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CALCULATION OF THE PLATE PLANE STRESS IN POLAR COORDINATE SYSTEM ON THE BASIS OF LAMÉ EQUATION GENERAL SOLUTION

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Summary. *The mathematical model describing plane stress-strain state of plane elastic bodies in polar coordinate system is offered. To describe its three-dimensional stress state, three harmonic functions expressing the general solution of Lamé equations in cylindrical coordinate system are used. After stresses integration on the thickness of the plate normal and tangential efforts are expressed through two two-dimensional harmonic and biharmonic functions. The closed equation system in partial derivatives is developed on the introduction of two-dimensional functions without using hypotheses about the geometric nature of plate deformation. Three-dimensional boundary conditions are reduced to two-dimensional form. The example of stress-strain state of disk is given.*

Key words*: disk, ring, polar coordinate system, effort, boundary conditions.*

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Introduction. Plates in the form of discs or rings described by plane stress state [1 – 4] are widely used in objects of transport, power engineering and construction engineering industry. The development of science and technology puts forward new high demands to the accuracy their strength and holding ability investigations. Therefore, there is the need for more complete consideration of the equations and relations of the elasticity theory in cylindrical coordinate system by simplifying the initial calculation models due to their reduction to two-dimensional case.

Analysis of the available investigation results. Although theoretical methods for stress calculation in circular or ring plates under the influence of loads on certain part of the lateral surface have been developed since the beginning of the 20-th century [1 – 5], this problem is still important nowadays, because of its importance for practical design and materials science [6]. The stressed state of thin plates is, in general, is calculated by the equations of the plane problem of the elasticity theory obtained in the Cartesian coordinate system, [1 – 4], and the thick ones – by uniform solutions and symbolic method [2, 7], harmonic and biharmonic functions [1 – 3], hypotheses about the behavior of the normal to the median surface [1, 2], the decomposition of three-dimensional stressed state by the normal to the median surface variable [2, 4].

The objective of the paper is to develop closed two-dimensional calculation model describing the plane stressed state of the rings in polar coordinate system based on general solution of Lamé equations, as well as to express the displacement and effort in plates by formulas that are consistent with three-dimensional elasticity theory.

Statement of the problem and presentation of the solution. Let us consider the plane problem of the elasticity theory for ring with radius R_j, $j=\overline{1,2}$, of constant thickness h, the plane medial surface of which coincides with the plane $Or\varphi$ of cylindrical coordinate system. On its flat surfaces ($z=h_j$, $j=\overline{1,2}$, $h_1=h/2$, $h_2=-h/2$) there are no normal and tangential stresses, and symmetric and parallel to the median surface loads are applied to the lateral surfaces:

$$\sigma_r(R_j,\varphi,z)=\sigma_1^j(\varphi,z)\,,\ \tau_{r\varphi}(R_j,\varphi,z)=\sigma_2^j(\varphi,z)\,,\ \tau_{rz}(R_j,\varphi,z)=0\,, \qquad (1)$$

where $\sigma_m^j(\varphi,-z)=\sigma_m^j(\varphi,z)$, $j,m=\overline{1,2}$ are known loads.

To develop the plate plane stressed state in polar coordinate system and integral satisfaction of conditions (1), we use the general representation of Lamé equations solution [8] in a cylindrical coordinate system

$$u_r=\frac{\partial P}{\partial r}+\frac{1}{r}\frac{\partial Q}{\partial\varphi}\,,\ u_\varphi=\frac{1}{r}\frac{\partial P}{\partial\varphi}-\frac{\partial Q}{\partial r}\,,\ u_z=\frac{\partial P}{\partial z}-4(1-\nu)\Phi\,, \qquad (2)$$

where $P=z\Phi+\Psi$ is biharmonic function; Φ, Ψ are introduced [8] harmonic functions of displacements; ν is the Poisson ratio. Function P satisfies the equation

$$\Delta P+\frac{\partial^2}{\partial z^2}P=2\frac{\partial}{\partial z}\Phi\,, \qquad (3)$$

where

$$\Delta P=[\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial\varphi^2}]P=[\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial\varphi^2}]P\,.$$

It follows from the relations (1), (2) and the stresses expression [8] that the functions P, Q are odd relatively to variable z, and function Φ is even.

