyuk University, Pershotravnevyi ave., 24, Poltava, Ukraine, 36000  
E-mail: [pugach\\_alexandra@ukr.net](mailto:pugach_alexandra@ukr.net)

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## USING THE METHOD OF IDEAL POINT TO SOLVE DUAL-OBJECTIVE PROBLEM FOR PRODUCTION SCHEDULING

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*In practice, there are often problems, which must simultaneously optimize several criterias. This so-called multi-objective optimization problem. In the article we consider the use of the method ideal point to solve the two-objective optimization problem of production planning. The process of finding solution to the problem consists of a series of steps where using simplex method, we find the ideal point. After that for solving a scalar problems, we use the method of Lagrange multipliers*

**Keywords:** *method of ideal point, dual-objective problem, scalarized problem, simplex process, Lagrange method of multipliers*

*На практиці часто зустрічаються задачі, в яких треба одночасно оптимізувати декілька критеріїв. Це задачі багатокритеріальної оптимізації. Ми застосовуємо методу ідеальної точки для розв'язання дво-критеріальної задачі планування виробництва. Розв'язок задачі складається з двох кроків, де за допомогою симплексного методу знаходимо ідеальну точку, а для розв'язування скаляризованої задачі, використовуємо метод множників Лагранжа*

**Ключові слова:** *метод ідеальної точки, двокритеріальна задача, скаляризована задача, метод множників Лагранжа*

### 1. Introduction

When solving various problems in economics, social sphere, information technologies, etc. the decision often needs to be made under certainty, uncertainty and conflict circumstances. There exists a set of methods to make decisions under such circumstances. In particular, under circumstances of uncertainty, in order to make a decision when solving multi-objective problems, methods of ideal point, successive approximation, additional constraints etc., are commonly used.

Let us review the use of ideal point method [1–3] to solve dual-objective problem of production schedul-

ing, where profit and demand for product being manufactured is taken as a criterion of optimality.

Suppose,  $n$  is the number of product items the enterprise may produce;  $m$  is the number of different resources used in manufacturing of products;  $a_{ij}$  is the number of units of the  $i$ -th resource used to manufacture one  $j$ -th unit of the product;  $b_i$  is the number of units of the  $i$ -th resource that may be used at the enterprise;  $p_j$  is the profit from the manufacture of product unit of the  $j$ -th type;  $r_j$  is the demand for the products of the  $j$ -th type;  $x_j$  is product manufacture schedule of the  $j$ -th type (thought-for values).

Then the mathematical model of the problem will look like this

$$P = \sum_{j=1}^n p_j x_j \rightarrow \max, \quad (1)$$

$$R = \sum_{j=1}^n r_j x_j \rightarrow \max, \quad (2)$$

if

$$\sum_{j=1}^n a_{ij} x_j \leq b_i, \quad i = 1, 2, \dots, m, \quad (3)$$

$$x_j \geq 0, \quad j = 1, 2, \dots, n. \quad (4)$$

To solve the problem (1)–(4), let us use the method of ideal point [1, 3]. Then the scalarized problem will look like this:

$$\left( \sum_{j=1}^n p_j x_j - a_1 \right)^2 + \left( \sum_{j=1}^n r_j x_j - a_2 \right)^2 \rightarrow \min,$$

if

$$\sum_{j=1}^n a_{ij} x_j \leq b_i, \quad i = 1, 2, \dots, m,$$

$$x_j \geq 0, \quad j = 1, 2, \dots, n,$$

where  $a = (a_1, a_2)$  is the ideal point, which is defined as follows:

$$a_1 = \max \sum_{j=1}^n p_j x_j,$$

$$a_2 = \max \sum_{j=1}^n r_j x_j$$

on the set that is defined by the irregularities (3), (4).

To find  $a_1, a_2$ , simplex process [4] may be used, since these are the linear programming problems. To solve a scalarized problem, Lagrange method of multipliers may be used [5, 6].

