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MATHEMATICAL MODEL AND CALCULATION METHOD QUASI-STATIONARY TRANSPORT AND DISTRIBUTION NATURAL GAS SYSTEMS

У статті наведено стохастичну модель квазістаціонарного неізотермічного режиму транспорту і розподілу природного газу в газотранспортних системах з багатонитковими лінійними ділянками та багатоцеховими компресорними станціями. Наведено метод розрахунку статистичних властивостей залежних змінних моделі від статистичних властивостей незалежних змінних.

В работе представлена стохастическая модель квазистационарного неизотермического режима транспорта и распределения природного газа в газотранспортных системах с многониточными линейными участками и многоцеховыми компрессорными станциями. Приведен метод расчёта статистических свойств зависимых переменных модели от статистических свойств независимых переменных.

The paper presents a stochastic model of a quasi-stationary non-isothermal mode of transport and distribution of natural gas transportation systems with multilinear linear plots and a lot of craft compressor stations A method for calculating the statistical properties of the dependent variables of the model from the statistical properties of the independent variables.

Keywords. Transmission system, mathematical model, a quasi-steady non-isothermal mode, natural gas, a linear plot, compressor station, the statistical properties.

1. Introduction

Solution of the problem of analysis and optimization of the actual modes of gas transportation systems (GTS) is associated with the development of mathematical models that more adequately and in a wider range describe the actual modes of the TCA. One such model is a stochastic model of quasi-stationary non-isothermal mode of transport and distribution of natural gas in the GTA with multithread linear sections (MLS), and many craft compressor stations (CS). In this article we give a general stochastic model of quasi-stationary non-isothermal mode of transport and distribution of natural gas in the GTS with MLS and compressor stations (CS). This model explicitly takes into account both the internal uncertainty of the parameters of technological elements of the GTS, and external uncertainty parameters of the processes of natural gas consumption by various categories of consumers. We consider the method of constructing the deterministic equivalent stochastic model of a quasi-stationary non-isothermal mode of transport and distribution of natural gas in the GTS and the approximate solutions obtained by co-nematode system of nonlinear and linear algebraic equations defined on a graph reflecting the structure of the GTS; and the method for calculating the statistical properties of the dependent variables model the statistical properties of the independent variables.

2. Mathematical modeling of stochastic quasi-stationary mode of natural gas transportation in GTS with MTS and MGP, CS

To build a general stochastic model of a quasi-stationary non-isothermal mode of transport and distribution of natural gas in the GTS with MLS and many CS will use the results obtained in [1]: stochastic models of the quasistationary mode of transport of natural gas pipeline and the stochastic model mode pumping unit (GPU). As a model of the structure of the GTS will use oriented connected graph G(V, E) [2], which is supplemented by a zero vertex and dummy arcs connecting this vertex with all inputs and outputs of the GTS, where V ($|V| = m$) - a set of vertices, E- the set of arcs ($|E| = n$). Choose a tree graph G(V, E) so that its branches have become real and fake parts of the arc corresponding to the input of GTS. Then the set of arcs of the graph E represented as a union of disjoint subsets of the following: the real sections M; fictitious sections on the network inputs L; fictitious sections on the network output , fictitious sections, connecting the input of the active elements with the zero point (fictitious additional network input) T; real tree branches ; real tree branches, which correspond to passive M_{11} and active M_{12} elements; real chords of the graph M_2; real chords of the graph which correspond to passive M_{21} and active M_{22} elements; fictitious branches of a tree, which correspond to inputs L_1; branches of a tree on the inputs of the network with a preset flow , pressure L_{12} and temperature L_{13}; chords of the graph, which correspond to inputs L_2; chords of the graph of the network inputs with the preset flow L_{21}, pressure L_{22}, temperature L_{23}; fictitious chords which correspond to outputs K_2 $(K_2 = K)$; fictitious chords on the outputs of the network with a preset flow K_{21} pressure K_{22}, temperature K_{23}; fictitious chords of the graph, corresponding to fictitious additional network input (arcs connecting the input of the active elements with the zero point) with a preset flow T_{21}. The quantity is considered preset if it is a normally distributed random variable with known expectation and variance.

