

# Synthesis of robust interconnected power system stabilizers for turbine generators in sugar factories

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## Abstract

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**Introduction.** This research aims to develop the method of synthesis of interconnected robust power system stabilizer (IRPSS) for power supply systems of sugar factories with own turbine generators.

**Materials and methods.** The mathematical tools of  $H_\infty$  synthesis with pole placement and the linear matrix inequalities (LMI) are used for developing the method of IRPSS synthesis. MATLAB Simulink is used to verify synthesized regulator by computer simulation.

**Results and discussion.** A model of power supply system with own generator, automatic exciter regulator, turbine and speed governor is constructed. The controlled object nonlinear differential equations model is linearized and simplified by reducing of its order using Schur's method. The model is extended with chosen weighting functions preparing for  $H_\infty$  synthesis procedure. The LMI region is chosen in the form of a conic sector for satisfying condition of lower oscillability of transient process, which is the main goal of such a regulator to optimize turbine generator operation process.

The IRPSS for extended model was synthesized, using  $H_\infty$  synthesis procedure with poles placement. The regulator is presented in the form of matrix continuous transfer function.

The transition process of 3-phase short-circuit fault behind the transformer and subsequent automatic re-closing is simulated with full-order nonlinear model with IRPSS and compared with standard system stabilizer and the system without stabilizer. The graphs of modelled transients demonstrate effectiveness of IRPSS.

**Conclusion.** The developed method can be used for operation process optimization of turbine generators in sugar factories. The synthesized IRPSS was found to have satisfying robust stability and performance qualities for given structure of power system.

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## Introduction

A power supply system of a sugar factory may enclose the turbine generators to cover the electricity requirements during peak loads. The availability of its own turbine units allows saving the heat and electrical energy in both production and standing periods of manufacturing process. As turbine generators in such a scheme work a lot of time in a state of transient processes, it is important to ensure their stable operation using power system stabilizers [10, 13].

In the mathematical model of electrical turbine generators connected to an electrical system, it is not always possible to determine exactly all the parameters. Moreover, because of simplification and linearization of models there is the so-called unmodeled dynamics [11, 12]. This leads to the fact that the system stabilizers that are synthesized for one set of model parameters do not always provide the desired level of stability and quality control for other parameters values within the permissible limits.

There are techniques to synthesize controllers that are insensitive to changes of model parameters in certain ranges, which are called robust control design methods. In works [1, 6, 9] some variants of the synthesis of robust controllers for power systems were proposed. In this work, the goal is to expand these results and apply these techniques for controllers that stabilize the operation of the turbine unit, using not only the traditional channel of excitation system, but also an additional channel of the steam turbine governing system.

## Materials and methods

**The method of  $H_\infty$  synthesis with pole placement.** Stability of the synthesized controller in this paper is based on the small gain theorem [7].

Let  $RH_\infty$  be the space of stable proper rational functions, i.e. functions of the form  $G(s) = A(s)/B(s)$ , where  $A$  is the polynomial,  $B$  – a polynomial with roots with negative real part, and  $\deg A \leq \deg B$ . If  $M(s)$  is the matrix with elements  $m_{ij}(s)$ , the expression  $M(s) \in RH_\infty$  means that  $m_{ij}(s) \in RH_\infty$  for all  $i, j$ .

The norm  $\|\cdot\|_\infty$  in space  $RH_\infty$  is defined as  $\|W(s)\|_\infty = \sup_{\omega \in R} \|W(j\omega)\|_2$ , where  $\|\cdot\|_2$  – is the spectral norm of the matrix, which is equal to its largest singular value  $\bar{\sigma}$ .

The small gain theorem states, that if  $M(s) \in RH_\infty$ , then matrix  $(I + M(s)\Delta(s))^{-1}$  exists and belongs to  $RH_\infty$  for all  $\Delta(s) \in RH_\infty$ ,  $\|\Delta(s)\|_\infty \leq 1/\gamma$  if and only if, when  $\|M(s)\|_\infty \leq \gamma$ . From this theorem appears that if we take the additive model of uncertainty  $G_{pert} = G + \Delta W_2$ , where  $G$  is the model with nominal parameters,  $\Delta$  is an arbitrary stable transfer function that corresponds to the condition  $\|\Delta\|_\infty < 1$ , and  $W_2$  is weighting function, which generally belongs  $RH_\infty$ , the robust stability of the closed-loop system with the controller  $K(s)$  for all  $\|\Delta(s)\|_\infty \leq 1$  takes place if  $\|W_2 K S\|_\infty \leq 1$ , where  $S = (I + M(s)K(s))^{-1}$  is so called output sensitivity function.

