APPROXIMATION OF THE DENSITY FUNCTION OF THE WEIBULL DISTRIBUTION USING ANALYSIS CUMULANT

L.R. Chupakhina, postgraduate student
N.V. Kireeva, Candidate of Technical Sciences, Associate Professor
Povolzhskiy State University of Telecommunications and Informatics, Russia

The application of the cumulant analysis for research of the density of probability of distribution in a multiservice network is considered in this article. Possibility of its representation through cumulants allows to consider properties of self-similarity of a traffic.
Keywords: queuing system, the traffic, the moments of functions, cumulants, the distribution of «heavy» tail.

Conference participants

In the modern telecommunication networks multiservice traffic is often described with the help of distributions with «heavy» tails, which allow to consider as a model system of mass service system type G/G/1.

In practice, in the study of real systems, are rarely known laws of distribution and service supplied to the input of the system of traffic. The study is based on representation of the distribution of time

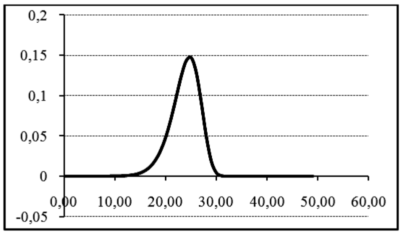

Fig. 1. The density function of the Weibull distribution withα=10andβ=25.

of service - a probability density function, which in turn presented with the help of approximation by means of cumulant analysis. This approach has a number of advantages, because cumulant functions are clear an independent statistical meaning and may be set to a certain extent independently of each other.

Consider the approximation of the probability density in a number of Edgeworth, the giver of decomposition of an arbitrary probability density for the derivative of a Gaussian distribution.

$$W(x) = W_G(x)-(\chi_3/3!)W_G^{(3)}(x) + (\chi_4/4!)W_G^{(4)}(x) - (\chi_5/5!)W_G^{(5)}(x)+ (\chi_6+10\chi_3^2/6!)W_G^{(6)}(x), \quad (1)$$

where $W_G^{(k)}$ - derivative of the density of normal function.

As a result collected probability density comparable with the known characteristics of the traffic that is being transmitted on multiservice network.

For the study will take a distribution function with «heavy» tail (Weibull distribution), according to the law which will come traffic to the input of the network element.

Function of the Weibull distribution has the form:

$$F(x) = 1-e^{-(x/\beta)\alpha},\ x>0,\ \alpha> 0,\ \beta> 0, \quad (2)$$

whereα- shape parameter, β- scale parameter.

The density of the Weibull distribution is as follows (Figure 1):

$$f(x)=\alpha\beta^{-\alpha}x^{\alpha-1}e^{-(x/\beta)\alpha}. \quad (3)$$

It is known that the moments m_k random variable$x$Weibull distribution are as follows:

$$m_k= \beta^{k/\alpha}G(1+k/\alpha), \quad (4)$$

where$G(z)$ - gamma function.

For a complete description $W(x)$find the first six points of the expression (4).

The connection between cumulants and moments of distributions given by the relations:

$$\chi_1= m_1,\ \chi_2= m_2- m_1^2, \chi_3= m_3- 3m_1m_2+2m_1^3,\ \chi_4 = m_4 - 2m_2^2 - 4m_1m_3 + 12 m_1^2m_2 - 6m_1^4,\ \chi_5=m_5 - 5m_1m_1- -10m_2m_3+20m_1^2m_3+30m_1m_2^2-60m_1^3m_2+24m_1^5,\ \chi_6=m_6-6m_1m_5-15m_2m_4+30m_1^2m_4-10m_3^2+120m_1m_3- -120m_1^3m_3+30m_2^3-270m_1^2m_2^2+ +360m_1^4m_2-120m_1^6. \quad (5)$$

Based on the foregoing, we can expand the function in a number of Edgeworth (1). From expression (1) is directly visible to the special value of the cumulants in the evaluation of the probability density deviation from a Gaussian distribution.

For the selected values αandβtaking into account (4) and (5) we can obtain an approximation of the distribution (3) as shown in Figure 2.

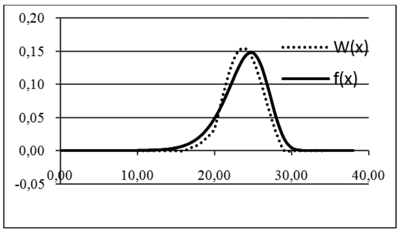

Fig. 2. Comparison of the two densities of the Weibull distribution

In the construction of the resulting density distribution (Fig. 2) takes into account that approximates the expression for the density must satisfy the normalization condition.

Thus, the resulting approximation of the density function of the Weibull distribution with cumulants can compare it with the theoretical distribution in the future to estimate the error variance.

Investigation of density function using cumulant analysis allows to take into account the properties of the self-similarity of traffic and service process. In practice, most simply realized the calculation of moments of time intervals between packets and time periods of service. After obtaining estimates of probability densities of the distributions of performance evaluation process unit may be obtained by a numerical (or approximate) solution of Lindley.

References:

1. Malakhov A.N. Cumulant analysis of random non-gaussian processes and their transformations. M., Sovetskoe radio, 1978, 376 p.
2. Korolyuk V.S., Portenko N.I., Skorokhod A.V., Handbook on probability theory and mathematical statistics, Publ. Nauka, M., 1985, 640 p.
3. Shelukhin O.I. Fractal processes in telecommunications./Shelukhin O.I., TenyakshevA.M., OsinA.V., M.: Radio engineering, 2003, 480 p.
4. Kleinrock L. Theory of mass service. Translation from English./ Grushko I.I., Neiman V.I.- M.: Machinery, 1979, 432 p.