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SOLUTION OF THE NON-AXISYMMETRIC QUASISTATIC THERMOELASTICITY PROBLEM FOR MULTILAYER CYLINDER WITH IDENTICAL LAMÉ COEFFICIENTS

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Summary. *The non-axisymmetric thermoelastisity state of the multilayer unlimited hollow cylinder with the identical Lame coefficients of layers under the action of internal and surface heat sources and non-uniform distribution of the initial temperature by means of the constructed Green's functions of the thermoelasticity quasistatic problem is defined. The thermoelastic state of two-layer cylinder caused by normally distributed heat stream on the external cylinder surface moving along the cylinder derivative is investigated.*
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Statement of the problem. Circular cylinders are widely used elements of structures of modern technology that undergo various thermal actions. To meet a wide range of requirements for such elements in terms of strength, reliability and durability is possible on the basis of theoretical investigations of their thermoelastic state taking into account the layered structure of the cylinder and the complex thermal load.

Analysis of available investigations. The analytical solution of an axially symmetrical quasistatic thermoelasticity problem for multi-layered long hollow cylinder, in the presence of surface and internal heat sources and nonuniform temperature distribution at the initial time moment is developed in [1]. Analytical solutions of non-axisymmetric quasistatic thermoelasticity problems for multilayered cylinders are obtained only for certain cases of thermal action, particularly in [2] – under the action of surface heat sources.

The objective of the paper. To develop using the Green's functions the analytical solution of non-axisymmetric quasistatic thermoelasticity problem for multi-layered long hollow cylinder with identical Lame coefficients for nonunoform initial temperature distribution and the combined action of surface and internal heat sources . To test the obtained solution on the thermoelasticity problem for two-layered cylinder heated by normally distributed moving heat source.

Statement of the problem. Let us consider in the cylindrical coordinate system r,φ,z, free from external loads the unlimited, on axial coordinate, multilayer hollow cylinder consisting of n concentrically located perfectly contacting isotropic layers with similar Lame coefficients. The cylinder is under convection heat exchange and is heated by internal sources of heat with density $W_T(r,\varphi,z,\tau)$ and sources of heat concentrated on the internal $r=r_0$ and external $r=r_n$ surfaces of the cylinder surface densities of which are $Q_0(\varphi,z,\tau)$ and $Q_n(\varphi,z,\tau)$ respectively. At the initial time moment $\tau=0$ the cylinder temperature equals $T_0(r,\varphi,z)$.

To determine the non-stationary temperature field $T(r,\varphi,z,\tau)$ we use the thermal conductivity equation

$$\frac{1}{r}\frac{\partial}{\partial r}\left[\lambda_t(r)r\frac{\partial T}{\partial r}\right]+\lambda_t(r)\left[\frac{1}{r^2}\frac{\partial^2 T}{\partial\varphi^2}+\frac{\partial^2 T}{\partial z^2}\right]=c_V(r)\frac{\partial T}{\partial\tau}-W_T(r,\varphi,z,\tau), \tag{1}$$

and boundary conditions

$$\left.\left(\lambda_t(r)\frac{\partial T}{\partial r}-\alpha_0 T\right)\right|_{r=r_0}=Q_0(\varphi,z,\tau),\ \left.\left(\lambda_t(r)\frac{\partial T}{\partial r}+\alpha_n T\right)\right|_{r=r_n}=Q_n(\varphi,z,\tau), \tag{2}$$

$$T(r,\varphi+2\pi,z,\tau)=T(r,\varphi,z,\tau), \qquad T\to 0,\ \text{if}\ z\to\pm\infty, \tag{3}$$

$$T|_{\tau=0}=T_0(r,\varphi,z), \tag{4}$$

where $\lambda_t(r)$ i $c_V(r)$ – are respectively, the piecewise constant coefficients of thermal conductivity and volume specific heat having the following form

$$p(r)=p_1+\sum_{i=1}^{n-1}(p_{i+1}-p_i)S(r-r_i), \tag{5}$$

$S(x)$ – is Heaviside function; $r=r_i$ – the surface of division of i and $i+1$ layers; α_0, α_n – coefficients of heat transfer from the internal and external surfaces of the cylinder, respectively; derivative for r – generalized.

To determine the thermoelastic state of the cylinder we use a system of differential equations relatively to displacements u_r, u_φ, u_z,

$$\Delta u_r+\frac{1}{1-2\nu}\frac{\partial e}{\partial r}-\frac{u_r}{r^2}-\frac{2}{r^2}\frac{\partial u_\varphi}{\partial\varphi}=\frac{1}{\mu}\frac{\partial}{\partial r}[\beta(r)T],$$

$$\Delta u_\varphi+\frac{1}{1-2\nu}\frac{1}{r}\frac{\partial e}{\partial\varphi}-\frac{u_\varphi}{r^2}+\frac{2}{r^2}\frac{\partial u_r}{\partial\varphi}=\frac{\beta(r)}{\mu}\frac{1}{r}\frac{\partial T}{\partial\varphi}, \qquad \Delta u_z+\frac{1}{1-2\nu}\frac{\partial e}{\partial z}=\frac{\beta(r)}{\mu}\frac{\partial T}{\partial z}, \tag{6}$$

dependencies between the components of the stress tensor σ_{rr}, $\sigma_{\varphi\varphi}$, σ_{zz}, $\sigma_{r\varphi}$, σ_{rz}, $\sigma_{z\varphi}$ and the components of the displacement vector

$$\sigma_{rr}=2\mu\frac{\partial u_r}{\partial r}+\lambda e-\beta(r)T,\ \sigma_{\varphi\varphi}=2\mu\frac{1}{r}\left(u_r+\frac{\partial u_\varphi}{\partial\varphi}\right)+\lambda e-\beta(r)T,$$

$$\sigma_{zz}=2\mu\frac{\partial u_z}{\partial z}+\lambda e-\beta(r)T,$$

$$\sigma_{r\varphi}=\mu\left(\frac{1}{r}\left(\frac{\partial u_r}{\partial\varphi}-u_\varphi\right)+\frac{\partial u_\varphi}{\partial r}\right),\quad \sigma_{rz}=\mu\left(\frac{\partial u_z}{\partial r}+\frac{\partial u_r}{\partial z}\right),\quad \sigma_{z\varphi}=\mu\left(\frac{\partial u_\varphi}{\partial z}+\frac{1}{r}\frac{\partial u_z}{\partial\varphi}\right), \tag{7}$$

and boundary conditions

$$\sigma_{rr} = \sigma_{r\varphi} = \sigma_{rz} = 0 \quad \text{at} \quad r = r_0 \,; \qquad \sigma_{rr} = \sigma_{r\varphi} = \sigma_{rz} = 0 \quad \text{at} \quad r = r_n \,. \tag{8}$$

Here $\Delta = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \varphi^2} + \frac{\partial^2}{\partial z^2}$; $e = \frac{\partial u_r}{\partial r} + \frac{1}{r}\left(\frac{\partial u_\varphi}{\partial \varphi} + u_r\right) + \frac{\partial u_z}{\partial z}$; $\beta(r) = [3\lambda + 2\mu]\alpha_t(r)$;

λ, μ and ν – are Lame and Poisson coefficients, $\alpha_t(r)$ – is the linear expansion coefficient having the form (5).