Taking into account the stresses representation [8], we write the boundary conditions for the plates free from the loads:

$$\sigma_z(\pm h_1)=\frac{\partial^2 P^\pm}{\partial z^2}-2(2-\nu)\frac{\partial\Phi^\pm}{\partial z}=0\,,$$

$$r\frac{\partial}{\partial r}[2\frac{\partial P^+}{\partial z}-\chi\Phi^+]+\frac{\partial}{\partial\varphi}\frac{\partial Q^+}{\partial z}=0,\qquad \frac{\partial}{\partial\varphi}[2\frac{\partial P^+}{\partial z}-\chi\Phi^+]-r\frac{\partial}{\partial r}\frac{\partial Q^+}{\partial z}=0\,, \qquad (4)$$

where signs «+», «–» describe the boundary values of the corresponding functions on the upper $z=h_1$ and lower $z=-h_1$ surfaces of the plate, $\chi=4(1-\nu)$.

Equations (4) result in the following harmonic conditions for the boundary values of the introduced functions:

$$\Delta[\frac{\partial P^+}{\partial z}-2(1-\nu)\Phi^+]=0\,,\ \Delta\frac{\partial Q^+}{\partial z}=0\,. \qquad (5)$$

Considering that the normal stresses σ_z [1, 3] for the plane loading plate are insignificant after their integration along the axis Oz, we find the following relation:

$$\frac{\partial P^+}{\partial z}=2(2-\nu)\Phi^+\,. \qquad (6)$$

Let us insert the dependence (6) in the first equation (5) and get:

$$\Delta\Phi^{+} = 0\,,\ \Delta\frac{\partial P^{+}}{\partial z} = 0\,. \tag{7}$$

We use formula (6) and simplify the second and third equations (4):

$$4r\frac{\partial}{\partial r}\Phi^{+} + \frac{\partial}{\partial\varphi}\frac{\partial Q^{+}}{\partial z} = 0, \qquad 4\frac{\partial\Phi^{+}}{\partial\varphi} - r\frac{\partial}{\partial r}\frac{\partial Q^{+}}{\partial z} = 0\,. \tag{8}$$

Let us use the found [8] stress components in cylindrical coordinate system and express the effort in the plate in polar coordinate system

$$T_{\text{r}} = 2G\{\frac{\partial^2\tilde{P}}{\partial r^2} - 4\nu\Phi^{+} + \frac{\partial}{\partial r}\frac{\partial\tilde{Q}}{r\partial\varphi}\}\,,\ \ T_{\varphi} = 2G\{\frac{1}{r^2}\frac{\partial^2\tilde{P}}{\partial\varphi^2} + \frac{1}{r}\frac{\partial\tilde{P}}{\partial r} - 4\nu\Phi^{+} - \frac{\partial}{\partial r}\frac{\partial\tilde{Q}}{r\partial\varphi}\}\,,$$

$$T_r + T_\varphi = 2G[\Delta\tilde{P} - 8\nu\Phi^{+}] = -4E\Phi^{+}\,,\ S_{rz} = S_{z\varphi} = 0\,, \tag{9}$$

$$S_{r\varphi} = \int_{-h_1}^{h_1}\tau_{\text{r}\varphi}dz = G\{\frac{2}{r}\frac{\partial^2\tilde{P}}{\partial r\partial\varphi} - \frac{2}{r^2}\frac{\partial\tilde{P}}{\partial\varphi} - r\frac{\partial}{\partial r}\frac{1}{r}\frac{\partial\tilde{Q}}{\partial r} + \frac{1}{r^2}\frac{\partial^2\tilde{Q}}{\partial\varphi^2}\}\,,$$

where $\tilde{P} = \int_{-h_1}^{h_1}Pdz$, $\tilde{Q} = \int_{-h_1}^{h_1}Qdz$.