We introduce the following notation: the number of nodes, in which pressure is preset – $m1 = |L_{12} \cup L_{22} \cup K_{12} \cup K_{22}|$, number of branches, in which flow is preset – $n1 = |L_{11} \cup L_{21} \cup K_{11} \cup K_{21}|$, the number of nodes, in which temperature is preset – the number of branches with active elements –

Given quantities are random variables with normal distribution law and represented by their mathematical expectations and variances.

Then the stochastic model of quasi-stationary non-isothermal mode of transport and distribution of natural gas in the GTS can be expressed as follows:

$$f_r = \underset{\omega}{M}\left\{\beta_r(\omega)q_r(\omega)^2 + \sum_{i\in M_{11}} b_{1ri}\beta_i(\omega)q_i(\omega)^2 + \sum_{i\in M_{12}} b_{1ri}\left\{\tilde{c}_i(\omega)\left(q_i(\omega) - \frac{\tilde{b}_i(\omega)P_{\text{н}i}(\omega)}{2\tilde{c}_i(\omega)}\right)^2 - \left(\tilde{a}_i(\omega) + \frac{\tilde{b}_i^2(\omega)}{4\tilde{c}_i(\omega)} - 1\right)P_{\text{н}i}(\omega)^2\right\}\right\} = 0, \qquad r \in M_{21}, \tag{1}$$

$$f_r = \underset{\omega}{M}\left\{\tilde{c}_r(\omega)\left(q_r(\omega) - \frac{\tilde{b}_r(\omega)P_{\text{н}r}(\omega)}{2\tilde{c}_r(\omega)}\right)^2 - \left(\tilde{a}_r(\omega) + \frac{\tilde{b}_r^2(\omega)}{4\tilde{c}_r(\omega)} - 1\right)P_{\text{н}r}(\omega)^2 + \sum_{i\in M_{11}} b_{lri}\beta_i(\omega)q_i(\omega)^2 + \sum_{i\in M_{12}} b_{lri}\left\{\tilde{c}_i(\omega)\left(q_i(\omega) - \frac{\tilde{b}_i(\omega)P_{\text{н}i}(\omega)}{2\tilde{c}_i(\omega)}\right)^2 - \left(\tilde{a}_i(\omega) + \frac{\tilde{b}_i^2(\omega)}{4\tilde{c}_i(\omega)} - 1\right)P_{\text{н}i}(\omega)^2\right\}\right\} = 0, \qquad r \in M_{22}, \tag{2}$$

$$f_r = \underset{\omega}{M}\left\{-P_{\text{к}r}(\omega)^2 - \sum_{i\in L_{11}} b_{lri}P_{\text{к}i}(\omega)^2 - \sum_{i\in L_{12}} b_{lri}P_{\text{к}}^{+2} + \sum_{i\in M_{11}} b_{lri}\beta_i(\omega)q_i(\omega)^2 + \sum_{i\in M_{12}} b_{lri}\times\left\{\tilde{c}_i(\omega)\left(q_i(\omega) - \frac{\tilde{b}_i(\omega)P_{\text{н}i}(\omega)}{2\tilde{c}_i(\omega)}\right)^2 - \left(\tilde{a}_i(\omega) + \frac{\tilde{b}_i^2(\omega)}{4\tilde{c}_i(\omega)} - 1\right)P_{\text{н}i}(\omega)^2\right\}\right\} = 0, \qquad r \in L_{21}, \tag{3}$$

$$f_r = \underset{\omega}{M}\left\{-P_{\text{к}r}^{+2} - \sum_{i\in L_{11}} b_{lri}P_{\text{к}i}(\omega)^2 - \sum_{i\in L_{12}} b_{lri}P_{\text{к}i}^{+2} + \sum_{i\in M_{11}} b_{lri}\beta_i(\omega)q_i(\omega)^2 + \sum_{i\in M_{12}} b_{lri}\times\left\{\tilde{c}_i(\omega)\left(q_i(\omega) - \frac{\tilde{b}_i(\omega)P_{\text{н}i}(\omega)}{2\tilde{c}_i(\omega)}\right)^2 - \left(\tilde{a}_i(\omega) + \frac{\tilde{b}_i^2(\omega)}{4\tilde{c}_i(\omega)} - 1\right)P_{\text{н}i}(\omega)^2\right\}\right\} = 0, \qquad r \in L_{22}, \tag{4}$$

