УДК 539.3

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ASYMPTOTIC METHOD OF CALCULATING OF STRUCTURAL EFFICIENCY OF VEHICLES STRUCTURE OF OPEN LONGITUDINALLY STIFFENED CYLINDRICAL SHELLS

Запропоновано асимптотичний метод розрахунку міцності конструкцій транспортних засобів, спроектованих на основі відкритих ребристих циліндричних оболонок.

Предложен асимптотический метод расчета прочности конструкций транспортных средств, спроектированных на основе открытых ребристых цилиндрических оболочек.

The asymptotic method of calculating of structural efficiency of vehicles designed on the base of open longitudinally stiffened cylindrical shells.

Key words. Method of calculating of structural efficiency, asymptotic integration, composite equations, cylindrical shells.

Introduction. The asymptotic method of calculation of the stress-strain state of vehicle structure of open cylindrical stringer shells is offered. In the correspondence with the offered approach a partial solution of a problem is created based on base equation, and the composite equations of a ground state and boundary effect are used for a construction of a common solution.

The asymptotic methods are one of the most effective tools for investigation of the stress-strain state of shell constructions. Using the method of asymptotic decomposition we can reduce our original task to some simplified limiting equations. This is attainable by eliminating from the source equation all the terms which have not much influence at the result, but which make calculations too difficult. Correspondingly to unstiffened shells, among the simplified limiting relations are the next: equations of the momentless theory; equations of the edge effect; equations of the shallow shell theory; equations describing tangential deformation and bending of plates. The solutions of those limiting equations may be applied successfully to investigation of a great number of practically important tasks. But this method of solving tasks has an essential disadvantage on account of unhomoheneity of the expansion at the variability parameter. This disadvantage results in necessity to use different limiting equations for different variability parameters, and, hence, we face a problem of linking their solutions to find an analytic solution effective for any value of the stress-strain state variability.

In order to escape the mentioned difficulties it is proposed to solve and to investigate boundary tasks in the open shell theory on the base of the method of composite equations. The idea of the method of composite equations is based on the next algorithm: first, we make asymptotic decomposition of the initial equation and find the limiting equations for different values of asymptotic integration parameters; then we construct the composite equations introducing into them the limiting relations effective for the large and for the small values of these parameters. Thus, the composite equation of general state includes relation of the half-momentless theory and that of tangential deformation of plates; the composite equation of edge effect includes relation of simple edge effect and that of bending of plates. The composite equations thus constructed are effective within the all range of values of the stress-strain state variability. This has been confirmed by comparison of roots of the characteristic equations corresponding to the initial equation and to the obtained composite equations.

Among the great variety of types of practically used shells, open shells belong to the most widely used ones. It is proposed here to use asymptotic methods on the base of composite equations for researches in the theory of such shells. Using composite equations in solving boundary tasks for open shells gave a possibility to obtain approximate analytic solutions for these tasks. The peculiar solution corresponding to the surface loading is obtained from the exact equation, and the composite equations are used in order to satisfy the boundary conditions on the shell borders. The solution of the composite equations is obtained in a formula form. The calculations that had been made, confirmed satisfactory precision of the proposed method.

The asymptotic methods are widely used for investigations of boundary value problems in the shell theory [1–3]. But the effectiveness of using them is in much restricted within the validity area of the limiting equations obtained after the asymptotic decomposition, and these results in the necessity to link the obtained solutions. In order to escape the mentioned difficulties the authors proposed the method of composite equations [2, 3]. The way how to use composite equations for solving calculation tasks in the theory of open cylindrical shells is described in the present work.

