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## THE PHASE DIFFERENCE BETWEEN COMPONENTS OF ELLIPTICAL OSCILLATIONS OF VIBRATORY CONVEYOR PROVIDING MAXIMUM CONVEYING VELOCITY

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**Abstract.** The piece goods conveying by the vibratory conveyor with elliptical oscillations is considered. Elliptical oscillations of the conveyor track are realized when conveyor has independent drives of oscillations in the direction of conveying (longitudinal oscillations) and oscillations in the direction perpendicular to conveying plane (normal oscillations) with phase difference between them. Elliptical oscillations allow increasing conveying velocity and inclination angle of conveyor track in comparison with the simplest linear oscillations. Conveying velocity of parts moving along the track of conveyor varies with the change of phase difference angle and reaches a maximum with a certain angle, depending on several different parameters. This angle is called the optimal phase difference angle, and it depends on the amplitudes and frequency of the component oscillations, the track inclination angle, the frictional properties, impact parameters of conveying parts.

As the experimental investigations have shown, the piece goods conveying is quite accurately described by the theory of a massive point particle moving on an inclined plane under the action of vibration. A system of nonlinear differential equations describing the conveying in continuous contact modes and in modes with hopping was composed. This system has been solved by gradual integration method with numerical calculation with any desired accuracy. This allowed us to study the dependence of conveying velocity on different parameters in the form of graphs. And for greater generality the study was conducted in dimensionless parameters. There were studied in detail the influence of dimensionless parameters on dimensionless velocity – the coefficient of velocity that is the ratio of conveying velocity to amplitude of longitudinal oscillations with a constant frequency.

To verify the obtained dependences, the experimental investigations of conveying velocity were carried out on vibratory conveyor with removable tracks. The conveyor was fastened on the turntable, the inclination of which was varied by a screw jack. The amplitude of the component oscillations was varied by changing voltage applied to the electromagnetic drives, the phase difference between component oscillations was varied by the phase shifter. Waveforms of oscillations were recorded by vibration measurement equipment. The coefficient of friction was measured directly during conveying. The velocity was measured by the stopwatch. The comparison of experimental results with theoretical data has shown the excellent agreement in continuous contact modes and the acceptable match in modes with hopping.

Based on the obtained graphs, the approximate formulas for calculating velocity and optimal phase difference angle were derived. The influence of frictional properties of the conveying parts, namely, the coefficient of friction on optimal in terms of velocity phase difference angle between the longitudinal and normal components of the elliptical oscillations is investigated. It is shown that the optimal phase difference angle decreases with the increase in the coefficient of friction. The approximate formula of optimal phase difference angle dependence on the coefficient of friction and track inclination angle is derived.

### Introduction

Vibratory conveyors with elliptical oscillations are used in industry for piece goods supply when a high rate is required. A high rate of conveyor is provided due a high velocity of conveying which is achieved with the respective phase difference between components of elliptical oscillations. The necessary

phase difference depends on many parameters: amplitudes and frequency of oscillations, inclination of conveyor track to horizon, the frictional properties of conveying material. Their influence on conveying velocity is researched in this paper.

### **Problem statement**

The use of elliptical oscillations of vibratory conveyor track, when the oscillations in the direction of conveying (longitudinal oscillations) and the oscillations in the direction perpendicular to conveying plane (normal oscillations) oscillate with the phase difference, allows increasing the conveying velocity of parts and the track inclination angle. Increase of conveying velocity  $V$  of parts compared with the simplest conveyors with linear oscillations is possible only at a certain so-called optimal phase difference angle between them [1]. Therefore the investigation of the influence of different parameters including coefficient of friction on this optimal angle is the actual problem.

### **Analysis of modern information sources on the subject of the article**

Vibratory conveying process within the framework of theory of massive point particle moving along a declined plane considered in monograph [2]. Differential equations describing process were obtained, on the basis of their decision by gradual integration method the graphs are obtained, different conveying modes are investigated. But the obtained results do not allow using them in engineering practice because of the need to consider the impact parameters, coefficient of sliding friction, coefficient of static friction and some other parameters that is not known in advance.