$$f_r = \underset{\omega}{M}\left\{P_{\text{н}r}(\omega)^2 - \sum_{i\in L_{11}} b_{lri}P_{\text{к}i}(\omega)^2 - \sum_{i\in L_{12}} b_{lri}P_{\text{к}i}^{+2} + \sum_{i\in M_{11}} b_{lri}\beta_i(\omega)q_i(\omega)^2 + \sum_{i\in M_{12}} b_{lri}\left\{\tilde{c}_i(\omega)\left(q_i(\omega) - \frac{\tilde{b}_i(\omega)P_{\text{н}i}(\omega)}{2\tilde{c}_i(\omega)}\right)^2 - \left(\tilde{a}_i(\omega) + \frac{\tilde{b}_i^2(\omega)}{4\tilde{c}_i(\omega)} - 1\right)P_{\text{н}i}(\omega)^2\right\}\right\} = 0, \qquad r \in K_{21}, \tag{5}$$

$$f_r = \underset{\omega}{M}\left\{P_{\text{н}r}^{+2} - \sum_{i\in L_{11}} b_{lri}P_{\text{к}i}(\omega)^2 - \sum_{i\in L_{12}} b_{lri}P_{\text{к}i}^{+2} + \sum_{i\in M_{11}} b_{lri}\beta_i(\omega)q_i(\omega)^2 + \sum_{i\in M_{12}} b_{lri}\left\{\tilde{c}_i(\omega)\left(q_i(\omega) - \frac{\tilde{b}_i(\omega)P_{\text{н}i}(\omega)}{2\tilde{c}_i(\omega)}\right)^2 - \left(\tilde{a}_i(\omega) + \frac{\tilde{b}_i^2(\omega)}{4\tilde{c}_i(\omega)} - 1\right)P_{\text{н}i}(\omega)^2\right\}\right\} = 0, \qquad r \in K_{22}, \tag{6}$$

$$f_r = \underset{\omega}{M}\left\{-P_{\text{к}r}(\omega)^2 - \sum_{i\in L_{11}} b_{lri}P_{\text{к}i}(\omega)^2 - \sum_{i\in L_{12}} b_{lri}P_{\text{к}i}^{+2} + \sum_{i\in M_{11}} b_{lri}\beta_i(\omega)q_i(\omega)^2 + \sum_{i\in M_{12}} b_{lri}\left\{\tilde{c}_i(\omega)\left(q_i(\omega) - \frac{\tilde{b}_i(\omega)P_{\text{н}i}(\omega)}{2\tilde{c}_i(\omega)}\right)^2 - \left(\tilde{a}_i(\omega) + \frac{\tilde{b}_i^2(\omega)}{4\tilde{c}_i(\omega)} - 1\right)P_{\text{н}i}(\omega)^2\right\}\right\} = 0, \qquad r \in T_{21}, \tag{7}$$

$$f_r = \underset{\omega}{M}\left\{\sum_{r\in M_2\cup L_{22}\cup K_{22}} b_{lri}q_r(\omega) + \sum_{r\in L_{21}\cup K_{21}} b_{lri}q_r^{+} - q_i^{+}\right\} = 0, \tag{8}$$

$$f_r = \underset{\omega}{M}\left\{T_{\text{кг}}(\omega) - T_{\text{гр}} - \left(T_{\text{нг}}(\omega) - T_{\text{гр}}\right)e^{-\theta_r(\omega)L}\right\} = 0, \quad r \in M_{11} \cup M_{21}, \tag{9}$$

$$f_r = \underset{\omega}{M}\left\{T_{\text{кг}}(\omega) - T_{\text{нг}}(\omega)\left(P_{\text{кг}}(\omega)/P_{\text{нг}}(\omega)\right)^{\frac{\mu_r(\omega)-1}{\mu_r(\omega)}}\right\} = 0, \tag{10}$$

$$f_r = \underset{\omega}{M}\left\{T_r(\omega)\sum_{i \in G_r^{+}} q_i(\omega) - \sum_{i \in G_r^{-}} q_i(\omega)T_{\text{к}i}(\omega)\right\} = 0, \tag{11}$$