Formulation of the problem. Let us consider an open longitudinally stiffened cylindrical shell of medium relative length that is loaded with unequal external pressure. In this case the stress-strain state is described with the rigidity differential equation [3]:

$$L\Phi = \frac{\partial^{\blacksquare}\Phi}{\partial\xi^{\blacksquare}} + 2\left(\varepsilon_1 + \varepsilon_4\varepsilon_1^{-1}\right)\frac{\partial^{\blacksquare}\Phi}{\partial\xi^{1}\partial\eta^{3}} + \left(\varepsilon_1 + 4\varepsilon_1\varepsilon_4\varepsilon_1^{-1}\right)\frac{\partial^{\blacksquare}\Phi}{\partial\xi^{4}\partial\eta^{4}} + 2\varepsilon_4\varepsilon_1\left(1 + \varepsilon_1^{-1}\right)\frac{\partial^{\blacksquare}\Phi}{\partial\xi^{3}\partial\eta^{1}} +$$

$$+\varepsilon_2\varepsilon_4\frac{\partial^8\Phi}{\partial\eta^8}+4\varepsilon_2\varepsilon_4\left(\varepsilon_3+\nu_{12}\varepsilon_2\varepsilon_4\right)\frac{\partial^8\Phi}{\partial\xi^6\partial\eta^2}+2\varepsilon_4\left(2\varepsilon_2\varepsilon_3^{-1}+\nu_{12}\varepsilon_3+\varepsilon_2\varepsilon_4\right)\frac{\partial^6\Phi}{\partial\xi^2\partial\eta^4}+ \quad (1)$$

$$+2\varepsilon_2\varepsilon_4\frac{\partial^6\Phi}{\partial\eta^6}+\varepsilon_1^{-2}\frac{\partial 4\Phi}{\partial\xi^4}+2\varepsilon_2\varepsilon_4\left(\nu_{12}+\varepsilon_3^{-1}\right)\frac{\partial^4\Phi}{\partial\xi^2\partial\eta^2}+\varepsilon_2\varepsilon_4\frac{\partial^4\Phi}{\partial\eta^4}=q.$$

The signification of the variables accepted here is the bollowing; ε_i (i = 1, 2, ... , 7) – geometric rigidity parameters of the shell; $\varepsilon_1^2 = D_1/(B_1R_2^2)$; $\varepsilon_2 = D_2/D_1$; $\varepsilon_3 = D_3/D_1$; $\varepsilon_4 = B_2/B_1$; $\varepsilon_5 = B_3/B_1$; $\varepsilon_6 = e_1/R_2$; B_j, D_j – membrane and bending solidity; j = 1..3; e_1 – distances from the weight centers of the stringers' cuts to the medium surface of the shell; h, R – thickness and radius of the shell; ν_{12} ,ν_{21} – Poisson's coefficients; ξ, η – relative coordinates; Φ – solving function; the rest of the significations correspond to those accepted in the paper [3] .

The deflections, strains and moments for the equation (1) are determined from the next relations:

$$u=\left(\nu_0\varepsilon_4^{-1}\varepsilon_1\frac{\partial^4}{\partial\xi^4}+2\nu_{10}\varepsilon_1\frac{\partial^4}{\partial\xi^2\partial\eta^2}+\nu_{12}\frac{\partial^2}{\partial\xi^2}-\frac{\partial^2}{\partial\eta^2}\right)\frac{\partial\Phi}{\partial\xi},$$

$$v=-\left[\left(2\nu_{12}\varepsilon_3^{-1}+\nu_{11}\varepsilon_4^{-1}\right)\varepsilon_1\frac{\partial^4}{\partial\xi^4}+\left(\varepsilon_3^{-1}+\nu_{10}\right)\frac{\partial^2}{\partial\xi^2}-\frac{\partial^2}{\partial\eta^2}\right]\frac{\partial\Phi}{\partial\eta},$$

$$w=\varepsilon_4^{-1}\nabla_2^4\Phi,\quad \nabla_2^4=\frac{\partial^4}{\partial\xi^4}+2\varepsilon_4\varepsilon_3^{-1}\frac{\partial^4}{\partial\xi^2\partial\eta^2}+\varepsilon_4\frac{\partial^4}{\partial\eta^4},$$