Vibratory conveying with elliptical oscillations of conveyor track was investigated in [1,3,4], it was shown that conveying velocity  $V$  changes with changing of phase difference angle reaching maximum when the phase difference angle between the longitudinal and normal oscillations is equal to its optimal value  $\varepsilon = \varepsilon_0$ , depending on several parameters. In [5,6] vibratory conveying with elliptical oscillations was researched for the most effective mode with one lap of sliding forward and one lap of sliding back by dint of approximate methods using the dimensionless parameters. The efficiency of the conveying process has been determined by value of the dimensionless coefficient of velocity. In these works the impact in hopping modes was taken inelastic and coefficient of sliding friction was taken equal to coefficient of static friction. This reduces the number of parameters which influence on velocity should be considered. But nevertheless these works are not given the opportunity to determine in advance the maximum velocity and values of all parameters that can provide it.

Vibratory conveying with elliptical oscillations of conveyor track was investigated in the author's dissertation [7]. The values of optimal phase difference angle between the longitudinal and normal oscillations depending on the dimensionless parameters were shown on the graphs. But the influence of coefficient of friction on these values didn't consider though according to the obtained graphs it is essential. Comparatively recently published monograph states that according experimental investigations the optimal phase difference angle almost doesn't depend on the coefficient of friction [8]. So availability of optimal angle depending on the friction coefficient requires the additional research.

### **Statement of purpose and problems of research**

The purpose of research is the investigation of the influence of different parameters: amplitudes and frequency of component oscillations and the frictional properties of conveying parts, first of all the coefficient of friction, on the optimal phase difference angle between the longitudinal and normal oscillations of vibratory conveyor with elliptical oscillations which provide the maximum conveying velocity.

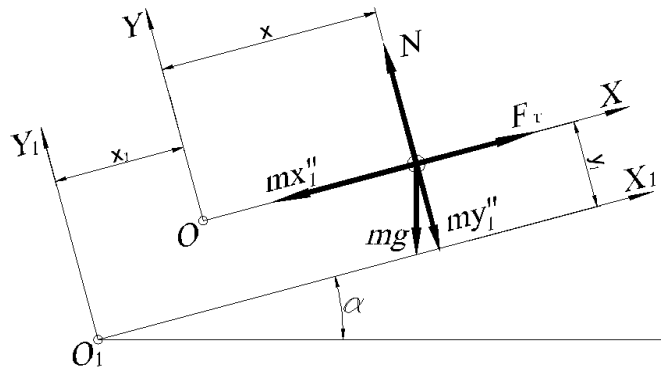
### **The main material presentation**

Let's consider the motion of a point particle with mass  $m$  on an inclined to the horizon at an angle  $\alpha$  plane, oscillating in the longitudinal direction under the law  $x_l = x_l(t)$  and in the normal direction under the law  $y_n = y_n(t)$ , where  $t$  – time. Fig. 1 shows all the forces acting on a particle in the coordinate system  $XOY$ , rigidly connected with the oscillating plane:  $F_T$  – force of friction,  $N$  – normal reaction,  $g$  – acceleration of gravity. To receive the elliptical oscillations of conveyor track drives must excite independent harmonic longitudinal and normal oscillations under the law

$$\begin{aligned} x_1 &= A \sin \omega t, \\ y_1 &= B \sin(\omega t - \epsilon). \end{aligned} \quad (1)$$

where  $A$  – amplitude of longitudinal oscillations,  $B$  – amplitude of normal oscillations,  $\omega$  – frequency of oscillations. Differential equations describing the motion of particle in coordinate system  $XOY$ , arising from the analysis of existing forces [2]

$$\begin{aligned} mx'' &= -mx_1'' - mg \sin \alpha + F_T, \\ my'' &= -my_1'' - mg \cos \alpha + N. \end{aligned} \quad (2)$$



**Fig. 1.** Diagram of the forces acting on a conveying particle

Substituting (1) in (2) provided  $y=0$  for continuous contact modes, we obtain [7]:

$$x'' = A\omega^2 \left[ \sqrt{1 + K_\beta^2} \pm K_\beta \cos \epsilon K_\beta^{-1} \sin(\omega t - \arctg \frac{\sin \epsilon}{\cos \epsilon \pm K_\beta}) - \frac{K_\alpha \pm 1}{K_\beta w} \right], \quad (3)$$