The solution of the nonstationary heat conductivity problem. Turning to dimensionless values

$$\rho = \frac{r}{r_n}, \ \zeta = \frac{z}{r_n}, \ Fo = \frac{a_1 \tau}{r_n^2}, \ Bi_0 = \frac{\alpha_0 r_n}{\lambda_t^{(1)}}, \ Bi_n = \frac{\alpha_n r_n}{\lambda_t^{(n)}}, \ \overline{\lambda}_t(\rho) = \frac{\lambda_t(r)}{\lambda_t^{(1)}}, \ \overline{c}_V(\rho) = \frac{c_V(r)}{c_V^{(1)}},$$

$$\overline{a}_i = \frac{a_i}{a_1},$$

$$\overline{Q_0}(\varphi, \zeta, Fo) = \frac{Q_0(\varphi, z, \tau)}{Q}, \ \overline{Q_n}(\varphi, \zeta, Fo) = \frac{Q_n(\varphi, z, \tau)}{Q}, \ w_t(\rho, \varphi, \zeta, Fo) = \frac{W_T(r, \varphi, z, \tau) r_n}{Q},$$

$$t(\rho, \varphi, \zeta, Fo) = \frac{\lambda_t^{(1)} T(r, \varphi, z, \tau)}{Q r_n}, \qquad t_0(\rho, \varphi, \zeta) = \frac{\lambda_t^{(1)} T_0(r, \varphi, z)}{Q r_n}, \tag{9}$$

the solution of the heat conductivity problem, in accordance with [3], is given in the form

$$t(M, Fo) = \int_0^{Fo}\int_{-\infty}^{+\infty}\int_0^{2\pi} G\left(M, \tilde{M}, Fo - \tilde{Fo}\right)\Big|_{\tilde{\rho}=1} \overline{Q}_n\left(\tilde{\varphi}, \tilde{\zeta}, \tilde{Fo}\right) d\tilde{\varphi}\, d\tilde{\zeta}\, d\tilde{Fo} -$$

$$- \rho_0 \int_0^{Fo}\int_{-\infty}^{+\infty}\int_0^{2\pi} G\left(M, \tilde{M}, Fo - \tilde{Fo}\right)\Big|_{\tilde{\rho}=\rho_0} \overline{Q_0}\left(\tilde{\varphi}, \tilde{\zeta}, \tilde{Fo}\right) d\tilde{\varphi}\, d\tilde{\zeta}\, d\tilde{Fo} +$$

$$+ \int_0^{Fo}\int_{\rho_0}^{1}\int_{-\infty}^{+\infty}\int_0^{2\pi} G\left(M, \tilde{M}, Fo - \tilde{Fo}\right) w_t\left(\tilde{\rho}, \tilde{\varphi}, \tilde{\zeta}, \tilde{Fo}\right) \tilde{\rho}\, d\tilde{\varphi}\, d\tilde{\zeta} d\tilde{\rho}\, d\tilde{Fo} +$$

$$+ \int_{\rho_0}^{1}\int_{-\infty}^{+\infty}\int_0^{2\pi} G\left(M, \tilde{M}, Fo\right) \overline{c}_V(\tilde{\rho}) t_0\left(\tilde{\rho}, \tilde{\varphi}, \tilde{\zeta}\right) \tilde{\rho}\, d\tilde{\varphi}\, d\tilde{\zeta}\, d\tilde{\rho}\,. \tag{10}$$

where

$$G\left(M, \tilde{M}, Fo\right) = \frac{2}{\pi^2} \sum_{k=0}^{\infty} \omega_k \cos k(\varphi - \tilde{\varphi}) \int_0^{+\infty} \cos \eta\left(\zeta - \tilde{\zeta}\right) \times$$

$$\times \sum_{m=1}^{\infty} \frac{\Phi^{(k)}(\rho, \mu, \eta) \Phi^{(k)}(\tilde{\rho}, \mu, \eta)}{N_k(\mu, \eta)}\Bigg|_{\mu = \mu_{k,m}(\eta)} \times \exp\left(-\mu_{k,m}^2(\eta) Fo\right) d\eta \tag{11}$$

– the Green's function [3] of the corresponding heat conductivity problem, $a_i = \lambda_t^{(i)} / c_V^{(i)}$ – the coefficient of the temperature conductivity of i - layer, Q has the same dimension as Q_0 and Q_n, (ρ,φ,ζ) and $(\tilde{\rho},\tilde{\varphi},\tilde{\zeta})$ – coordinates of points M and $\tilde{M}$ accordingly, $\omega_k = \begin{cases} 0{,}5, & k=0 \\ 1, & k=1,2,3,\ldots \end{cases}$,

$$N_k(\mu,\eta) = \bar{c}_V^{(n)}\left(\frac{Bi_n^2}{\varepsilon_n^2} + 1 - \frac{k^2}{\varepsilon_n^2}\right)\left(\Phi_n^{(k)}(1,\mu,\eta)\right)^2 - \rho_0^2\left(\frac{Bi_0^2}{\varepsilon_1^2} + 1 - \frac{k^2}{\varepsilon_1^2\rho_0^2}\right)\left(\Phi_1^{(k)}(\rho_0,\mu,\eta)\right)^2 +$$

$$+\sum_{i=1}^{n-1}\rho_i^2\left\{\frac{\bar{c}_V^{(i)}}{\varepsilon_i^2}\left[1 - \frac{\bar{c}_V^{(i+1)}}{\bar{c}_V^{(i)}}\left(\frac{\bar{\lambda}_t^{(i)}}{\bar{\lambda}_t^{(i+1)}}\frac{\varepsilon_i}{\varepsilon_{i+1}}\right)^2\right]\left(\Phi_i'^{(k)}(\rho_i,\mu,\eta)\right)^2 +\right.$$

$$\left.+\left[\bar{c}_V^{(i)}\left(1 - \frac{k^2}{\varepsilon_i^2\rho_i^2}\right) - \bar{c}_V^{(i+1)}\left(1 - \frac{k^2}{\varepsilon_{i+1}^2\rho_i^2}\right)\right]\left(\Phi_i^{(k)}(\rho_i,\mu,\eta)\right)^2\right\},$$