## 2. Analysis of published data

Scientific-theoretical and methodological aspects of the nature of decision-making methods, optimizing financial decisions and mathematical economics investigated domestic and foreign scientists, including Voloshin A. F. [1], Tsehelyk H. H. [3], Kigelia V. R. [7], and et al. Dobulyak L. P. and Tsehelyk H. H. [8] make a mathematical model of the problem for production scheduling for maximum profit. They used a method ideal point for solving dual-objective problem. To find the maximum of compromise they solved the problem of quadratic programming. However, methodological and applied problems of the method ideal point to solve the multi-objective optimization problem of production planning require research and system development [9, 10].

## 3. The purpose and objectives of research

The main goal is to solve dual-objective problem for production scheduling using ideal point method.

We make the production plan to make the most of available resources and at the same time at the same time to ensure maximum profit and maximum output with the greatest demand.

## 4. Materials and methods of research

The method of research is the method ideally point. This method is based on the fact that postulated the existence of "ideal point" for the solution of the problem, which all reached the extreme criteria. Thus, to solve the problem using the method of "ideal point" must first determine its coordinates. To determine the coordinates of the "ideal point"  $n$ -th solve single-objective problems for each of the criteria optimization.

## 5. Research results

Solve the following problem using the ideal point method:

$$P = 2x_1 + 3x_2 + 2x_3 + x_4 \rightarrow \max, \quad (5)$$

$$R = x_1 + 2x_2 + 3x_3 + 4x_4 \rightarrow \max, \quad (6)$$

if

$$\begin{cases} 3x_1 + 2x_2 + x_3 + 2x_4 \leq 100, \\ x_1 + 3x_2 + 2x_3 + x_4 \leq 200, \\ 2x_1 + x_2 + 3x_3 + 2x_4 \leq 250, \end{cases} \quad (7)$$

$$x_j \geq 0, \quad j = 1, 2, 3, 4. \quad (8)$$

*Solution.* First, let us find the ideal point  $a=(a_1, a_2)$ . To do so, we need to solve two problems separately using simplex method of linear programming problems solution (5), (7), (8) and (6)–(8). Vector  $x_1=(0; 10; 80; 0)$ ,  $\max P=190$  is a solution of the problem (5), (7), (8) (Table 1). Vector  $x_2=(0; 0; 75; 12,5)$ ,  $\max R=275$  is a solution of the problem (6), (7), (8) (Table 2).  $a=(190, 275)$  is the ideal point.

Now let us solve the scalarized problem.

$$\begin{aligned} & (2x_1 + 3x_2 + x_3 + x_4 - 190)^2 + \\ & + (x_1 + 2x_2 + 3x_3 + 4x_4 - 275)^2 \rightarrow \min, \end{aligned}$$

if

$$\begin{cases} 3x_1 + 2x_2 + x_3 + 2x_4 \leq 100, \\ x_1 + 3x_2 + 2x_3 + x_4 \leq 200, \\ 2x_1 + x_2 + 3x_3 + 2x_4 \leq 250, \\ x_j \geq 0, \quad j = 1, 2, 3, 4. \end{cases}$$

Lagrange method of multipliers may be used to do it [5, 6].

Let us generate Lagrange function

$$\begin{aligned} L = & (2x_1 + 3x_2 + x_3 + x_4 - 190)^2 + \\ & + (x_1 + 2x_2 + 3x_3 + 4x_4 - 275)^2 + \\ & + \lambda_1 (3x_1 + 2x_2 + x_3 + 2x_4 - 100) + \\ & + \lambda_2 (x_1 + 3x_2 + 2x_3 + x_4 - 200) + \\ & + \lambda_3 (2x_1 + x_2 + 3x_3 + 2x_4 - 250). \end{aligned}$$

Table 1

Simplex method for solving linear programming problem (6), (7), (8).