$$T_1=\frac{B_1}{R}\left(\nu_{12}\varepsilon_1\frac{\partial^2}{\partial\xi^2}-\varepsilon_1\frac{\partial^2}{\partial\eta^2}-1\right)\frac{\partial^4\Phi}{\partial\xi^2\partial\eta^2},$$

$$T_2=\frac{B_1}{R}\left(\nu_{12}\varepsilon_1\frac{\partial^2}{\partial\xi^2}-\varepsilon_1\frac{\partial^2}{\partial\eta^2}-1\right)\frac{\partial^4\Phi}{\partial\xi^4},$$

$$S=-\frac{B_1}{R}\left(\nu_{12}\varepsilon_1\frac{\partial^2}{\partial\xi^2}-\varepsilon_1\frac{\partial^2}{\partial\eta^2}-1\right)\frac{\partial^4\Phi}{\partial\xi^3\partial\eta}, \quad (2)$$

$$M_1=B_1\left\{\left[\left(\varepsilon_1^2+\nu_{12}\varepsilon_1^2\right)\frac{\partial^2}{\partial\xi^2}+\nu_{12}\varepsilon_1^2\varepsilon_4\frac{\partial^2}{\partial\eta^2}\right]\nabla_2^4\Phi+\right.$$

$$\left.+\left[\left(1+2\nu_{12}\varepsilon_4\varepsilon_3^{-1}\right)\varepsilon_1\frac{\partial^4}{\partial\xi^2\partial\eta^2}+\nu_{12}\varepsilon_4\varepsilon_1^2\frac{\partial^4}{\partial\eta^4}+\varepsilon_1\frac{\partial^2}{\partial\xi^2}\right]\left(\frac{\partial^2}{\partial\eta^2}-\nu_{12}\frac{\partial^2}{\partial\xi^2}\right)\Phi\right\},$$

$$M_2=B_2\varepsilon_1^2\varepsilon_2\varepsilon_4^{-1}\left(\frac{\partial^2}{\partial\eta^2}+\nu_{12}\frac{\partial^2}{\partial\xi^2}\right)\nabla_2^4\Phi,\quad H=\nu_{12}B_1\varepsilon_2\varepsilon_4^{-1}\nabla_2^4\frac{\partial^2\Phi}{\partial\xi\partial\eta}.$$

The boundary task for the equation (1) includes the next boundary condition on the each edge:

$$S = T_1 = W = M_1 = 0,\ \text{when}\ \xi = 0,l; \quad (3)$$

$$U = V = W = W_\eta = 0,\ \text{when}\ \eta = 0,1. \quad (4)$$

Research results. For making a research result problem let us assume the external loading in the next form:

$$q = P \sin(m\pi\eta)\sin(\lambda_n\xi). \quad (5)$$

Then the partial solution of the equation (1) may be written:

$$\Phi = Q_{mn}\sin(m\pi\eta)\sin(\lambda_n\xi). \quad (6)$$

Introducing the solution (6) into the equation (1) we determine the next relation for the coefficients Q_{mn}:

$$Q_{mn}=P/\left[\lambda_n^8+2\left(\varepsilon_3+\varepsilon_4\varepsilon_3^{-1}\right)\lambda_n^6(m\pi)^2+\left(\varepsilon_3+4\varepsilon_2\varepsilon_4\varepsilon_3^{-1}\right)\lambda_n^4(m\pi)^4+\right.$$

$$+2\varepsilon_2\varepsilon_4(1+\varepsilon_3^{-1})\lambda_n^2(m\pi)^6+\varepsilon_2\varepsilon_4(m\pi)^8-4\varepsilon_2\varepsilon_4(\varepsilon_3+\nu_{12}\varepsilon_2\varepsilon_4)\lambda_n^4(m\pi)^2-$$

$$-2\varepsilon_4\left(2\varepsilon_2\varepsilon_3^{-1}+\nu_{12}\varepsilon_3+\varepsilon_2\right)\lambda_n^2(m\pi)^4-2\varepsilon_3\varepsilon_4(m\pi)^6+\varepsilon_1^{-2}\lambda_n^4+ \tag{7}$$

$$+2\varepsilon_3\varepsilon_4\left(\lambda_{12}+\varepsilon_3^{-1}\right)\lambda_n^2(m\pi)^2+\varepsilon_3\varepsilon_4(m\pi)^4.$$