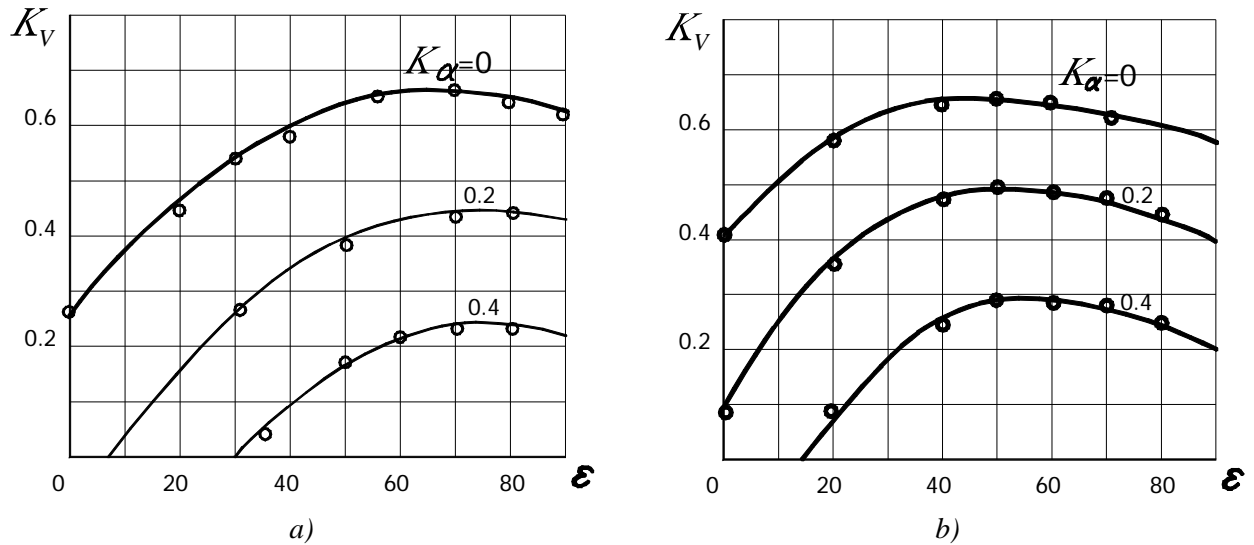
where signs “+” and “-” are respectively for lap of sliding forward and lap of sliding back,  $K_\alpha$  – track angle inclination parameter,  $K_\beta$  – vibration angle parameter,  $w$  – parameter of overload, and

$$K_\alpha = \frac{\tan \alpha}{f}, \quad K_\beta = \frac{A}{B \cdot f}, \quad w = \frac{B\omega^2}{g \cos \alpha}. \quad (4)$$