$$\Phi^{(k)}(\rho,\mu,\eta) = \Phi_1^{(k)}(\rho,\mu,\eta) + \sum_{i=1}^{n-1}\left(\Phi_{i+1}^{(k)}(\rho,\mu,\eta) - \Phi_i^{(k)}(\rho,\mu,\eta)\right)S(\rho-\rho_i),$$

$$\Phi_1^{(k)}(\rho,\mu,\eta) = -2\pi^{-1}\left\{\left(k\rho_0^{-1} - Bi_0\right)\psi_{k,k}(\varepsilon_1,\rho,\rho_0) + \psi_{k+1,k}(\varepsilon_1,\rho_0,\rho)\right\},$$

$$\Phi_1'^{(k)}(\rho,\mu,\eta) = -2\pi^{-1}\left\{\left(k\rho_0^{-1} - Bi_0\right)\left(k\rho^{-1}\psi_{k,k}(\varepsilon_1,\rho,\rho_0) -\right.\right.$$

$$\left.\left.-\psi_{k+1,k}(\varepsilon_1,\rho,\rho_0)\right) + k\rho^{-1}\psi_{k+1,k}(\varepsilon_1,\rho_0,\rho) + b_1\psi_{k+1,k+1}(\varepsilon_1,\rho,\rho_0)\right\},$$

$$\Phi_i^{(k)}(\rho,\mu,\eta) = \Phi_{i-1}^{(k)}(\rho_{i-1},\mu,\eta)\rho_{i-1}\left\{k\rho_{i-1}^{-1}\psi_{k,k}(\varepsilon_i,\rho,\rho_{i-1}) +\right.$$

$$\left.+\psi_{k+1,k}(\varepsilon_i,\rho_{i-1},\rho)\right\} - \frac{\bar{\lambda}_t^{(i-1)}}{\bar{\lambda}_t^{(i)}}\Phi_{i-1}'^{(k)}(\rho_{i-1},\mu,\eta)\rho_{i-1}\psi_{k,k}(\varepsilon_i,\rho,\rho_{i-1}),$$

$$\Phi_i'^{(k)}(\rho,\mu,\eta) = \Phi_{i-1}^{(k)}(\rho_{i-1},\mu,\eta)\left\{k\left(k\psi_{k,k}(\varepsilon_i,\rho,\rho_{i-1})/\rho -\right.\right.$$

$$\left.\left.-\psi_{k+1,k}(\varepsilon_i,\rho,\rho_{i-1})\right) + \rho_{i-1}\left(k\psi_{k+1,k}(\varepsilon_i,\rho_{i-1},\rho)/\rho + b_i\psi_{k+1,k+1}(\varepsilon_i,\rho,\rho_{i-1})\right)\right\} -$$

$$-\frac{\bar{\lambda}_t^{(i-1)}}{\bar{\lambda}_t^{(i)}}\Phi_{i-1}'^{(k)}(\rho_{i-1},\mu,\eta)\rho_{i-1}\left(k\psi_{k,k}(\varepsilon_i,\rho,\rho_{i-1})/\rho - \psi_{k+1,k}(\varepsilon_i,\rho,\rho_{i-1})\right);$$

$$\psi_{k,l}(\varepsilon_i,x,y) = \frac{\pi}{2}\varepsilon_i^{|k-l|}\left[J_k(\varepsilon_i x)Y_l(\varepsilon_i y) - Y_k(\varepsilon_i x)J_l(\varepsilon_i y)\right], \text{ if } b_i > 0;$$

$$\psi_{k,k}(\varepsilon_i,x,y) = -\left[I_k(\varepsilon_i x)K_k(\varepsilon_i y) - K_k(\varepsilon_i x)I_k(\varepsilon_i y)\right],$$

$$\psi_{k,k+1}(\varepsilon_i,x,y) = -\varepsilon_i\left[I_k(\varepsilon_i x)K_{k+1}(\varepsilon_i y) + K_k(\varepsilon_i x)I_{k+1}(\varepsilon_i y)\right],$$

$$\psi_{k+1,k}(\varepsilon_i, x, y) = \varepsilon_i\left[I_{k+1}(\varepsilon_i x)K_k(\varepsilon_i y) + K_{k+1}(\varepsilon_i x)I_k(\varepsilon_i y)\right], \text{ if } b_i < 0;$$

$$\varepsilon_i = \sqrt{|b_i|};\ b_i = \mu^2/\bar{a}_i - \eta^2;\ 0 < \mu_{k,1}(\eta) < \mu_{k,2}(\eta) < \mu_{k,3}(\eta) < \ldots < \mu_{k,m}(\eta) < \ldots$$

- the roots of the transcendental equation $\left(\Phi_n'^{(k)}(\rho,\mu,\eta) + Bi_n \Phi_n^{(k)}(\rho,\mu,\eta)\right)\Big|_{\rho=1} = 0$;

$J_k(x)$, $Y_k(x)$ – Bessel functions of k order; $I_k(x)$, $K_k(x)$ – modified Bessel functions of k -order; derivative on ρ is marked by prime.

After substituting (11) into (10), we obtain the following expression for the temperature field

$$t(M, Fo) = \frac{2}{\pi^2}\sum_{k=0}^{\infty}\omega_k \int_0^{\infty}\int_{-\infty}^{\infty}\int_0^{2\pi}\sum_{m=1}^{\infty} \frac{\theta_k(\mu,\eta,\tilde{\varphi},\tilde{\zeta},Fo)\Phi^{(k)}(\rho,\mu,\eta)}{N(\mu,\eta)}\Bigg|_{\mu=\mu_{k,m}(\eta)} \times$$

$$\times \cos k(\varphi - \tilde{\varphi})\cos\eta(\zeta - \tilde{\zeta})d\tilde{\varphi}d\tilde{\zeta}d\eta, \qquad (12)$$

where

$$\theta_k(\mu,\eta,\tilde{\varphi},\tilde{\zeta},Fo) = \sum_{j=1}^{4} {}_j\tilde{\Phi}_k(\mu,\eta,\tilde{\varphi},\tilde{\zeta},Fo),$$

$${}_1\tilde{\Phi}_k(\mu,\eta,\tilde{\varphi},\tilde{\zeta},Fo) = e^{-\mu^2 Fo}\int_{\rho_0}^{1}\bar{c}_\upsilon(\tilde{\rho})t_0(\tilde{\rho},\tilde{\varphi},\tilde{\zeta})\Phi^{(k)}(\tilde{\rho},\mu,\eta)\tilde{\rho}d\tilde{\rho},$$