Based on the above, one of the quality criteria, which appears to be the condition of searching for the optimal robust controller, will be the criterion  $\min \|W_2KS\|_\infty$ . It should be noted that this criterion is responsible for control signal bounds. Consequently, if there are clear requirements to the control signal, such that they expressed the weighting function  $W_2'$ , then for the specified criteria the weighting function which is need to be selected among  $W_2$  and  $W_2'$ , will be that the graph of maximum singular number  $\bar{\sigma}(i\omega)$  of which is greater.

To meet the requirements of the usual performance specifications there is another criterion that should be optimized along with the above one:  $\min \|W_1S\|_\infty$ , where  $W_1 \in RH_\infty$  is the another weighting function. It follows from requirements of minimisation of the control error and, as a rule, should provide a high quality of tracking and disturbance attenuation at the object's output.

But using the weighting function  $W_1$  is very difficult to specify some transient process specifications, for example, its dumping coefficient, so in addition to the restrictions that can be put together in this way:  $\min \left\| \begin{matrix} W_1S \\ W_2KS \end{matrix} \right\|_\infty$ , in [1] it was proposed to introduce an additional constraint on the poles of the closed-loop system in the form of regions that can be described in the form of linear matrix inequalities (LMI).

A region in the complex plane can be described using expressions of the form

$$D = \{z \in C \mid L + zM + \bar{z}M^T < 0\},$$

where  $L$  is some symmetric matrix  $m \times m$ ,  $M$  is an arbitrary matrix  $m \times m$ . Here the matrix-valued function  $f_D = L + zM + \bar{z}M^T$  is denoted as the characteristic function of the LMI region  $D$ .

Consider an example of such a region, which limits the dumping coefficient of closed-loop object (Fig. 1). This area is called conic sector and guarantees that the damping ratio is not less than  $\xi = \cos \theta$  or the oscillability is not more than  $\mu = \text{tg} \theta$ , where  $\mu = \frac{\beta}{\alpha}$  and  $a + j\beta$  are the complex roots of the closed-loop system. The region  $D$  of the conic sector can be written as

$$D = \left\{ z \mid \text{Re } z < 0, \frac{|\text{Im } z|}{|\text{Re } z|} < \text{tg} \theta \right\},$$

or through matrix inequalities

$$D = \left\{ z \in C \mid \begin{pmatrix} \sin \theta & \cos \theta \\ -\cos \theta & \sin \theta \end{pmatrix} z + \begin{pmatrix} \sin \theta & -\cos \theta \\ \cos \theta & \sin \theta \end{pmatrix} \bar{z} < 0 \right\}.$$

where the matrix  $L = 0$  and  $M = \begin{pmatrix} \sin \theta & \cos \theta \\ -\cos \theta & \sin \theta \end{pmatrix}$ .

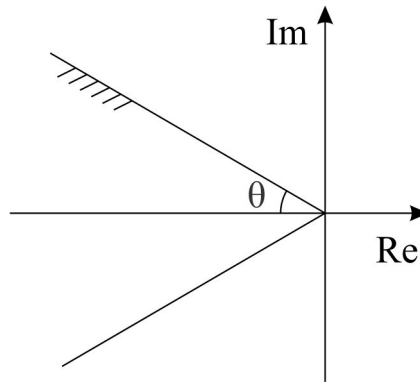


Figure 1. LMI region – conic sector

### Mathematical model of controlled object

Steam turbines mathematical models in calculations of transients in electrical systems should reflect the impact of speed governing system to turbine output power when rotational frequency is changing [3]. The models of steam turbines are generally based on the assumption of the constancy of the steam pressure before the control valves. Thus the control system of the boiler is not taken into account. This determines the scope of the mathematical model of the steam turbine: it is adequate to those transients which endure no more than 5-10 seconds from the moment of frequency change.