In the common case the obtained solution (6) does not satisfy the boundary conditions (3), (4) on the borders of the shell. The corresponding declensions on the longitudinal edges may be compensated upon solving the composite equation of general state [3]:

$$\varepsilon_4\left[\frac{\partial^4}{\partial\xi^4}+2\varepsilon_2\frac{\partial^4}{\partial\xi^2\partial\eta^2}+\varepsilon_3\left(1+\frac{\partial^4}{\partial\eta^4}\right)^2\right]\frac{\partial^4\Phi_0}{\partial\eta^4}+\varepsilon_1^{-2}\left(1+\varepsilon_1\frac{\partial^2}{\partial\eta^2}\right)\frac{\partial^4\Phi_0}{\partial\xi^4}=0. \tag{8}$$

The deflections, strains and moments for the equation (8) are determined from the relations:

$$u=-\frac{\partial^3\Phi}{\partial\xi\partial\eta^2},\quad v=\frac{\partial^3\Phi}{\partial\eta^3},\quad w=\frac{\partial^4\Phi}{\partial\eta^4},$$

$$T_1=-\frac{B_1}{R}\nabla_{2\eta}\frac{\partial^4\Phi}{\partial\xi^2\partial\eta^2},\qquad T_2=-\frac{B_1}{R}\nabla_{2\eta}\frac{\partial^4\Phi}{\partial\xi^4},$$

$$S=\frac{B_1}{R}\nabla_{2\eta}\frac{\partial^4\Phi}{\partial\xi^3\partial\eta},\qquad \nabla_{2\eta}=1+\varepsilon_1\frac{\partial^2\Phi}{\partial\eta^2}, \tag{9}$$

$$M_1=B_1(\varepsilon_1^2\varepsilon_4+\varepsilon_1)\left(\frac{\partial^2}{\partial\xi^2}+\nu_{12}\frac{\partial^2}{\partial\eta^2}\right)\frac{\partial^4\Phi}{\partial\eta^4},$$

$$M_2=B_2\varepsilon_1^2\varepsilon_3\frac{\partial^8\Phi}{\partial\eta^8},\qquad H=B_2\varepsilon_1^2\varepsilon_3\frac{\partial^8\Phi}{\partial\xi\partial\eta^7}.$$

The equation (8) allows satisfying only two boundary conditions on each of the curved edges of the shell. Thus, in the extremities of the shell we have the stress-strain state of the type of simple edge effect, which may be described with the equation:

$$\nabla_2^4\Phi+\varepsilon_1^{-2}\left[\varepsilon_1\left(\nu_{12}\frac{\partial^2}{\partial\xi^2}-\frac{\partial^2}{\partial\eta^2}\right)-1\right]\Phi=0. \tag{10}$$

The equation (10) defines a stress-strain state of the type of edge effect, which occurs in the extremities of the shell.

The deflections, strains and moments for this equation are determined from the next relations:

$$u=-\Phi_{\xi\eta\eta},\quad v=\Phi_{\eta\eta\eta},\quad w=\Phi_{\eta\eta\eta\eta},$$

$$T_1=-B_1/R\nabla_{2\eta}\Phi_{\xi\xi\eta\eta},\; T_2=-B_1/R\nabla_{2\eta}\Phi_{\xi\xi\xi\xi},$$