Replacing the load action (1) with normal and tangential efforts (9) we get the following boundary conditions for their determination:

$$T_r(R_j,\varphi) = T^j(\varphi), \quad S_{r\varphi}(R_j) = S^j(\varphi)\,,\ j = \overline{1,2} \tag{10}$$

where $T^j = \int_{-h_1}^{h_1}\sigma_{1n}^j dz$, $S^j = \int_{-h_1}^{h_1}\sigma_2^j dz$. Using equations (3), (6), the harmonious condition of displacement functions, we find the following key equations of the plane elasticity theory plates:

$$\Delta\tilde{P} = -4(1-\nu)\Phi^{+}\,,\ \Delta\tilde{Q} = -2\frac{\partial Q^{+}}{\partial z}\,, \tag{11}$$

where functions Φ^{+} and $\frac{\partial Q^{+}}{\partial z}$ are harmonic, and $\tilde{P}$, $\tilde{Q}$ are biharmonic functions.

It should be noted that the use of relations (8), (9), (11) results in satisfaction of the known plate equilibrium equations in efforts [1, 2, 4].

Let us represent the general solution of the first harmonic equation (7) in the following way

$$\Phi^{+} = \frac{\partial f(r,\varphi)}{\partial\varphi} + h\text{b}_0^1\,, \tag{12}$$

where $\frac{\partial f(r,\varphi)}{\partial\varphi}$ is unknown harmonic function. We use the expression (12), the relation (8) between the harmonic functions and derive the dependence:

$$\frac{\partial Q^{+}}{\partial z}=-4r\frac{\partial f}{\partial r}, \tag{13}$$

where $\Delta r\frac{\partial f}{\partial r}=0$. Let us consider relations (12), (13) and specify the representation of the second equation (11)

$$\Delta\tilde{Q}=8r\frac{\partial f}{\partial r}. \tag{14}$$

To simplify further presentation we will consider symmetric normal stresses relatively to angle φ. Let us represent the desired functions f, Φ^{+} and biharmonic parts of functions $\tilde{P}$, $\tilde{Q}$ in the form of series, find partial solutions of the equations (11), (14) and derive

$$\mathrm{f}=h\sum_{\mathrm{k}=1}^{\infty}(\mathrm{b}_{\mathrm{k}}^{1}r^{k}+b_{\mathrm{k}}^{2}r^{-k})\sin k\varphi, \tag{15}$$

$$\Phi^{+}=h\sum_{\mathrm{k}=1}^{\infty}\mathrm{k}(\mathrm{b}_{\mathrm{k}}^{1}r^{k}+b_{\mathrm{k}}^{2}r^{-k})\cos k\varphi+h\mathrm{b}_{0}^{1}, \tag{16}$$

$$\tilde{\mathrm{P}}_{1}=h\delta_{0}^{1}\mathrm{b}_{0}^{1}r^{2}+h(\delta_{1}^{1}\mathrm{b}_{1}^{1}r^{3}+\delta_{1}^{2}\mathrm{b}_{1}^{2}r\ln r)\cos\varphi+h\sum_{\mathrm{k}=2}^{\infty}(\delta_{\mathrm{k}}^{1}\mathrm{b}_{\mathrm{k}}^{1}r^{k+2}+\delta_{\mathrm{k}}^{2}\mathrm{b}_{\mathrm{k}}^{2}r^{-k+2})\cos k\varphi, \tag{17}$$

$$\tilde{Q}_{1}=h(\delta_{\mathrm{k}}^{3}\mathrm{b}_{\mathrm{k}}^{1}r^{3}+\delta_{1}^{4}\mathrm{b}_{1}^{2}r\ln r)\sin\varphi+h\sum_{\mathrm{k}=2}^{\infty}(\delta_{\mathrm{k}}^{3}\mathrm{b}_{\mathrm{k}}^{1}r^{k+2}+\delta_{\mathrm{k}}^{4}\mathrm{b}_{\mathrm{k}}^{2}r^{-k+2})\sin k\varphi, \tag{18}$$