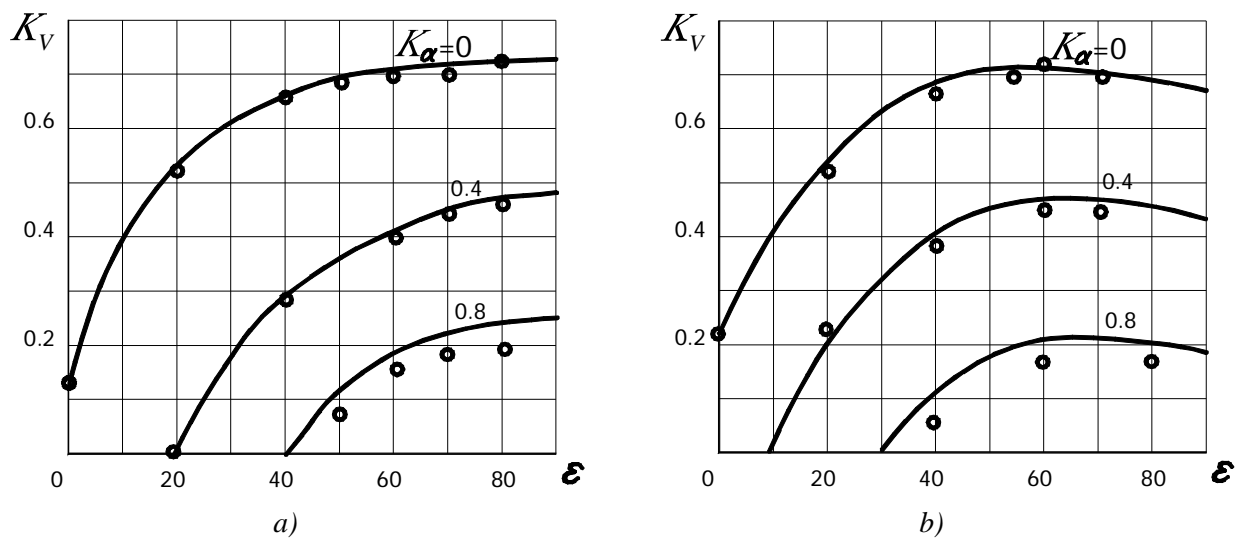
For solving equation (3) by gradual integration method using dimensionless coefficient of velocity  $K_v = V/A\omega$ , the Fortran programs [7] and later MathCAD programs [10] were created that calculate the dependence of coefficient of velocity  $K_v$  on phase difference angle  $\epsilon$  with different values of parameters  $K_\alpha$ ,  $K_\beta$ ,  $w$ . And the optimal value of phase difference angle was determined for each combination of parameters  $K_\alpha$ ,  $K_\beta$  and  $w$ . The programs have been composed both for continuous contact modes and the modes with hopping. The laps of sliding forward, sliding back and flying were considered separately, coefficient of sliding friction and static friction were considered the same, the impact in hopping modes was taken inelastic and longitudinal velocity of particle after flying was taken unchanging. The value of  $K_v$  increases with increasing  $w$ , therefore the continuous contact modes are advisable to apply with parameter of overload value  $w=1$ . The results of calculations are presented in a form of graphs; some of them are shown on Fig. 2 and 3.

To verify the obtained dependences the experimental investigations of conveying velocity were carried out on vibratory conveyor with the removable tracks (Fig. 4). Conveyor with independent electromagnetic drives of longitudinal and normal oscillations was fastened on the turntable, the inclination of which to horizon varied by a screw jack. Power to the electromagnetic drives was supplied through laboratory transformers PHO-250-2, the amplitudes of longitudinal and normal component oscillations varied by changing voltage applied to the electromagnetic drives independently by each transformer. The phase difference between component oscillations was varied by the phase shifter МАФ-22-00. The measurement of longitudinal and normal amplitudes was carried out with a help of two sensors ДУ-5С connected to the vibration measurement equipment ВИ6-6ТН. The waveforms of oscillations were recorded by vibration measurement equipment which was connected to the DC power source Б5-47, output of which was connected to the oscilloscope С1-34. Calibration of vibration measurement equipment ВИ6-6ТН was performed with a

help of microscope MIB-1 with accuracy 0.025 mm. A zero value of the phase difference angle on the phase shifter was exposed with an overlap of waveforms on the oscilloscope screen.



**Fig. 2.** The coefficient of velocity  $K_V$  dependence on phase difference angle  $\varepsilon$  with different values of track angle inclination parameter  $K_\alpha$  and parameter of overload  $w=1$  and vibration angle parameter a)  $K_\beta=10$ ; b)  $K_\beta=5$