$${}_2\tilde{\Phi}_k(\mu,\eta,\tilde{\varphi},\tilde{\zeta},Fo) = \int_0^{Fo}\int_{r_0}^{r_n} w_t(\tilde{\rho},\tilde{\varphi},\tilde{\zeta},\xi)e^{-\mu^2(Fo-\xi)}\Phi^{(k)}(\tilde{\rho},\mu,\eta)\tilde{\rho}d\tilde{\rho}d\xi,$$

$${}_3\tilde{\Phi}_k(\mu,\eta,\tilde{\varphi},\tilde{\zeta},Fo) = -\rho_0\Phi^{(k)}(\rho_0,\mu,\eta)\int_0^{Fo}\overline{Q}_0(\tilde{\varphi},\tilde{\zeta},\xi)e^{-\mu^2(Fo-\xi)}d\xi,$$

$${}_4\tilde{\Phi}_k(\mu,\eta,\tilde{\varphi},\tilde{\zeta},Fo) = \Phi^{(k)}(1,\mu,\eta)\int_0^{Fo}\overline{Q}_n(\tilde{\varphi},\tilde{\zeta},\xi)e^{-\mu^2(Fo-\xi)}d\xi. \qquad (13)$$

Solution of the problem of thermoelasticity. The corresponding dimensionless displacements $\bar{u}_i = \frac{u_i}{\alpha_t^{(1)}}\cdot\frac{\lambda_t^{(1)}}{Qr_n^2}$ $(i = r, \varphi, z)$ will be worked out in the form

$$\bar{u}_r = u + \frac{\partial\Psi}{\partial\rho}, \qquad \bar{u}_\varphi = \upsilon + \frac{1}{\rho}\frac{\partial\Psi}{\partial\varphi}, \qquad \bar{u}_z = w + \frac{\partial\Psi}{\partial\zeta}. \qquad (14)$$

Substituting (14) into (6), we obtain the equation for determining the thermoelastic displacement potential Ψ

$$\overline{\Delta}\Psi = \bar{\gamma}(\rho)t \qquad (15)$$

and the system of equations for determining u, υ, w,

$$\bar{\Delta}u+\frac{1}{1-2\nu}\frac{\partial e^*}{\partial\rho}-\frac{u}{\rho^2}-\frac{2}{\rho^2}\frac{\partial\upsilon}{\partial\varphi}=0,\quad \bar{\Delta}\upsilon+\frac{1}{1-2\nu}\frac{1}{\rho}\frac{\partial e^*}{\partial\varphi}-\frac{\upsilon}{\rho^2}+\frac{2}{\rho^2}\frac{\partial u}{\partial\varphi}=0,$$

$$\bar{\Delta}w+\frac{1}{1-2\nu}\frac{\partial e^*}{\partial\zeta}=0, \tag{16}$$

where

$$\bar{\gamma}(\rho)=\frac{\alpha_t(\rho)}{\alpha_t^{(1)}}\frac{1+\nu}{1-\nu},\quad \bar{\Delta}=\frac{\partial^2}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial}{\partial\rho}+\frac{1}{\rho}\frac{\partial^2}{\partial\varphi^2}+\frac{\partial^2}{\partial\zeta^2},\quad e^*=\frac{\partial u}{\partial\rho}+\frac{1}{\rho}\left(\frac{\partial\upsilon}{\partial\varphi}+u\right)+\frac{\partial w}{\partial\zeta}.$$

The solutions of equation (15) and the system of equations (16) are worked out in accordance with (12) in the form

$$\begin{Bmatrix}\Psi\\ u\\ w\end{Bmatrix}=\frac{2}{\pi^2}\sum_{k=0}^{\infty}\omega_k\int_0^{\infty}\int_{-\infty}^{\infty}\int_0^{2\pi}\sum_{m=1}^{\infty}\left.\frac{\theta_k(\mu,\eta,\tilde{\varphi},\tilde{\zeta},Fo)}{N(\mu,\eta)}\right|_{\mu=\mu_{k,m}(\eta)}\times$$

$$\times\begin{Bmatrix}\Psi^{(k)}\left(\rho,\mu_{k,m}(\eta),\eta\right)\cos k(\varphi-\tilde{\varphi})\cos\eta(\zeta-\tilde{\zeta})\\ u^{(k)}\left(\rho,\mu_{k,m}(\eta),\eta\right)\cos k(\varphi-\tilde{\varphi})\cos\eta(\zeta-\tilde{\zeta})\\ w^{(k)}\left(\rho,\mu_{k,m}(\eta),\eta\right)\cos k(\varphi-\tilde{\varphi})\sin\eta(\zeta-\tilde{\zeta})\end{Bmatrix}d\tilde{\varphi}d\tilde{\zeta}d\eta,$$

$$\upsilon=\frac{2}{\pi^2}\sum_{k=1}^{\infty}\int_0^{\infty}\int_{-\infty}^{\infty}\int_0^{2\pi}\sum_{m=1}^{\infty}\left.\frac{\theta_k(\mu,\eta,\tilde{\varphi},\tilde{\zeta},Fo)}{N(\mu,\eta)}\right|_{\mu=\mu_{k,m}(\eta)}\times$$

$$\times\upsilon^{(k)}\left(\rho,\mu_{k,m}(\eta),\eta\right)\sin k(\varphi-\tilde{\varphi})\cos\eta(\zeta-\tilde{\zeta})d\tilde{\varphi}d\tilde{\zeta}d\eta. \tag{17}$$

Taking into account (12), (17) from (15), (16) we obtain an equation for determination $\Psi^{(k)}$

$$\tilde{\Delta}\Psi^{(k)}=\bar{\gamma}(\rho)\Phi^{(k)}\qquad\left(\tilde{\Delta}=\frac{d^2}{d\rho^2}+\frac{1}{\rho}\frac{d}{d\rho}-\frac{k^2}{\rho^2}-\eta^2\right) \tag{18}$$

and the system of equations for determination $u^{(k)}$, $\upsilon^{(k)}$ and $w^{(k)}$

$$\tilde{\Delta}u^{(k)}+\frac{1}{1-2\nu}\frac{de^{(k)}}{d\rho}-\frac{u^{(k)}}{\rho^2}-\frac{2k}{\rho^2}\upsilon^{(k)}=0,\quad \tilde{\Delta}\upsilon^{(k)}-\frac{1}{1-2\nu}\frac{k}{\rho}e^{(k)}-\frac{\upsilon^{(k)}}{\rho^2}-\frac{2k}{\rho^2}u^{(k)}=0,$$

$$\tilde{\Delta}w^{(k)}-\frac{1}{1-2\nu}\eta e^{(k)}=0. \tag{19}$$

The partial solution of equation (18) can be found in the form

$$\Psi^{(k)}(\rho,\mu,\eta)=\Psi_1^{(k)}+\sum_{i=1}^{n-1}(\Psi_{i+1}^{(k)}-\Psi_i^{(k)})S(\rho-\rho_i)\,, \tag{20}$$