$$f_r = \underset{\omega}{M}\left\{T_{\text{срг}}(\omega) - T_{\text{гр}} + \left[(T_{\text{нг}}(\omega) - T_{\text{гр}})/\theta_r(\omega)L\right](1 - e^{-\theta_r(\omega)L})\right\} = 0, \tag{12}$$

$$f_r = \underset{\omega}{M}\left\{P^2_{\text{нг}}(\omega) - P^2_{\text{кг}}(\omega) - \beta_r(\omega)q_r^2(\omega)\right\} = 0, \tag{13}$$

$$r \in M_{12} \cup M_{22} \tag{14}$$

where:

($P^{+}_{\text{н}i}, P^{+}_{\text{к}i}, T^{+}_{\text{н}i}, T^{+}_{\text{к}i}, q^{+}_r$ – marks the preset quantities);

G_j^{+}, – the set of elements on which the gas comes into the j -th node, and is bled from it, respectively;

b_{ri} – Cyclomatic matrix element, located at the intersection of the r-th row and the i-th column;

– Random variables, characterizing the pressure at the beginning and the end of the i-th arc;

$T_{\text{н}i}(\omega), T_{\text{к}i}(\omega)$ – Random variables, characterizing the temperature at the beginning and the end of the i-th arc;

$q_i(\omega)$ – Random variable characterizing the commercial flow of i-th arc;

$\beta_i(\omega)$ – Random variable characterizing the assessment ratio of hydraulic resistance of pipeline:

$$\beta_i(\omega) = \frac{\Delta(\omega)\,L\,T_{\text{ср}i}(\omega)\cdot Z_{\text{ср}i}(\omega)}{\tau_i\,\alpha_i^2\phi_i^2E_i^2(\omega)\,D_i^{5.2}}, \tag{15}$$

where $\Delta(\omega)$ – random variable characterizing the assessment ratio of the relative density of natural gas in the air, $T_{\text{ср}i}(\omega), Z_{\text{ср}i}(\omega)$ – random variable characterizing the estimation of the average temperature and average density of natural gas of i-th arc, $E_i(\omega)$ – random variable characterizing the assessment of effectiveness ratio i-th pipeline.

$\theta_i(\omega)$ – random variable defined by the expression:

$$\theta_i(\omega) = 62.6K_{T_i}(\omega)D_{H_i}/10^6\,q_i(\omega)\Delta(\omega)B(\omega), \tag{16}$$

where $K_{T_i}(\omega)$ – random variable characterizing the estimate of the average values of the coefficient of heat transfer from the gas in the ground on the i – th section of the pipeline, $B(\omega)$ - a random variable characterizing the estimate of the coefficient of the specific heat of natural gas;

$\tilde{a}_i(\omega), \tilde{b}_i(\omega), \tilde{c}_i(\omega)$ – random variable characterizing the approximation estimates for the coefficients describe the degree of compression of GPA from the commercial flow for GPA-owned i-th arc:

$$\tilde{a}_i(\omega) = a_{2i}(\omega)\,, \tag{17.1}$$

$$, \tag{17.2}$$

$$\tilde{c}_i(\omega) = c_{2i}(\omega)\left(\frac{n}{n_0}\,\frac{\gamma_0 Z(\omega) R T_{\text{н}i}(\omega)}{1440}\right)^2, \tag{17.3}$$

where:

$$a_{2i}(\omega) = n'^4_i(\omega)\cdot a_{1i}(\omega) + 2n'^2_i(\omega)\left(1 - n'^2_i(\omega)\right)a_{0i}(\omega) + \left(1 - n'^2_i(\omega)\right)^2, \tag{18.1}$$

$$b_{2i}(\omega) = n'^4_i(\omega)\cdot b_{1i}(\omega) + 2n'^2_i(\omega)\left(1 - n'^2_i(\omega)\right)b_{0i}(\omega), \tag{18.2}$$

$$c_{2i}(\omega) = n'^4_i(\omega)\cdot c_{1i}(\omega) + 2n'^2_i(\omega)\left(1 - n'^2_i(\omega)\right)c_{0i}(\omega), \tag{18.3}$$

where $a_{0i}(\omega)$, $b_{0i}(\omega)$, $c_{0i}(\omega)$ and $a_{1i}(\omega)$, $b_{1i}(\omega)$, $c_{1i}(\omega)$ – random variables characterizing the estimates of coefficients of approximation polynomials of the degree of compression GPA first and second degree, respectively, at $\left(\frac{n}{n_0}\right)_{\text{пр}} = 1,$