i	Б	c	P <sub>0</sub>	2	3	2	1	0	0	0
				P <sub>1</sub>	P <sub>2</sub>	P <sub>3</sub>	P <sub>4</sub>	P <sub>5</sub>	P <sub>6</sub>	P <sub>7</sub>
1	P <sub>5</sub>	0	100	3	<b>2</b>	1	2	1	0	0
2	P <sub>6</sub>	0	200	1	3	2	1	0	1	0
3	P <sub>7</sub>	0	250	2	1	3	2	0	0	1
4			0	-2	-3	-2	-1	0	0	0
1	P <sub>2</sub>	3	50	3/2	1	1/2	1	1/2	0	0
2	P <sub>6</sub>	0	50	-7/2	0	1/2	-2	-3/2	1	0
3	P <sub>7</sub>	0	200	1/2	0	<b>5/2</b>	1	-1/2	0	1
4			150	5/2	0	-1/2	2	3/2	0	0
1	P <sub>2</sub>	3	10	7/5	1	0	4/5	3/5	0	-1/5
2	P <sub>6</sub>	0	10	-18/5	0	0	-11/5	-7/5	1	-1/5
3	P <sub>3</sub>	2	80	1/5	0	1	2/5	-1/5	0	2/5
4			190	13/5	0	0	11/5	7/5	0	1/5

Table 2

Simplex method for solving linear programming problem (5), (7), (8).

i	Б	c	P <sub>0</sub>	1	2	3	4	0	0	0
				P <sub>1</sub>	P <sub>2</sub>	P <sub>3</sub>	P <sub>4</sub>	P <sub>5</sub>	P <sub>6</sub>	P <sub>7</sub>
1	P <sub>5</sub>	0	100	3	2	1	<b>2</b>	1	0	0
2	P <sub>6</sub>	0	200	1	3	2	1	0	1	0
3	P <sub>7</sub>	0	250	2	1	3	2	0	0	1
4			0	-1	-2	-3	-4	0	0	0
1	P <sub>4</sub>	4	50	3/2	1	1/2	1	1/2	0	0
2	P <sub>6</sub>	0	150	-1/2	2	3/2	0	-1/2	1	0
3	P <sub>7</sub>	0	150	-1	-1	<b>2</b>	0	-1	0	1
4			200	5	2	-1	0	2	0	0
1	P <sub>4</sub>	4	12.5	7/4	5/4	0	1	3/4	0	-1/4
2	P <sub>6</sub>	0	37.5	1/4	11/4	0	0	1/4	1	-3/4
3	P <sub>3</sub>	3	75	-1/2	-1/2	1	0	-1/2	0	1/2
4			275	-9/2	3/2	0	0	3/2	0	1/2

Then, the required and sufficient condition for the existence of the function saddle point  $L = (X, \Lambda)$  is the following:

$$\frac{\partial L}{\partial x_j} \geq 0, \quad j = 1, 2, 3, 4;$$

$$\frac{\partial L}{\partial \lambda_i} \leq 0, \quad i = 1, 2, 3;$$

$$x_j \frac{\partial L}{\partial x_j} = 0, \quad j = 1, 2, 3, 4;$$

$$\lambda_i \frac{\partial L}{\partial \lambda_i} = 0, \quad i = 1, 2, 3;$$

$$x_j \geq 0, \quad j = 1, 2, 3, 4; \quad \lambda_i \geq 0, \quad i = 1, 2, 3.$$

**6. SWOT- analysis of research results**

– Strengths – we can make a plan of production to make the most of available resources.

– Weaknesses – the method of ideal point does not use a supporting information superiority on the set criteria. Solving this problems we assumes the existence of so-called optimal solution multi-objective optimization problem, which can be found by converting dual-objective problem in an appropriate single-objective problem.

– Opportunities – in the future using this method, we cannot just make a plan of production but also to ensure maximum profit and maximum output with the greatest demand.

– Threats – at this stage of solution this problem I do not faced threats.

**7. Conclusions**

The proposed optimization model allows the manufacturer to make a plan of production to ensure maximum profit and maximum output with the greatest demand or the highest quality simultaneously, which will improve the chances of survival on the competition.

In the example we have demonstrated the use of the method ideal point to solve the two-objective optimization problem of production planning. We have found the greatest demand and profits for this problem.

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**Mariia Marko**, Applicant, Department of Mathematical Modeling of Social and Economics Processes, Ivan Franko National Lviv University of Lviv, Universytetska str., 1, Lviv, Ukraine, 79000  
E-mail: mariiamarko@gmail.com