where $b_{\mathrm{k}}^{\mathrm{j}}$ are unknown constants,

$$\delta_{0}^{1}=\nu-1,\ \delta_{1}^{1}=-\frac{1-\nu}{2},\ \delta_{1}^{2}=-2(1-\nu),\ \delta_{\mathrm{k}}^{1}=-\frac{\mathrm{k}(1-\nu)}{k+1},\ \delta_{\mathrm{k}}^{2}=-\frac{\mathrm{k}(1-\nu)}{k-1},$$

$$\delta_{1}^{3}=1,\ \delta_{1}^{4}=-4,\ \delta_{\mathrm{k}}^{3}=\frac{2\mathrm{k}}{k+1},\ \delta_{\mathrm{k}}^{4}=\frac{2\mathrm{k}}{k-1},\ k=2,3...$$

Taking into account representation (12) – (18) we express the general solution of equations (11)

$$\tilde{P}=\tilde{\mathrm{P}}_{1}+g_{1}(r,\varphi)+\mathrm{a}_{0}^{1}\ln\mathrm{r},\ \tilde{Q}=\tilde{Q}_{1}+g_{2}(r,\varphi), \tag{19}$$

where $\tilde{\mathrm{P}}_{1}$, $\tilde{Q}_{1}$ are biharmonic, g_j are harmonic functions that can be, exactly to the constant, submitted in the following way:

$$g_{1}=r\frac{\partial}{\partial r}[\phi-\psi],\quad g_{2}=\frac{\partial}{\partial\varphi}[\phi+\psi], \tag{20}$$

where ϕ, ψ are harmonic functions.

Let us apply functions (12) – (20) to relation (9) and express efforts through the introduced functions

$$T_{\rm r} = 2G\{\frac{\partial^2}{\partial r^2}[\tilde{\rm P}_1 - 2r\frac{\partial}{\partial r}\psi] - 4\nu\Phi^+ + \frac{\partial}{\partial r}\frac{\partial}{r\partial\varphi}\tilde{\rm Q}_1] - {\rm a}_0^1 r^{-2},$$

$$T_\varphi = 4E\Phi^+ - T_r, \tag{21}$$

$$S_{r\varphi} = G\{\frac{2}{r}\frac{\partial^2}{\partial r\partial\varphi}\tilde{\rm P}_1 - \frac{2}{r^2}\frac{\partial}{\partial\varphi}\tilde{\rm P}_1 - r\frac{\partial}{\partial r}\frac{1}{r}\frac{\partial}{\partial r}\tilde{\rm Q}_1 + \frac{1}{r^2}\frac{\partial^2}{\partial\varphi^2}\tilde{\rm Q}_1 - 4\frac{\partial}{\partial r}\frac{\partial^2}{\partial r\partial\varphi}\psi\}.$$

Hence, function ϕ is not part of the efforts representation (21) and can not be taken into account while calculating the plate stress state. The relation (20) is simplified to the form

$$g_1 = r\frac{\partial\psi}{\partial r}, \quad g_2 = \frac{\partial\psi}{\partial\varphi}.$$

Let us take into account relations (2), (6) and find lateral displacements and deformations of the plate plane surfaces: $u_z^+ = 2\nu\Phi^+$, $e_z^+ = 2\nu\frac{\partial\Phi^+}{\partial z}$. For thin plate, the dependence $\frac{\partial\Phi^+}{\partial z} = \frac{2}{h}\Phi^+$ which is consistent with three-dimensional elasticity theory is derived.

Since all relations of the elasticity theory of are exactly satisfied, then we can determine the plane displacements and deformations in the plate after formulas (2) averaging

$$u_{\rm r} = \frac{1}{h}[\frac{\partial\tilde{P}}{\partial r} + \frac{1}{r}\frac{\partial\tilde{Q}}{\partial\varphi}],\ u_\varphi = \frac{1}{h}[\frac{1}{r}\frac{\partial\tilde{P}}{\partial\varphi} - \frac{\partial\tilde{Q}}{\partial r}], \tag{22}$$

and stresses by dividing the relative efforts (21) on the plate thickness. Relations (21), (22) completely describe the plane stressed-deformed state of plates in polar coordinate system.