where

$$\Psi_i^{(k)}=-K_k(\eta\rho)\sum_{j=1}^{i-1}\bar{\gamma}_j\bar{a}_j\frac{\Omega_j^{(k)}\Big|_{\rho=\rho_j}-\Omega_j^{(k)}\Big|_{\rho=\rho_{j-1}}}{\mu^2}+$$

$$I_k(\eta\rho)\sum_{j=i+1}^{n}\bar{\gamma}_j\bar{a}_j\frac{\Omega_j^{*(k)}\Big|_{\rho=\rho_j}-\Omega_j^{*(k)}\Big|_{\rho=\rho_{j-1}}}{\mu^2}-$$

$$-\frac{\bar{\gamma}_i\bar{a}_i}{\mu^2}\left[\Phi_i^{(k)}(\rho,\mu,\eta)-\Omega_i\Big|_{\rho=\rho_{i-1}}K_k(\eta\rho)-\Omega_i^*\Big|_{\rho=\rho_i}I_k(\eta\rho)\right],$$

$$\Omega_i^{(k)}=\rho\eta\Phi_i^{(k)}(\rho,\mu,\eta)I_{k+1}(\eta\rho)-\left[\rho\Phi_i'^{(k)}(\rho,\mu,\eta)-k\Phi_i^{(k)}(\rho,\mu,\eta)\right]I_k(\eta\rho)\,,$$

$$\Omega_i^{*(k)}=\rho\eta\Phi_i^{(k)}(\rho,\mu,\eta)K_{k+1}(\eta\rho)+\left[\rho\Phi_i'^{(k)}(\rho,\mu,\eta)-k\Phi_i^{(k)}(\rho,\mu,\eta)\right]K_k(\eta\rho)\,.$$

The general solution of the system of equations (19) is found in the form

$$u^{(k)}=\sum_{p=1}^{6}C_p^{(k)}(\eta)y_{2p}^{(k)}(\eta,\rho)\,,\quad \upsilon^{(k)}=\sum_{p=1}^{6}C_p^{(k)}(\eta)y_{3p}^{(k)}(\eta,\rho)\,,\quad w^{(k)}=\sum_{p=1}^{4}C_p^{(k)}(\eta)y_{1p}^{(k)}(\eta,\rho)\,, \tag{21}$$

where

$$y_{11}^{(k)}(\eta,\rho)=I_k(\eta\rho)\,,\quad y_{12}^{(k)}(\eta,\rho)=K_k(\eta\rho)\,,\quad y_{13}^{(k)}(\eta,\rho)=\eta\rho I_{k+1}(\eta\rho)\,,$$

$$y_{14}^{(k)}(\eta,\rho)=-\eta\rho K_{k+1}(\eta\rho)\,;\quad y_{21}^{(k)}(\eta,\rho)=-I_{k+1}(\eta\rho)\,,\quad y_{22}^{(k)}(\eta,\rho)=K_{k+1}(\eta\rho)\,,$$

$$y_{23}^{(k)}(\eta,\rho)=-\eta\rho I_k(\eta\rho)+[k+4(1-\nu)]I_{k+1}(\eta\rho)\,,\quad y_{25}^{(k)}(\eta,\rho)=-\frac{1}{\eta\rho}I_k(\eta\rho)\,,$$

$$y_{24}^{(k)}(\eta,\rho)=-\eta\rho K_k(\eta\rho)-[k+4(1-\nu)]K_{k+1}(\eta\rho)\,,\quad y_{26}^{(k)}(\eta,\rho)=-\frac{1}{\eta\rho}K_k(\eta\rho)\,;$$

$$y_{31}^{(k)}(\eta,\rho)=-I_{k+1}(\eta\rho)\,,\quad y_{32}^{(k)}(\eta,\rho)=K_{k+1}(\eta\rho)\,,\quad y_{33}^{(k)}(\eta,\rho)=[k+4(1-\nu)]I_{k+1}(\eta\rho)\,,$$

$$y_{34}^{(k)}(\eta,\rho)=-[k+4(1-\nu)]K_{k+1}(\eta\rho)\,,\quad y_{35}^{(k)}(\eta,\rho)=\frac{1}{k}I_{k+1}(\eta\rho)+\frac{1}{\eta\rho}I_k(\eta\rho)\,,$$

$$y_{36}^{(k)}(\eta,\rho)=-\frac{1}{k}K_{k+1}(\eta\rho)+\frac{1}{\eta\rho}K_k(\eta\rho)\,.$$

When $k\neq 0$ the values $C_p^{(k)}(\eta)$ $(p=\overline{1,6})$ are defined from boundary conditions

$$\sigma_{rr}^{(k)}=\sigma_{r\varphi}^{(k)}=\sigma_{rz}^{(k)}=0\,,\quad \rho=\rho_0\,;\qquad \sigma_{rr}^{(k)}=\sigma_{r\varphi}^{(k)}=\sigma_{rz}^{(k)}=0\,,\quad \rho=1\,. \tag{22}$$

When $k=0$ the values $C_5^{(0)}(\eta)=C_6^{(0)}(\eta)=0$, a $C_p^{(0)}(\eta)$ $(p=\overline{1,4})$ are defined from boundary conditions

$$\sigma_{rr}^{(0)}=\sigma_{rz}^{(0)}=0, \quad \rho=\rho_0; \qquad \sigma_{rr}^{(0)}=\sigma_{rz}^{(0)}=0, \quad \rho=1. \tag{23}$$

It should be noted that the expressions derived for $u^{(0)}$, $w^{(0)}$ coincide with the corresponding expressions for the axisymmetric problem [1] for the same Lame coefficients.

When $u^{(k)}$, $\upsilon^{(k)}$, $w^{(k)}$ and $\Psi^{(k)}$ are known non-dimensional displacements $\overline{u}_i$ and stresses $\overline{\sigma}_{ij}=\dfrac{\sigma_{ij}}{2\mu\alpha_t^{(1)}}\cdot\dfrac{\lambda_t^{(1)}}{Qr_n}$ $(i,j=r,\varphi,z)$ are determined according to the following relations

$$V_s(M,Fo)=\frac{2}{\pi^2}\sum_{k=0}^{\infty}\omega_k\int_0^{\infty}\int_{-\infty}^{\infty}\int_0^{2\pi}\sum_{m=1}^{\infty}\frac{\theta_k(\mu,\eta,\tilde{\varphi},\tilde{\zeta},Fo)V_s^{(k)}(\rho,\mu,\eta)}{N(\mu,\eta)}\Bigg|_{\mu=\mu_{k,m}(\eta)}\times$$