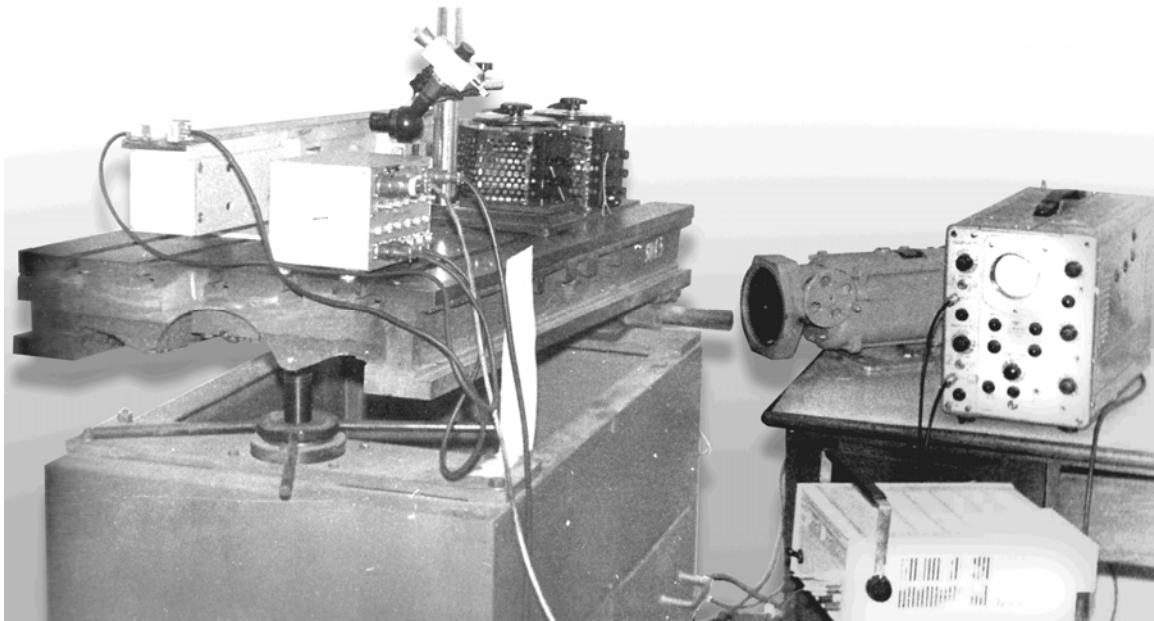


**Fig. 3.** The coefficient of velocity  $K_V$  dependence on phase difference angle  $\varepsilon$  with different values of track angle inclination parameter  $K_\alpha$  and parameter of overload  $w=1.2$  and vibration angle parameter a)  $K_\beta=50$ ; b)  $K_\beta=10$

Some removable tracks with a length 1m from different materials (steel, PCB, rubber, aluminum, plexiglass) were installed on the conveyer. Each track was provided with several longitudinal V-shaped paths in which conveying details from different materials moved. For experiments there were used cylindrical details with diameter 6 mm and length 30 mm and flat details with size from 20x10x1 mm to 30x25x8 mm that were made of steel, aluminum, rubber, lead. All of it allowed changing the value of coefficient of friction over a wide range.

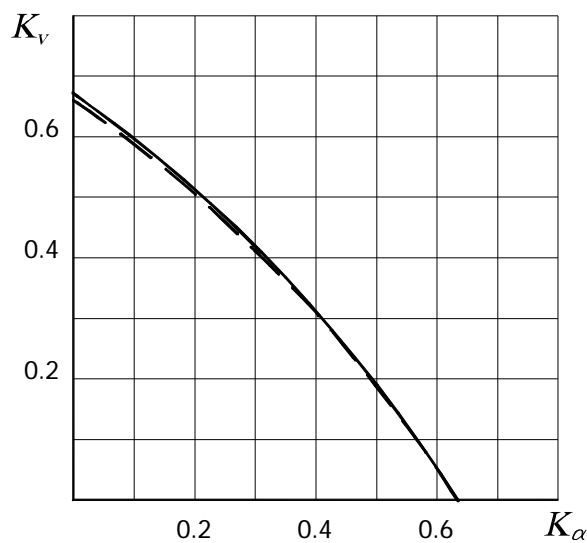
Coefficient of friction was measured directly during conveying. Sensor ДУ-5С was mounted on conveying detail that allows to measure on a waveform the acceleration of detail  $x_d''$  with pure longitudinal oscillations. And then coefficient of friction  $f = x_d''/g$  [9]. The conveying velocity was determined of measuring by a stopwatch the time in which detail passed a certain distance 800 mm marked on a track. The received results were recalculated in values of dimensionless parameters. Some of obtained

dependences are shown in graphs on Fig. 2 and 3; theoretical results are marked by continues lines and experimental results – by circles.