Numerical implementation of the method. As an example of the developed model use, we determine the stressed state of the disk with radius $\rm R$ under the action of normal loads distributed symmetrically relatively to the axis Ox ($\rm x = r\cos\varphi$)

$$\sigma_r(R,\varphi) = \sigma_1(\varphi),\ \ \tau_{r\varphi}(R,\varphi) = \sigma_2(\varphi), \quad \varphi\in[0,2\pi], \tag{23}$$

where known loads $\sigma_1(\varphi)$ are odd and $\sigma_2(\varphi)$ are even relatively to angle φ and can be submitted in the form of the following series:

$$\sigma_1(\varphi) = \sum_{\rm k=0}^{\infty}{\rm d}_{\rm k}^1\cos k\varphi,\ \sigma_2(\varphi) = \sum_{\rm k=1}^{\infty}{\rm d}_{\rm k}^2\sin k\varphi. \tag{24}$$

While calculating the stressed state of disk coefficients $b_{\rm k}^2$ will be equal to zero in the formulas (15) – (18), and function ψ has the form

$$\psi = h\sum_{\rm k=1}^{\infty}{\rm a}_{\rm k}^1 r^k\cos k\varphi. \tag{25}$$

After substitution of functions (15) – (18), (25) into the relation (22) we define the displacement

$$u_{\mathrm{r}} = \sum_{\mathrm{k}=0}^{\infty}\{[(k+2)\delta_{\mathrm{k}}^{1} + k\delta_{\mathrm{k}}^{3}]\mathrm{b}_{\mathrm{k}}^{1}(R\alpha)^{k+1} - 2\mathrm{k}^{2}\mathrm{a}_{\mathrm{k}}^{1}(R\alpha)^{k-1}]\cos k\varphi ,$$

$$u_{\varphi} = -\sum_{\mathrm{k}=0}^{\infty}\{[k\delta_{\mathrm{k}}^{1} + (k+2)\delta_{\mathrm{k}}^{3}]\mathrm{b}_{\mathrm{k}}^{1}(R\alpha)^{k+1} - 2\mathrm{k}^{2}\mathrm{a}_{\mathrm{k}}^{1}(R\alpha)^{k-1}]\sin k\varphi\} , \qquad (26)$$

and from relations (21) we determine the stress

$$\sigma_{\mathrm{r}} = 2G\{\sum_{\mathrm{k}=0}^{\infty}\{[\chi_{\mathrm{k}}^{1}\mathrm{b}_{\mathrm{k}}^{1}\alpha^{k} - \chi_{\mathrm{k}}^{2}\mathrm{a}_{\mathrm{k}}^{1}\alpha^{k-2}]\cos k\varphi - 4\nu\mathrm{b}_{0}^{1}\} ,$$

$$\sigma_{\varphi} = 4E\{\sum_{\mathrm{k}=1}^{\infty}\mathrm{kb}_{\mathrm{k}}^{1}r^{k}\cos k\varphi + \mathrm{b}_{0}^{1}\} - \sigma_{r} , \qquad (27)$$

$$\tau_{r\varphi} = 2G\{\sum_{\mathrm{k}=1}^{\infty}\{\chi_{\mathrm{k}}^{3}\mathrm{b}_{\mathrm{k}}^{1}\alpha^{k} + \chi_{\mathrm{k}}^{2}\mathrm{a}_{\mathrm{k}}^{1}\alpha^{k-2}]\}\sin k\varphi ,$$

where $\mathrm{b}_{\mathrm{k}}^{1}$, $\mathrm{a}_{\mathrm{k}}^{1}$ are unknown coefficients,

$$\alpha = \frac{r}{R},\ \chi_{\mathrm{k}}^{1} = \{(\mathrm{k}+1)[(k+2)\delta_{\mathrm{k}}^{1} + k\delta_{\mathrm{k}}^{3}] - 4\nu\mathrm{k}\}R^{k} ,$$

$$\chi_{\mathrm{k}}^{2} = 2(\mathrm{k}-1)\mathrm{k}^{2}\mathrm{a}_{\mathrm{k}}^{1}R^{k-2},\ \chi_{\mathrm{k}}^{3} = -k(k+1)(\delta_{\mathrm{k}}^{1} + \delta_{\mathrm{k}}^{3})R^{k} .$$