The equation of steam turbine governing system [8]

$$\frac{d\mu_*}{dt} = \frac{1}{T_c} \left( \frac{s}{\sigma} - \mu_* + \mu_{0*} \right);$$

$$\mu_{\min} \leq \mu_* \leq \mu_{\max}; \mu_* = \mu / \mu_{nom}; \mu_{0*} = \mu_0 / \mu_{nom}; s = (\omega_{nom} - \omega) / \omega_{nom},$$

where  $\sigma$  – the droop of the governing system of the turbine;  $T_c$  – time constant of the servomotor that moves the valves of the turbine;  $\mu_*$  – the current value of the servomotor displacement that equals in per unit system to the displacement of equivalent responsive valve of the turbine;  $\mu_{nom}$  – is the nominal value of the servomotor displacement that corresponds to the nominal power of the turbine at the nominal mode parameters of the unit;  $\mu_{0*}$  – the initial value of the servo motor position;  $s$  – slip;  $\omega$ ,  $\omega_{nom}$  – current and nominal frequency value;  $\mu_{\min}$ ,  $\mu_{\max}$  – the restrictions of the servomotor displacement, minimal and maximal respectively.

The equation of the turbine

$$D = \mu_* p_T; P_{HP} = Dk_{HP}; \frac{dP_{LMP}}{dt} = \frac{1}{T_{PP}} (D(1 - k_{HP}) - P_{LMP}); P_T = P_{HP} + P_{LMP},$$

where  $D$  – the current value of the flow rate of steam passing through the turbine;  $p_T$  – steam pressure before the turbine;  $T_{PP}$  – time constant of intermediate steam reheater;  $k_{HP}$

– part of the power that produced by turbine high pressure stage;  $P_{HP}$  – output power of the turbine high pressure stage;  $P_{LMP}$  – output power of the turbine middle pressure and low pressure stages;  $P_T$  – full output power of the turbine.

The equation of the turbine speed governor simplistically describes the measuring part of the governor (assuming ideal and is replaced by the gain  $1/\sigma$ ) and hydraulic servomotor of turbine control valves [transfer function  $1/(T_c p + 1)$ ]. The input of the unit is a signal of the slip  $s$ , the output is a signal of displacement of the servomotor. The displacement of the servomotor has upper and lower limits.

Turbine with intermediate steam reheating, which has a large storage capacity, is modelled by two parallel elements, one of which is the gain block, and the second is first-order system. The gain block is related to the turbine high pressure stage (HP), which is located between the control valve and the intermediate reheater. Variation of power  $P_{HP}$  by changing the position of HP control valves actually lasts for 0.2-0.4 s, which is determined by volume of steam behind the valve, but in this simplified model this time lag is ignored and the transfer function is taken to be equal  $k_{HP}$ . The other part of the power which is produced by turbine middle pressure and low pressure stages changes with a time lag determined by the capacitance of the intermediate reheater. This part of the turbine's dynamic property is modelled as the first-order system  $(1 - k_{HP}) / (T_{PP} p + 1)$ .

The mathematical model of turbine generator with a rigid shaft can be represented as follows:

$$\begin{aligned} U_{gd} &= r_s i_d + \frac{d\psi_d}{dt} - \omega \psi_q; \quad U_{gq} = r_s i_q + \frac{d\psi_q}{dt} + \omega \psi_d; \quad U_f = r_f i_f + \frac{d\psi_f}{dt}; \\ 0 &= r_{kd} i_{kd} + \frac{d\psi_{kd}}{dt}; \quad 0 = r_{kq} i_{kq} + \frac{d\psi_{kq}}{dt}; \\ \psi_d &= x_d i_d + x_{ad} i_f + x_{ad} i_{kd}; \quad \psi_q = x_q i_d + x_{aq} i_{kq}; \quad \psi_f = x_{ff} i_f + x_{ad} i_d + x_{ad} i_{kd}; \\ \psi_d &= x_d i_d + x_{ad} i_f + x_{ad} i_{kd}; \quad \psi_q = x_q i_d + x_{aq} i_{kq}; \\ J \frac{d^2 \theta}{dt^2} &= M_t - (\psi_d i_q - \psi_q i_d); \quad \frac{d\theta}{dt} = \omega, \end{aligned}$$

where  $x_s, r_s$  – stator resistance,  $M_t = P_T / \omega$  – turbine torque.