$$S=\frac{B_1}{R}\nabla_{2\eta}\Phi_{\xi\xi\xi\eta},\quad M_1=B_1\left(\varepsilon_1^2\varepsilon_4+\varepsilon_1\right)\left(\frac{\partial^2}{\partial\xi^2}+\nu_{12}\frac{\partial^2}{\partial\eta^2}\right)\nabla_2^4\Phi, \tag{11}$$

$$M_2=B_2\varepsilon_1^2\varepsilon_3\varepsilon_4^{-1}\left(\frac{\partial^2}{\partial\eta^2}+\nu_{12}\frac{\partial^2}{\partial\xi^2}\right)\nabla_2^4\Phi,$$

$$H=-\nu_{12}B_1\varepsilon_4^{-1}\left(\varepsilon_1^2\varepsilon_4+\varepsilon_1\right)\nabla_2^4\Phi_{\xi\eta}.$$

In the common case the solution of the equation (8) does not satisfy the boundary conditions on the longitudinal edges of the shell. The appearing declensions are quickly oscillating by the coordinate η (∂/∂η~ε) and disappear at the distance ξ~ε from the edges. Thus, near the corners of the panel we have the stress- strain state of the type of two-dimensional boundary layer, whose effective areas are rather small. In the present work the two-dimensional boundary layers are not considered, but they may be constructed follow the methods described in the paper [4].

Boundary tasks for the equations (8), (10) include the decomposited boundary conditions (3). The decomposition of the boundary conditions was made on the base of characteristic parameters of all variables in the boundary conditions. In the correspondence with the method proposed in the work [3], the characteristic parameter $\mu(Q)$ is defined from the order of relation of the strain or of the deflection for the decomposited equations of general state and of edge effect:

$$\varepsilon^{\mu(Q)} \sim Q^{(k)}\Phi^{(o)} / \left(Q^{(o)}\Phi^{(k)}\right) \qquad (12)$$

For the case of boundary conditions of the type (3) we obtain the next values of the characteristic parameters of the variables in the boundary conditions:

$$\mu(S) = -3 + 7\beta_1 - \beta_2, \qquad \mu(T) = -2 + 6\beta_1 - 2\beta_2,$$

$$\mu(w) = -2 + 4\beta_1, \qquad \mu(M) = -3 + 6\beta_1. \qquad (13)$$

The view of canonical sequences depends on the relation of parameters β_1, β_2 characterizing the variability of the stress-strain state in the circular direction:

$$\beta_1 < \beta_2 \qquad\qquad \beta_1 = \beta_2 \qquad\qquad \beta_1 > \beta_2$$

$$w,\ T_1,\ M,\ S; \qquad w \sim T_1,\ S \sim M; \qquad T_1,\ w,\ S,\ M. \qquad (14)$$

Since the composite equations are valid within the whole range of the variability parameters of the stress-strain state, we choose the case $\beta_1 = \beta_2$. Then the corresponding decomposition of the boundary conditions on the curved edges of the shell may be written as follows:

$$S^{(k)} = \bar{S}, \quad M^{(k)} = 0, \quad T_1^{(o)} = -T_1^{(k)}, \quad w^{(o)} = -w^{(k)} \ \text{ when } \ \xi = 0, 1. \qquad (15)$$

Here the indexes (o), (k) correspond to the states described by the equations (7), (8); s – declension in the boundary conditions on account of the partial solution (6). Let us note also that the methods proposed in the paper [5] are quite applicable to solving boundary tasks; in the correspondence with those methods, at first, we have to solve boundary task for the equation of general state, and then, the components of this type are to be eliminated from the boundary conditions and we have to solve the boundary task for the equation of supplementary state.

The construction of the general solution should be commenced from calculation of the state of simple edge effect (10). The general solution of the equation (10) that satisfies the boundary conditions (15) may be written as following:

$$\Phi^{(k)} = 2Q_{mn}\lambda_n^3 b_1^{-3}\left[\exp(-b_1\xi)\cos(b_1\xi) + \right. \qquad (16)$$

$$\left. + (-1)^n \exp\left(b_1(\xi - 1)\right)\cos\left(b_1(\xi - 1)\right)\right]\sin(m\pi\eta),$$

where $b_1 = (2\varepsilon_1)^{-1/2}$.