$$\times\cos k(\varphi-\tilde{\varphi})\begin{Bmatrix}\cos\eta(\zeta-\tilde{\zeta})d\tilde{\varphi}d\tilde{\zeta}d\eta, s=2,4,7,8\\ \sin\eta(\zeta-\tilde{\zeta})d\tilde{\varphi}d\tilde{\zeta}d\eta, \quad s=1,6\end{Bmatrix},$$

$$V_s(M,Fo)=\frac{2}{\pi^2}\sum_{k=1}^{\infty}\int_0^{\infty}\int_{-\infty}^{\infty}\int_0^{2\pi}\sum_{m=1}^{\infty}\frac{\theta_k(\mu,\eta,\tilde{\varphi},\tilde{\zeta},Fo)V_s^{(k)}(\rho,\mu,\eta)}{N(\mu,\eta)}\Bigg|_{\mu=\mu_{k,m}(\eta)}\times$$

$$\times\sin k(\varphi-\tilde{\varphi})\begin{Bmatrix}\cos\eta(\zeta-\tilde{\zeta})d\tilde{\varphi}d\tilde{\zeta}d\eta, \quad s=3,5\\ \sin\eta(\zeta-\tilde{\zeta})d\tilde{\varphi}d\tilde{\zeta}d\eta, \quad s=9\end{Bmatrix}, \tag{24}$$

where

$$[V_1,V_2,V_3,V_4,V_5,V_6,V_7,V_8,V_9]=[\overline{u}_z,\overline{u}_r,\overline{u}_\varphi,\overline{\sigma}_{rr},\overline{\sigma}_{r\varphi},\overline{\sigma}_{rz},\overline{\sigma}_{\varphi\varphi},\overline{\sigma}_{zz},\overline{\sigma}_{z\varphi}];$$

$$[V_1^{(k)},V_2^{(k)},V_3^{(k)},V_4^{(k)},V_5^{(k)},V_6^{(k)},V_7^{(k)},V_8^{(k)},V_9^{(k)}]$$
$$=[u_z^{(k)},u_r^{(k)},u_\varphi^{(k)},\sigma_{rr}^{(k)},\sigma_{r\varphi}^{(k)},\sigma_{rz}^{(k)},\sigma_{\varphi\varphi}^{(k)},\sigma_{zz}^{(k)},\sigma_{z\varphi}^{(k)}];$$

$$u_r^{(k)}=u^{(k)}+\frac{d\Psi^{(k)}}{d\rho}, \qquad u_\varphi^{(k)}=\upsilon^{(k)}-\frac{k}{\rho}\Psi^{(k)}, \qquad u_z^{(k)}=w^{(k)}-\eta\Psi^{(k)};$$

$$\sigma_{rr}^{(k)}=\frac{du^{(k)}}{d\rho}+\frac{ve^{(k)}}{1-2v}-\frac{1}{\rho}\frac{d\Psi^{(k)}}{d\rho}+\left(\frac{k^2}{\rho^2}+\eta^2\right)\Psi^{(k)},$$

$$\sigma_{\varphi\varphi}^{(k)}=\frac{1}{\rho}\left(k\upsilon^{(k)}+u^{(k)}\right)+\frac{ve^{(k)}}{1-2v}+\frac{1}{\rho}\frac{d\Psi^{(k)}}{d\rho}-\frac{k^2}{\rho^2}\Psi^{(k)}-\overline{\gamma}(\rho)\Phi^{(k)},$$

$$\sigma_{zz}^{(k)} = \eta w^{(k)} + \frac{ve^{(k)}}{1-2v} - \eta^2 \Psi^{(k)} - \bar{\gamma}(\rho)\Phi^{(k)},\ e^{(k)} = \frac{du^{(k)}}{d\rho} + \frac{1}{\rho}\left(k\upsilon^{(k)} + u^{(k)}\right) + \eta w^{(k)},$$

$$\sigma_{r\varphi}^{(k)} = \frac{1}{2}\left(\frac{d\upsilon^{(k)}}{d\rho} - \frac{1}{\rho}\left(\upsilon^{(k)} + ku^{(k)}\right) - \frac{2k}{\rho}\left(\frac{d\Psi^{(k)}}{d\rho} - \frac{1}{\rho}\Psi^{(k)}\right)\right),$$

$$\sigma_{rz}^{(k)} = \frac{1}{2}\left(\frac{dw^{(k)}}{d\rho} - \eta u^{(k)} - 2\eta\frac{d\Psi^{(k)}}{d\rho}\right),\ \sigma_{z\varphi}^{(k)} = \frac{1}{2}\left(-\eta\upsilon^{(k)} - \frac{k}{\rho}w^{(k)} + \frac{2\eta k}{\rho}\Psi^{(k)}\right).$$

Substituting in (24) expressions for $\theta_k(\mu,\eta,\tilde{\varphi},\tilde{\zeta},Fo)$ from (13), we obtain the desired solution of the thermoelasticity problem similar to (10):

$$V_s(M,Fo) = \int_0^{Fo}\int_{-\infty}^{+\infty}\int_0^{2\pi} U_s\left(M,\tilde{M},Fo-\tilde{F}o\right)\Big|_{\tilde{\rho}=1}\overline{Q}_n\left(\tilde{\varphi},\tilde{\zeta},\tilde{F}o\right)d\tilde{\varphi}\,d\tilde{\zeta}\,d\tilde{F}o -$$

$$-\rho_0\int_0^{Fo}\int_{-\infty}^{+\infty}\int_0^{2\pi} U_s\left(M,\tilde{M},Fo-\tilde{F}o\right)\Big|_{\tilde{\rho}=\rho_0}\overline{Q}_0\left(\tilde{\varphi},\tilde{\zeta},\tilde{F}o\right)d\tilde{\varphi}\,d\tilde{\zeta}\,d\tilde{F}o +$$

$$+\int_0^{Fo}\int_{\rho_0}^{1}\int_{-\infty}^{+\infty}\int_0^{2\pi} U_s\left(M,\tilde{M},Fo-\tilde{F}o\right)w_t\left(\tilde{\rho},\tilde{\varphi},\tilde{\zeta},\tilde{F}o\right)\tilde{\rho}\,d\tilde{\varphi}\,d\tilde{\zeta}d\tilde{\rho}\,d\tilde{F}o +$$

$$+\int_{\rho_0}^{1}\int_{-\infty}^{+\infty}\int_0^{2\pi} U_s\left(M,\tilde{M},Fo\right)\bar{c}_V(\tilde{\rho})t_0\left(\tilde{\rho},\tilde{\varphi},\tilde{\zeta}\right)\tilde{\rho}\,d\tilde{\varphi}\,d\tilde{\zeta}\,d\tilde{\rho}\,, \tag{25}$$