The model of external power system:

$$\begin{aligned} E_s \cos(\Theta) &= x_v \frac{di_q}{dt} + r_v i_q + \omega x_v i_d + U_{gq} \\ E_s \cos(\Theta) &= x_v \frac{di_q}{dt} + r_v i_q + \omega x_v i_d + U_{gq} \end{aligned}$$

where  $x_v, r_v$  – resistance of external power system,  $E_s$  – external system electromotive force.

The model of automatic excitation proportional regulator without stabilization channels, dynamics of the exciter is ignored:

$$U_g = \sqrt{U_{gd}^2 + U_{gq}^2}; E_f = -K_{0u} (U_g - U_{gnom}).$$

### Model linearization and simplification

First step of robust control system design is linearizing of turbine generator model with automatic excitation regulator (AER) and a turbine speed governor (TSG) in the neighbourhood of the nominal operation point to obtain the linear system  $G$  with inputs  $u1$  and  $u2$  to AER and TSG, respectively, and with the output  $y$  as slip  $d\omega$  (Fig. 2).

As a result of the controller synthesis for complex high-order models with the help of robust methods one can usually obtain the high-order regulators. Such regulators are not only difficult to implement but sometimes overly precise. There are different ways to obtain a lower-order controller [2]: to lower the order of the original object model with subsequent synthesis of controller, to synthesize the controller for the initial model and then reduce its order or to synthesize at once a reduced order controller for the full model.

In this work we have chosen the method of lowering the order of the original model. First we simplify the model to a 4-th order with the help of Schur's method [2]. The resulting transfer matrix  $G$  is:

$$G = \begin{pmatrix} \frac{-0.005465s^3 + 0.1145s^2 + 1.295s + 1.001}{s^4 + 6.899s^3 + 168.9s^2 + 794.4s + 4231} & \frac{0.000057s^3 - 0.818s^2 - 34.45s - 11.73}{s^4 + 6.899s^3 + 168.9s^2 + 794.4s + 4231} \end{pmatrix}$$

The model can be represented schematically as shown in fig. 2.



Figure 2. Schematic representation of the model

### Uncertainty of the model

In the model of power system with turbine generators there are many sources of uncertainty. In this study, for example of using the method we will consider model uncertainty with respect to one parameter –AER gain for voltage channel  $K_{0u}$ , which can range from 50 to 150 p. u. Nominal value is selected to be  $K_{0u} = 100$ . We accept the model of unstructured additive uncertainty which was described above. The weighting function  $W_2$  in this model is chosen based on the model uncertainty and ranges of parameters in such a way: we build a set of frequency response of difference  $G_{param} - G_{nom}$  with the parameters from the range of their variations, in this case for  $K_{0u} \in [50, 150]$ . By the maximum of this frequency response family we can restore a minimum-phase transfer

function  $w_2$ , which makes a matrix weighting function  $W_2 = \begin{pmatrix} w_2 & 0 \\ 0 & w_2 \end{pmatrix}$ . Another simplification could be the replacement of  $w_2$  with a constant function that equals to the maximum point of the set of frequency response of differences  $G_{param} - G_{nom}$ .

For application of the  $H_\infty$  norm optimization theory we construct extended model  $P$  based on the original system  $G$ , which is shown in Fig. 3.

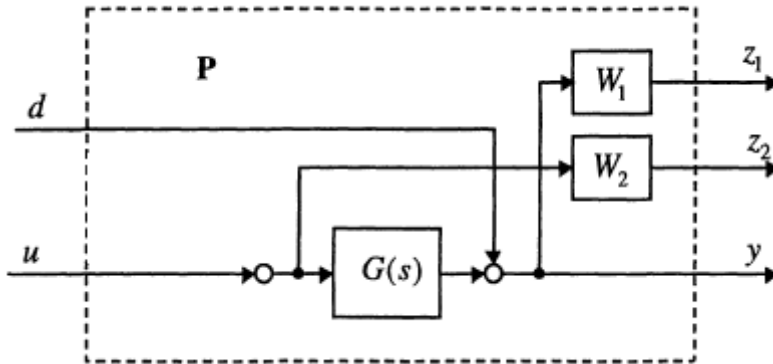


Figure 3. Extended model

The place of the controller  $K$  in the system is shown in Fig. 4.