$n'_i(\omega)$ – random variables characterizing the above assessment of the relative speed drive of i-th GPA:

$$n'_i(\omega) = \left(\frac{n}{n_0}\right)_{i\text{пр}}(\omega) = \frac{n}{n_0}\sqrt{\frac{Z_{\text{пр}} R_{\text{пр}} T_{\text{пр}}}{Z_{\text{н}}(\omega) R_{\text{н}} T_{\text{н}i}(\omega)}}, \tag{19}$$

$$\mu'_i(\omega) = \frac{\mu_i(\omega)}{\mu_i(\omega) - 1} = \eta_{\text{пол}\,i}(\omega)\,\frac{k}{k-1}, \tag{20}$$

$\eta_{i\text{пол}}(\omega)$ – random variable characterizing the assessment polytropic efficiency:

$$\eta_{\text{пол ш}}(\omega) = d_{0i}(\omega) + d_{1i}(\omega) Q_{\text{пр}i}(\omega) + d_{2i}(\omega) Q^2_{\text{пр}i}(\omega) + d_{3i}(\omega) Q^3_{\text{пр}i}(\omega), \tag{21}$$

$Q_{\text{пр}i}(\omega)$ – random variable characterizing the performance evaluation of the reduced volume of i-th GPA:

$$Q_{\text{пр}i}(\omega) = \frac{n_0}{n}\gamma_0\,\frac{Z(\omega) R T_{\text{н}i}(\omega)}{1440}\,\frac{q_i(\omega)}{P_{\text{н}i}(\omega)}. \tag{22}$$

Stochastic model of quasi-stationary non-isothermal mode of transport and distribution of natural gas transportation systems (1)–(22) takes into account almost all sources of internal and external uncertainties modes TCA and allows enough to adequately analyze and simulate a wide class of quasi-stationary modes of the GTS. Of greatest interest, this model is to optimize the planned modes GTS. In this case the optimal plan of GTS at a given time interval is represented as mathematical expectations and variances of parameters of flows of natural gas (pressure, flow, temperature) on the inputs and outputs of the GTS, expectations and variances of operational parameters (speed drives) GPA. To calculate the mathematical expectation of flow parameters of natural gas for each real portion, and at every entrance and exit of the GTS is necessary to construct a deterministic equivalent stochastic model of a quasi-stationary non-isothermal mode of transport and distribution of natural gas transportation systems (1)–(14).

3. Construction of the deterministic equivalent stochastic model of quasi-stationary non-isothermal mode of transport and distribution of natural gas transportation systems

To construct a deterministic equivalent stochastic model of a quasi-stationary non-isothermal mode of transport and distribution of natural gas transportation systems ((1)–(14) will replace all the random quantities in the system of equations (1)–(14) of their assessments in the form of conditional expectations. In because of the nonlinearity of the system of equations (1)–(14), such replacement would result in the right side of equations (1)–(14) of non-zero residuals, the sign and magnitude of which, according to Jensen's inequality [3], determined by the degree of convexity (concavity) of implicit functions of the variables defining the system of equations (1)–(14). As shown by our studies, the numerical value of these residuals is comparable with the magnitude of error in the numerical solution of equations (1)–(14). Therefore, without loss of generality , residuals of the deterministic equivalent stochastic model of a quasi-stationary non-isothermal mode of transport and distribution of natural gas transportation systems (1)–(14) will be neglected. It can be shown [3] that in this case, the deterministic equivalent stochastic model of a quasi-stationary non-isothermal mode of transport and distribution of natural gas in the transmission system (1)–(14) will coincide with the steady-state model of the flow of gas pipeline networks with active elements, in which the boundary conditions and unknown parameters are represented by their conditional expectation. A numerical algorithm for solving systems of equations of the deterministic equivalent stochastic model of a quasi-stationary non-isothermal mode of transport and distribution of natural gas transportation systems is given in [3].