where

$$U_s\left(M,\tilde{M},Fo\right) = \frac{2}{\pi^2}\sum_{k=0}^{\infty}\omega_k\int_0^{\infty}\sum_{m=1}^{\infty}\frac{V_s^{(k)}(\rho,\mu,\eta)\Phi^{(k)}(\tilde{\rho},\mu,\eta)e^{-\mu^2 Fo}}{N(\mu,\eta)}\Bigg|_{\mu=\mu_{k,m}(\eta)}\times$$

$$\times\cos k(\varphi-\tilde{\varphi})\begin{Bmatrix}\cos\eta(\zeta-\tilde{\zeta})d\tilde{\varphi}d\tilde{\zeta}d\eta\,, s=2,4,7,8\\ \sin\eta(\zeta-\tilde{\zeta})d\tilde{\varphi}d\tilde{\zeta}d\eta\,, \quad s=1,6\end{Bmatrix},$$

$$U_s\left(M,\tilde{M},Fo\right) = \frac{2}{\pi^2}\sum_{k=1}^{\infty}\int_0^{\infty}\sum_{m=1}^{\infty}\frac{V_s^{(k)}(\rho,\mu,\eta)\Phi^{(k)}(\tilde{\rho},\mu,\eta)e^{-\mu^2 Fo}}{N(\mu,\eta)}\Bigg|_{\mu=\mu_{k,m}(\eta)}\times$$

$$\times\sin k(\varphi-\tilde{\varphi})\begin{Bmatrix}\cos\eta(\zeta-\tilde{\zeta})d\tilde{\varphi}d\tilde{\zeta}d\eta\,, s=3,5\\ \sin\eta(\zeta-\tilde{\zeta})d\tilde{\varphi}d\tilde{\zeta}d\eta\,, \quad s=9\end{Bmatrix} \tag{26}$$

- Green's function of a non-axisymmetric quasistatic termoelasticity problem for multi-layered long hollow cylinder when there is no power loads.

Investigation of the thermoelastic state of two-layer cylinder, caused by the moving normally distributed heat stream. As an example, we consider the cylinder heated by moving, normally distributed on the external surface thermal flow with density described by the relation

$$\overline{Q}_n(\varphi,\zeta,Fo)=\exp\left(-\bar{k}\left[\sin^2\varphi+(\zeta-\upsilon_\zeta Fo)^2\right]\right)S(\cos\varphi)\cos\varphi\,, \qquad (27)$$

where $\bar{k}$ – the coefficient of thermal flow concentration; υ_ζ – the velocity of the center of the heating spots in the axial direction.

Nondimensional temperatures and stresses in the two-layer cylinder ($n=2$) were investigated in the cross-sections going through the moving centre of the heating spot ($\zeta=\upsilon_\zeta Fo$) perpendicular to the cylinder axis at $\varphi=0$, $\pi/12$, $\pi/6$, $\pi/4$, $\pi/3$ for different time moments. The calculations are carried out according to the following parameters: $\upsilon_\zeta=5$; $\overline{\lambda}_t^{(1)}=1$; $\overline{\lambda}_t^{(2)}=0{,}1622$; $\overline{a}_1=1$; $\overline{a}_2=0{,}1313$; $Bi_0=1$, $Bi_n=0$; $\rho_0=0{,}8$; $\rho_1=0{,}9$; $\rho_2=1$; $\bar{k}=4$; $\alpha_t^{(2)}/\alpha_t^{(1)}=0{,}339$; $\nu=0{,}33$.

Certain results are presented in Figures 1 – 8, where curves 1 – 4 correspond to $Fo=0{,}01$; $0{,}05$; $0{,}1$; $0{,}2$.

It is evident from the investigations that the nature of the temperature distribution for $\varphi=0$ (given in [3]) and at other φ values is the same. When φ increases the corresponding temperatures decrease. At $Fo>0{,}2$ the quasi-stationary mode occurs.

If $0\le\varphi\le\pi/12$ and $Fo\le 0{,}01$ the radial stresses $\overline{\sigma}_{rr}$ (their behavior at $\varphi=0$ is shown in Figure 1) in the middle of each layer are tensile. In course of time, they change into compression stresses in the first layer, including the contact area. Moreover, the compression area in the second layer increases with the time increment. At other φ values, the behavior of radial stress is significantly different. Particularly when $\varphi=\pi/3$ stress in the middle of each layer is tensile to $Fo=0{,}1$ value inclusive.

The tangential stresses $\overline{\sigma}_{r\varphi}$ at $Fo\ge 0{,}05$ for considered values φ are compressive in the first layer (except $\varphi=0$, because in this case $\overline{\sigma}_{r\varphi}=0$). At $Fo\le 0{,}01$ depending on φ they can be both compressive and tensile. In the second layer at $0<\varphi\le\pi/6$ they are tensile in the middle of the area adjacent to the external surface for all time moments. When $\varphi=\pi/4$, $\pi/3$ and $Fo\le 0{,}01$ these stresses are tensile in the middle of each layer. The behavior of stresses $\overline{\sigma}_{r\varphi}$ is shown in Fig. 2

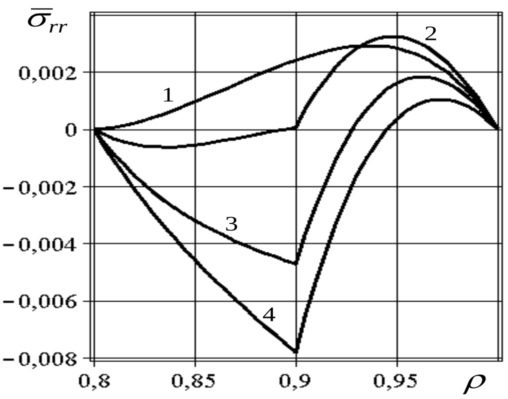

Figure 1. Dependencies of stresses $\overline{\sigma}_{rr}$ on the coordinate ρ at $\varphi=0$

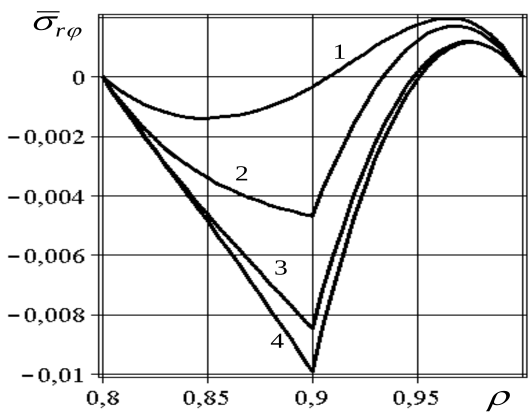

Figure 2. Dependencies of stresses $\overline{\sigma}_{r\varphi}$ on the coordinate ρ at $\varphi=\pi/12$

The behavior of tangential stresses $\overline{\sigma}_{rz}$ and $\overline{\sigma}_{z\varphi}$, which are mostly compressible, is depicted in Figures 3 and 4. Note $\overline{\sigma}_{z\varphi}=0$ when $\varphi=0$.