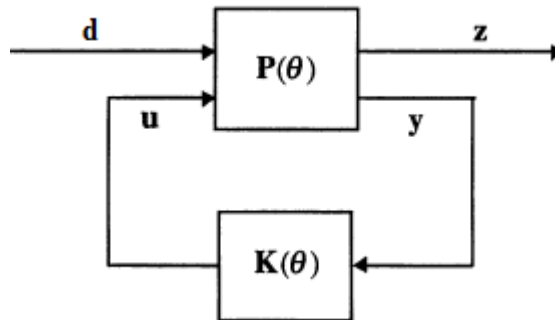


Figure 4. Closed-loop model with controller  $K$

The function  $W_1$  is responsible for the constraint on the sensitivity function  $S$  and has the form [5]:

$$W_1 = \frac{1}{S_\infty} \frac{s + \omega_1}{s + \omega_\varepsilon}$$

We select the region of the poles placement in the form of a conic sector with an angle  $\theta = 30^\circ$  from requirements of reducing process oscillability and the existence of a stable transient process in the simulation on the original model that is found by numerical calculations.

## Results and discussion

**Controller synthesis.** We synthesize the controller  $K$  by the procedure described above using the MATLAB function `hinfmix`. The form of the obtained regulator is:

$$K = \left( \begin{array}{c} \frac{15.33s^5 + 4.535 \cdot 10^5 s^4 + 1.018 \cdot 10^7 s^3 + 4.622 \cdot 10^7 s^2 + 1.691 \cdot 10^8 s + 1.69 \cdot 10^7}{s^5 + 3782s^4 + 1.786 \cdot 10^5 s^3 + 2.956 \cdot 10^6 s^2 + 5.535 \cdot 10^5 s - 5240} \\ \frac{-89.17s^5 - 5.623 \cdot 10^5 s^4 + 1.826 \cdot 10^6 s^3 + 2.716 \cdot 10^6 s^2 + 1.311 \cdot 10^8 s - 2.772 \cdot 10^8}{s^5 + 3782s^4 + 1.786 \cdot 10^5 s^3 + 2.956 \cdot 10^6 s^2 + 5.535 \cdot 10^5 s - 5240} \end{array} \right)$$

Pole-zero map of the closed-loop system (Fig. 5) shows that the poles of that system with nominal parameters lie within the selected sector.

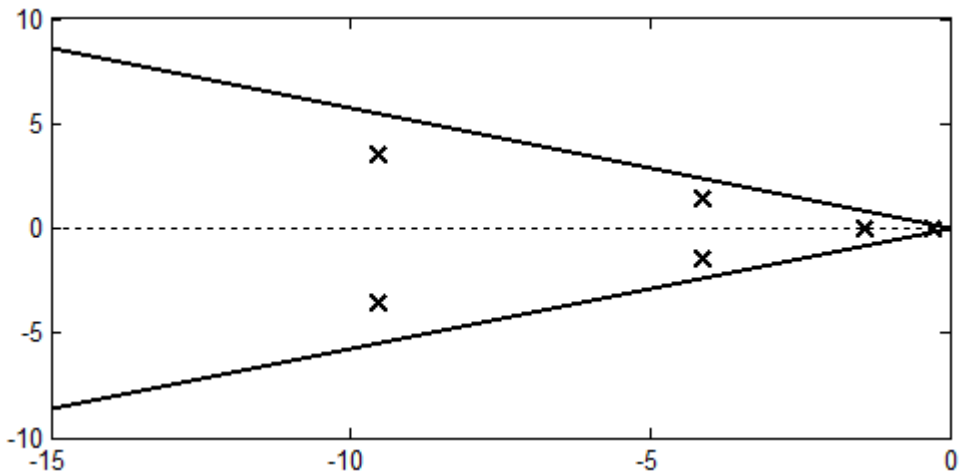


Figure 5. Pole-zero map of the closed-loop system

**Simulation of the transient process on the full-order nonlinear model.** The scheme of simulation of transient processes with interconnected system stabilizer is shown in Fig. 6.

We simulate the transient process of 3-phase short-circuit fault (0.2 s) behind the transformer and subsequent automatic re-closing for the system with the turbine generator without stabilizer, with the standard [4] and interconnected robust system stabilizer, drawing them on the same graph (Fig. 7). This graph shows lower oscillability of transient process with the use of robust controller with poles placement technique. To demonstrate the robust properties of the synthesized controller, the graphs were generated for different values of  $K_{ou}$ . In Fig. 7 a) for  $K_{ou}$  100 and 150, b)  $K_{ou}$  100 and 50.