$$f_r=\bar{\beta}_r\bar{q}_r^{\,2}+\sum_{i\in M_{11}} b_{lri}\bar{\beta}_i\bar{q}_i^{\,2}+\sum_{i\in M_{12}} b_{lri}\left\{\bar{\tilde{c}}_i\left(\bar{q}_i-\frac{\bar{\tilde{b}}_i\bar{P}_{\text{нi}}}{2\bar{\tilde{c}}_i}\right)^2-\left(\bar{\tilde{a}}_i+\frac{\bar{\tilde{b}}_i^{\,2}}{4\bar{\tilde{c}}_i}-1\right)\bar{P}_{\text{нi}}^{\,2}\right\}=0, r\in M_{11},$$

$$f_r=\bar{\tilde{c}}_r\left(\bar{q}_r-\frac{\bar{\tilde{b}}_r\bar{P}_{IH}}{2\bar{\tilde{c}}_r}\right)^2-\left(\bar{\tilde{a}}_r+\frac{\bar{\tilde{b}}_r^{\,2}}{4\bar{\tilde{c}}_r}-1\right)\bar{P}_{\text{нг}}^{\,2}+\sum_{i\in M_{11}} b_{lri}\bar{\beta}_i\bar{q}_i^{\,2}++\sum_{i\in M_{12}} b_{lri}\left\{\bar{\tilde{c}}_i\left(\bar{q}_i-\frac{\bar{\tilde{b}}_i\bar{P}_{\text{нi}}}{2\bar{\tilde{c}}_i}\right)^2-\left(\bar{\tilde{a}}_i+\frac{\bar{\tilde{b}}_i^{\,2}}{4\bar{\tilde{c}}_i}-1\right)\bar{P}_{\text{нi}}^{\,2}\right\}=$$

$$=0, r\in M_{22},$$

$$f_r=-\bar{P}_{\text{кг}}^{\,2}-\sum_{i\in L_{11}} b_{lri}\bar{P}_{\text{кi}}^{\,2}-\sum_{i\in L_{12}} b_{lri}P_{\text{ік}}^{+2}+\sum_{i\in M_{11}} b_{lri}\bar{\beta}_i\bar{q}_i^{\,2}+$$

$$+\sum_{i\in M_{12}} b_{lri}\times\left\{\bar{\tilde{c}}_i\left(\bar{q}_i-\frac{\bar{\tilde{b}}_i\bar{P}_{\text{нi}}}{2\bar{\tilde{c}}_i}\right)^2-\left(\bar{\tilde{a}}_i(\omega)+\frac{\bar{\tilde{b}}_i^{\,2}}{4\bar{\tilde{c}}_i}-1\right)\bar{P}_{\text{нi}}^{\,2}\right\}=0, r\in L_{21},$$

$$f_r=-P_{\text{кг}}^{+2}-\sum_{i\in L_{11}} b_{lri}\bar{P}_{\text{кi}}^{\,2}-\sum_{i\in L_{12}} b_{lri}P_{\text{кi}}^{+2}+\sum_{i\in M_{11}} b_{lri}\bar{\beta}_i\bar{q}_i^{\,2}+$$

$$+\sum_{i\in M_{12}} b_{lri}\times\left\{\bar{\tilde{c}}_i\left(\bar{q}_i-\frac{\bar{\tilde{b}}_iP_{\text{нi}}}{2\bar{\tilde{c}}_i}\right)^2-\left(\bar{\tilde{a}}_i+\frac{\bar{\tilde{b}}_i^{\,2}}{4\bar{\tilde{c}}_i}-1\right)\bar{P}_{\text{нi}}^{\,2}\right\}=0, r\in L_{22},$$

$$f_r=\bar{P}_{\text{нг}}^{\,2}-\sum_{i\in L_{11}} b_{lri}\bar{P}_{\text{кi}}^{\,2}-\sum_{i\in L_{12}} b_{lri}P_{\text{кi}}^{+2}+\sum_{i\in M_{11}} b_{lri}\bar{\beta}_i\bar{q}_i^{\,2}+$$