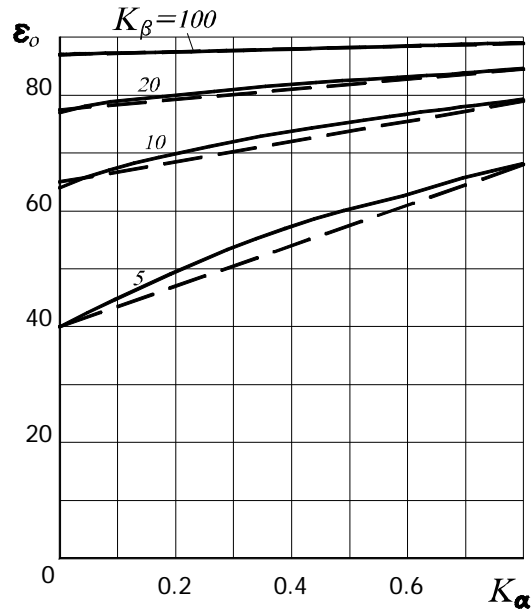
**Fig. 4.** Experimental installation for researching the dependence of conveying velocity on different parameters

The comparison of experimental results with theoretical data showed the excellent agreement in continuous contact modes and the acceptable match in modes with hopping. The results of calculations allowed to build the graphs of dependence of coefficient of velocity  $K_V$  on parameters  $K_\alpha$ ,  $K_\beta$  and  $w$  as well as graphs of dependence of optimal phase difference angle  $\varepsilon_o$  on the same parameters. When conveying occurs without hopping of parts the maximum velocity is achieved when parameter of overload  $w = 1$ . As experimental investigations show [8] in hopping modes the value of coefficient of overload is limited  $w = 1.2 \dots 2$ . If the value of  $w$  will be greater, the hopping takes on an unstable erratic and the motion of parts will be unstable. This instability has a deleterious effect on a conveying velocity. And with sufficiently intense oscillations when  $A \gg B$  (when  $K_\beta > 10$ ) the value of  $K_V$  is almost unchanged with change of  $K_\beta$ .



**Fig. 5.** The coefficient of velocity  $K_V$  dependence on the track angle inclination parameter  $K_\alpha$  with the parameter of overload  $w=1$ .

Fig. 5 shows the dependence of coefficient of velocity  $K_V$  on the track angle inclination parameter  $K_\alpha$  with  $K_\beta > 10$  and the parameter of overload  $w = 1$ . Fig. 6 shows the optimal phase difference angle  $\varepsilon_o$  (in degrees) dependence on the track angle inclination parameter  $K_\alpha$ , the vibration angle parameter  $K_\beta$  (its values are in italics) with  $w = 1$ .



**Fig. 6.** The optimal phase difference angle  $\varepsilon_o$  dependence on the track angle inclination parameter  $K_\alpha$  with different values of the vibration angle parameter  $K_\beta$  and the parameter of overload  $w = 1$

Based on the obtained graphs and analysis of approximate description of vibratory conveying process author derived the approximate formulas of coefficient of velocity  $K_V$  dependence and optimal phase difference angle  $\varepsilon_o$  (in degrees) dependence on parameters  $K_\alpha$  and  $K_\beta$  for the most important case  $w = 1$  [10]:

$$K_V = \left(1 - \frac{1}{K_b^2}\right) \sin\left(\sqrt{3 - p \cdot K_\alpha \left(1 - \frac{1}{K_b^2}\right)} - 1\right). \quad (5)$$

$$e_o = 90 - \frac{25}{K_b} (10 - 7K_\alpha). \quad (6)$$

Graphs of dependence according to formula (5) with  $K_\beta > 10$  are shown by dashed lines on Fig. 5 and dependence according to formula (6) – by dashed lines on Fig. 6. Analysis of these graphs indicates that optimal phase difference angle  $\varepsilon_o$  increases with increasing of parameters  $K_\alpha$  and  $K_\beta$ . Since the coefficient of friction  $f$  according to formula (6) is in the denominator of expressions for  $K_\alpha$  and  $K_\beta$ , the angle  $\varepsilon_o$  decreases with increasing of coefficient of friction  $f$ .