The solution of the boundary task for the composite equation (8) is obtained in the next form:

$$\Phi^{(o)} = \sum_{k=1}^{N}\varphi_k(\eta)\sin(\lambda_k\xi) + \left[1 + \left((-1)^n - 1\right)\frac{\xi}{1}Q_{mn}b_1\lambda_n^3/(\varepsilon_4 m\pi)\right]\sin(m\pi\eta). \qquad (17)$$

Here
$$\varphi_k(\eta) = \exp(-\alpha_k\eta)\left[A_{1k}\cos(\beta_k\eta) + A_{2k}\sin(\beta_k\eta)\right] +$$

$$+ \exp(-\alpha_k\eta)\left[A_{3k}\cos(\alpha_k\eta) + A_{4k}\sin(\alpha_k\eta)\right] +$$

$$+ (-1)^m\left\{\exp\left[\alpha_k(\eta - 1)\right]\left[A_{1k}\cos\left(\beta_k(1 - \eta)\right) + A_{2k}\sin\left(\beta_k(1 - \eta)\right)\right] +\right.$$

$$\left. + \exp\left[\beta_k(\eta - 1)\right]\left[A_{3k}\cos\left(\alpha_k(1 - \eta)\right) + A_{4k}\sin\left(\alpha_k(1 - \eta)\right)\right]\right\} - \psi_k,$$

$$\psi_k = \frac{4\varepsilon_2 m^4\pi^3 Q_{mn} b_1\lambda_k\left[1 - (-1)^{n+k}\right]}{k\varepsilon_4\left[\lambda_k^4 + 2\varepsilon_2 m^2\pi^2\lambda_k^2 + \varepsilon_3\left(1 + m^2\pi^2\right)^2\right]m^4\pi^4 + \varepsilon_1^{-2}\left(1 + \varepsilon_1 m^2\pi^2\right)\lambda_k^4},$$

$$\alpha_k = \sqrt{\sqrt{2} + 1}\,R_k, \quad \beta_k = \sqrt{\sqrt{2} - 1}\,R_k, \quad R_k = \frac{\sqrt{\lambda_k}}{2\sqrt{2}\,\sqrt[4]{\varepsilon_1^3\varepsilon_2\varepsilon_4}}.$$

The constants A_{ik} are determined from the boundary conditions on the longitudinal edges. The series (12), whose partial sum is determined from the relation (17), is convergent as exp $(-k^{1/2}\eta)/k^3$, hence, in our calculations it is sufficient to take into account just some of its first terms.

The results of the digital calculations of the axial bending moment are presented in the table 1. The calculations have been made with the next values of the parameters: $\nu = 0{,}3$, $m = 1$, $n = 3$, $l = 3$, $\eta = 0{,}3$, $E_c F/(BR) = 0{,}3$, $E_c J/(BR^3) = 0{,}3\ 10^{-6}$.

Table 1

The results of the digital calculations of the axial bending moment

ξ	0	0,05	0,1	0,15	0,2	0,25	0,3	0,5	0,75	1,0
$M_{(1)1}$	0,0	0,163	0,292	0,297	0,301	0,311	0,320	0,319	0,172	0,0
$M_{(2)1}$	0,02	0,625	0,51	0,633	0,648	0,667	0,736	0,723	0,522	0,0
$M_{(3)1}$	0,0	1,187	2,583	2,10	2,495	0,973	0,907	0,726	0,529	0,0

In the table index 1 corresponds to the partial solution on the base of the equation (1), index 2 – to the solution on the base of the composite equation of general state, index 3 – to the general solution after the described method.

Summary. The analysis of the obtained results shows that it is necessary to take into account the boundary effect relations only for calculations of characteristics of the stress-strain state in extremities of open shell.

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