Let us substitution stress (27) into the boundary conditions (23), (24) and define the unknown coefficients

$$k = 0:\ \mathrm{b}_{0}^{1} = \frac{d_{0}^{1}}{(\chi_{0}^{1} - 4\nu)2\mathrm{G}} ;$$

$$k > 0:\ \mathrm{b}_{\mathrm{k}}^{1} = \frac{d_{\mathrm{k}}^{1} + d_{\mathrm{k}}^{2}}{2G(\chi_{\mathrm{k}}^{1} + \chi_{\mathrm{k}}^{3})},\ \mathrm{a}_{\mathrm{k}}^{1} = \frac{2G\chi_{\mathrm{k}}^{1}\mathrm{b}_{\mathrm{k}}^{1} - d_{\mathrm{k}}^{1}}{2G\chi_{\mathrm{k}}^{2}} . \qquad (28)$$

Introducing coefficients (28) into the formulas (27), (28) we can determine the displacement and the stress in the circular disk with required accuracy.

Conclusions. It is determined that on the basis of the general solution of Lamé equations in cylindrical coordinate system, it is possible to develop two-dimensional plane elasticity theory of plates in polar coordinate system without the application of hypotheses about the distribution of displacements and stresses. Mathematical and physical rigor while developing the calculation formulas of the plane problem theory of elasticity is observed. Normal and tangent efforts exactly satisfy the plate equilibrium equation. It is shown that the found stresses and displacements are exactly equal to the relative averaged values of the stresses of three-dimensional elasticity theory. The obtained results can be used in calculating the plane stressed state of both thick and thin disks and rings.

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РОЗРАХУНОК ПЛОСКОГО НАПРУЖЕНОГО СТАНУ ПЛАСТИН У ПОЛЯРНІЙ СИСТЕМІ КООРДИНАТ НА ОСНОВІ ЗАГАЛЬНОГО РОЗВ'ЯЗКУ РІВНЯНЬ ЛЯМЕ

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Резюме. *Розглянуто плоску задачу теорії пружності для кільця сталої товщини, плоска серединна поверхня якого збігається з координатною площиною циліндричної системи координат. На зовнішніх плоских поверхнях кільця відсутні нормальні та дотичні навантаження, а до бічних циліндричних поверхонь прикладені навантаження, симетричні й паралельні серединній поверхні. Побудова плоского напруженого стану пластин у полярній системі координат ґрунтується на використанні загального розв'язку рівнянь теорії пружності. Для описування її тривимірного напруженого стану використано три гармонічних функції, які виражають загальний розв'язок рівнянь Ляме в циліндричній системі координат. Після інтегрування напружень по товщині пластини виражено нормальні й дотичні зусилля через дві двовимірні гармонічні та бігармонічні функції. Враховано крайові умови на вільних від навантажень поверхнях пластини і побудовано замкнену систему рівнянь у часткових похідних на введені двовимірні функції без використання гіпотез про геометричний характер деформування пластини. Встановлено, що використання отриманих співвідношень призводить до задоволення рівнянь рівноваги пластини в зусиллях. Для інтегрального задоволення крайових умов дію навантажень на зовнішніх циліндричних поверхнях кільця замінено нормальними та дотичними зусиллями. Записано подавання напружень і переміщень плоскої задачі теорії пружності за формулами, які узгоджуються з тривимірною теорією пружності. Тривимірні крайові умови зведено до двовимірного вигляду. Розроблено математичну модель, яка описує плоский напружено-деформований стан пластин у полярній системі координат. Наведено приклад розрахунку напружено-деформованого стану диска.*

Ключові слова: *диск, кільце, полярна система координат, зусилля, крайові умови.*

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