If the character of behaviour $\bar{\sigma}_{\varphi\varphi}$ (Fig. 5) and $\bar{\sigma}_{zz}$ (Fig. 6) at $\varphi = 0$ is the same, then the numerical values of the corresponding stresses differ significantly. Thus, on the distribution surface, the maximum values of the compressive stresses $\bar{\sigma}_{\varphi\varphi}$ are almost half, and the tensile ones are almost twice as large as the values of axial stresses. Moving apart from the heating centre the redistribution of stresses occurs. It is shown in Fig. 7.8.

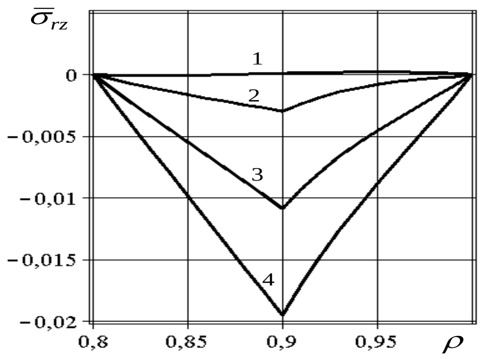

Figure 3. Dependencies of stresses $\bar{\sigma}_{rz}$ on the coordinate ρ at $\varphi = 0$

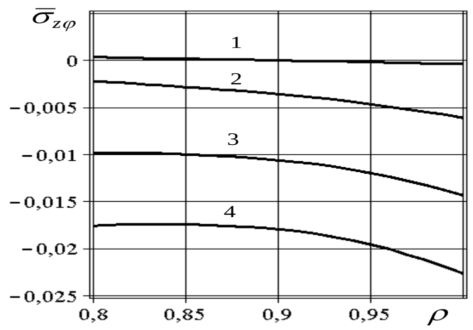

Figure 4. Dependencies of stresses $\bar{\sigma}_{z\varphi}$ on the coordinate ρ at $\varphi = \pi/12$

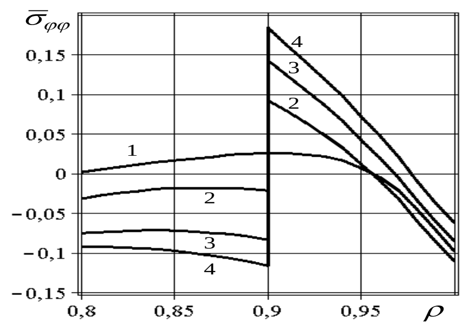

Figure 5. Dependencies of stresses $\bar{\sigma}_{\varphi\varphi}$ on the coordinate ρ at $\varphi = 0$

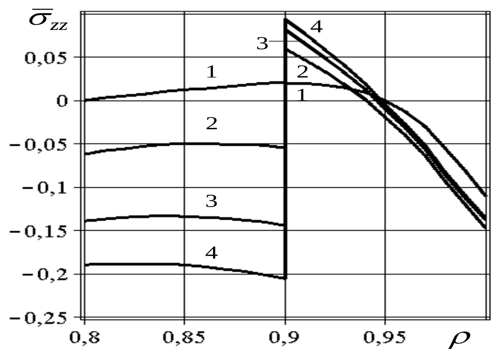

Figure 6. Dependencies of stresses $\bar{\sigma}_{zz}$ on the coordinate ρ at $\varphi = 0$

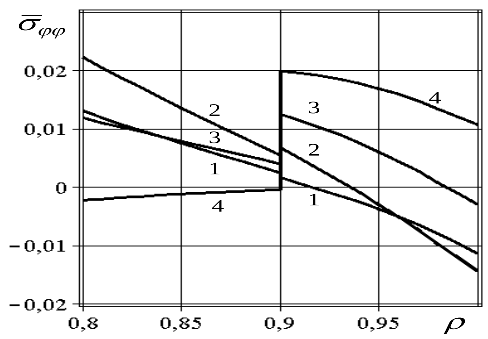

Figure 7. Dependencies of stresses $\bar{\sigma}_{\varphi\varphi}$ on the coordinate ρ at $\varphi = \pi/3$

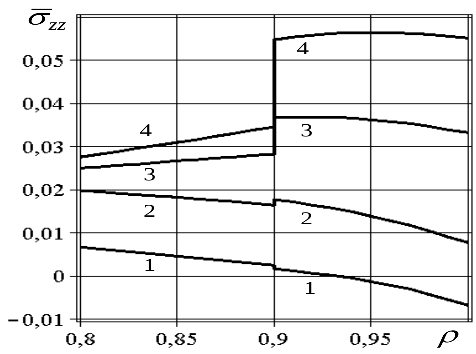

Figure 8. Dependencies of stresses $\bar{\sigma}_{zz}$ on the coordinate ρ at $\varphi = \pi/3$

The obtained investigation results conform to results presented in [2].

Conclusions. The solution of a non-axisymmetric quasistatic thermoelasticity problem for the multi-layered long hollow cylinder with identical Lame coefficients which enables to investigate the influence of geometric and thermomechanical characteristics on the thermoelastic state of a cylinder under different laws of environment temperature changes, density of heat sources and initial temperature is obtained. The solution is given through the Green's functions for the corresponding thermoelasticity problem. Numerical results are given for two-layer cylinder heated by normally distributed on the external surface heat stream moving along the cylinder derivative. The subject of further investigations is the construction of the solution of the corresponding non-axisymmetric thermoelasticity problem for the cylinder under different Lame coefficients.

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УДК 539.3

РОЗВ'ЯЗОК НЕОСЕСИМЕТРИЧНОЇ КВАЗІСТАТИЧНОЇ ЗАДАЧІ ТЕРМОПРУЖНОСТІ ДЛЯ БАГАТОШАРОВОГО ЦИЛІНДРА ЗА ОДНАКОВИХ КОЕФІЦІЄНТІВ ЛАМЕ

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Резюме. *За допомогою побудованих функцій Гріна квазістатичної задачі термопружності визначено неосесиметричний термопружний стан багатошарового необмеженого порожнистого циліндра з однаковими коефіцієнтами Ламе шарів за дії внутрішніх та поверхневих джерел тепла і нерівномірного розподілу початкової температури. Досліджено термопружний стан двошарового циліндра, зумовлений нормально розподіленим на зовнішній поверхні циліндра потоком тепла, який рухається по твірній циліндра.*

Ключові слова: *багатошаровий циліндр, термопружність, функції Гріна.*

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