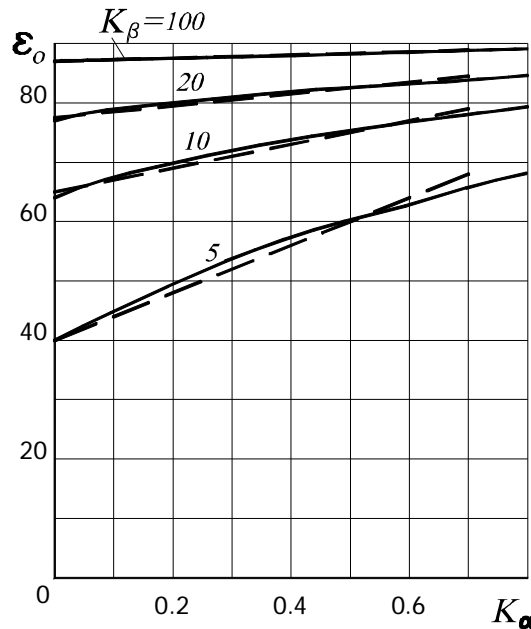
The more accurate approximate formula for angle  $\varepsilon_o$  with account of parameter of overload was derived by the author in [11]:

$$e_o = 90 - \frac{25}{K_b} (9 - 11K_\alpha + 4.5K_\alpha^2 + 1/w). \quad (7)$$

Error of values  $\varepsilon_o$  calculated by this formula does not exceed 2%, but if we limit the values  $0 \leq K_\alpha < 0.6$  and  $K_\beta > 5$  and  $1 \leq w < 1.5$ , used in practice, considering that in these limits value of  $w$  has a little effect on angle  $\varepsilon_o$ , the next more simple formula is applicable for practical use [11]:

$$e_o = 90 - \frac{50}{K_b} (5 - 4K_\alpha). \quad (8)$$

Graphs, counted by equation (7), are shown in Fig. 7 by continuous line and counted by equation (8) – by dashed lines. As we can see the difference between them when  $0 \leq K_\alpha < 0.6$  and  $K_\beta > 10$  is less than  $1^\circ$ . And it is understood from Fig. 2 and 3 that with a slight deviation of phase difference angle from optimal value the velocity is almost unchanged. That's why the approximate formula for calculating phase difference angle is quite acceptable for practical use.



**Fig. 7.** The optimal phase difference angle  $\varepsilon_o$  dependence on the track angle inclination parameter  $K_\alpha$  with different values of the vibration angle parameter  $K_\beta$  on the basis of approximate formulas

Substituting the values  $K_\alpha$  and  $K_\beta$  from expressions (4) to equation (8), we obtain

$$e_o = 90 - \frac{50B}{A}(5f - 4 \tan \alpha). \tag{9}$$

Equation (9) allows calculating optimal phase difference angle if we know the coefficient of friction, track inclination angle and amplitudes of component oscillations. As it seen from equation and graphs with the increase in amplitude of longitudinal oscillation the influence of coefficient of friction on optimal phase difference angle is reduced. Possibly the conclusion that the optimal phase difference angle doesn't depend on the coefficient of friction [8] is explained to the fact that in experimental investigations the amplitudes of longitudinal oscillation were quite large.

### Conclusions

The influence of parameters of elliptical oscillations and the frictional properties of parts moving along the vibratory conveyor on optimal by velocity phase difference angle between the longitudinal and normal oscillation is investigated. Theoretical and experimental investigations were carried out; theoretical results were compared with experimental ones. The comparison between them shows the excellent agreement in continuous contact modes and the acceptable match in modes with hopping. The dependences of coefficient of velocity and the optimal phase difference angle on the dimensionless track angle inclination parameter, vibration angle parameter and the parameter of overload are researched. The approximate formulas for calculating the values of parameters of these dependences are derived on the basis of obtained graphs. A formula of optimal phase difference angle dependence on the coefficient of friction and track inclination angle is derived. It is shown that the optimal phase difference angle decreases with the increase in the coefficient of friction.

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