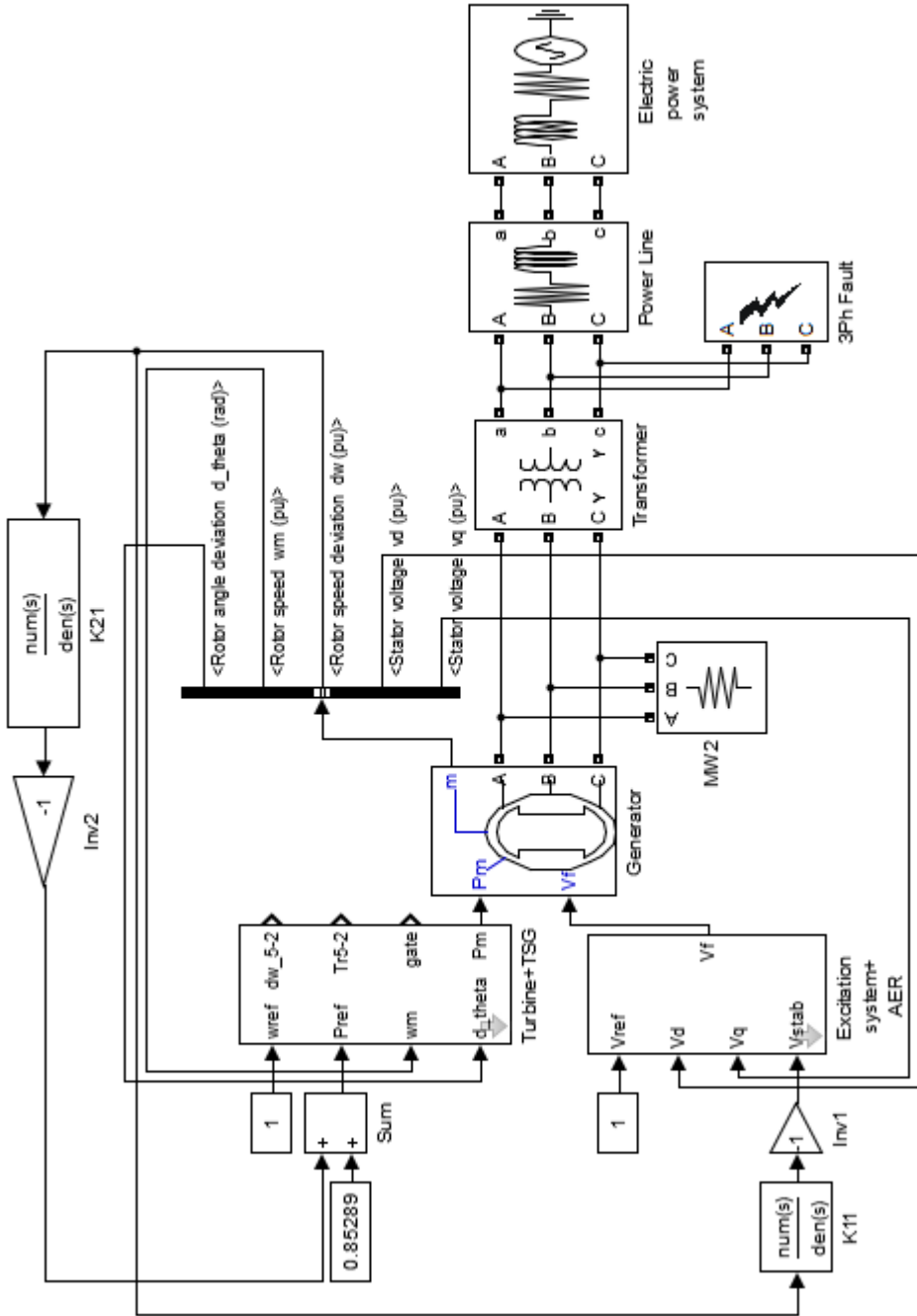


Figure 6. Simulation scheme

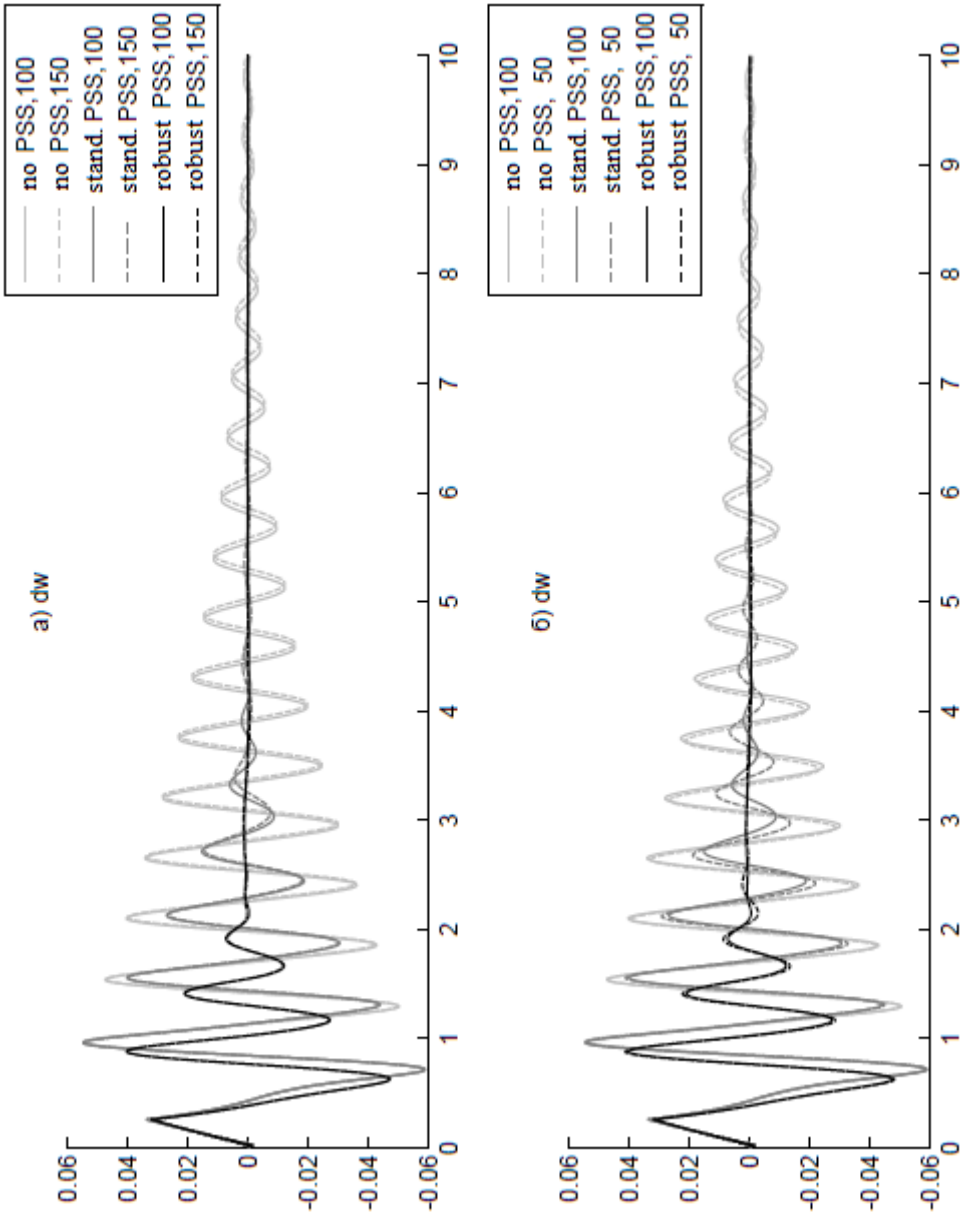


Figure 7. Simulation graphs

## Conclusion

To ensure the performance specifications of automatic control of the turbine-generator unit it is expedient to use interconnected robust controller, which is synthesized using  $H_\infty$ -optimization method with the closed-loop system poles placements with the linear matrix inequalities regions. As numeric simulation graphs show this kind of controller has better performance specifications of transients than the standard power system stabiliser which was reported by P. Kundur in [4].

It was shown that the method of model simplification before the synthesis of controller can produce sufficient accuracy for resulting system stabiliser. In addition, the selection method of weighting functions for chosen uncertainty model leads not only to robust stabilization of closed-loop system but to robust satisfying of performance specifications of transients for different model parameters within their predefined ranges.

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