$$+\sum_{i\in M_{12}} b_{lri}\left\{\bar{\tilde{c}}_i\left(\bar{q}_i-\frac{\bar{\tilde{b}}_i\bar{P}_{\text{нi}}}{2\bar{\tilde{c}}_i}\right)^2-\left(\bar{\tilde{a}}_i+\frac{\bar{\tilde{b}}_i^{\,2}}{4\bar{\tilde{c}}_i}-1\right)\bar{P}_{\text{нi}}^{\,2}\right\}=0, r\in K_{21},$$

$$f_r=P_{\text{нг}}^{+2}-\sum_{i\in L_{11}} b_{lri}\bar{P}_{\text{кi}}^{\,2}-\sum_{i\in L_{12}} b_{lri}P_{\text{кi}}^{+2}+\sum_{i\in M_{11}} b_{lri}\bar{\beta}_i\bar{q}_i^{\,2}+ \qquad f_r=-\bar{P}_{\text{кг}}^{\,2}-\sum_{i\in L_{11}} b_{lri}\bar{P}_{\text{кi}}^{\,2}-\sum_{i\in L_{12}} b_{lri}P_{\text{кi}}^{+2}+\sum_{i\in M_{11}} b_{lri}\bar{\beta}_i\bar{q}_i^{\,2}+$$

$$+\sum_{i\in M_{12}} b_{lri}\left\{\bar{\tilde{c}}_i\left(\bar{q}_i-\frac{\bar{\tilde{b}}_i\bar{P}_{\text{нi}}}{2\bar{\tilde{c}}_i}\right)^2-\left(\bar{\tilde{a}}_i+\frac{\bar{\tilde{b}}_i^{\,2}}{4\bar{\tilde{c}}_i}-1\right)\bar{P}_{\text{нi}}^{\,2}\right\}=0, r\in K_{22}, +\sum_{i\in M_{12}} b_{lri}\left\{\bar{\tilde{c}}_i\left(\bar{q}_i-\frac{\bar{\tilde{b}}_i\bar{P}_{\text{нi}}}{2\bar{\tilde{c}}_i}\right)^2-\left(\bar{\tilde{a}}_i+\frac{\bar{\tilde{b}}_i^{\,2}}{4\bar{\tilde{c}}_i}-1\right)\bar{P}_{\text{нi}}^{\,2}\right\}=0, r\in T_{21},$$

$$f_r=\bar{T}_{\text{кг}}-T_{\text{гр}}-\left(\bar{T}_{\text{нг}}-T_{\text{гр}}\right)e^{-\bar{\theta}_rL}=0,$$

$$f_r=\bar{T}_{\text{кг}}-\bar{T}_{\text{нг}}\left(\bar{P}_{\text{кг}}/\bar{P}_{\text{нг}}\right)^{\frac{\bar{\mu}_r-1}{\bar{\mu}_r}}=0, r\in M_{12}\cup M_{22},$$

$$, r\in V,$$

$$f_r=\bar{T}_{\text{срг}}-T_{\text{гр}}+\left[\left(\bar{T}_{\text{нг}}-T_{\text{гр}}\right)/\bar{\theta}_rL\right]\left(1-e^{-\bar{\theta}_rL}\right)=0,$$

$$f_r=\bar{P}_{\text{нг}}^{\,2}-\bar{P}_{\text{кг}}^{\,2}-\bar{\beta}_r\bar{q}_r^{\,2}=0,$$

$$f_r=\bar{\tilde{a}}_r\bar{P}_{\text{нг}}^{\,2}-\bar{P}_{\text{кг}}^{\,2}+\bar{\tilde{b}}_r\bar{P}_{\text{нг}}\bar{q}_r-\bar{\tilde{c}}_r\bar{q}_r^{\,2}=0, r\in M_{12}\cup M_{22},$$

where the parameters marked feature of the above are estimates in the form of mathematical expectation of the random variables model presented in section 2.

4. Assessing the relation of statistical properties of the dependent and the independent variables in the stochastic model of the quasi-stationary non-isothermal natural gas transportation mode in the gas transportation

systems

Formal statement of the problem of assessing the statistical properties of the dependent variables in the stochastic model of the quasi-stationary non-isothermal natural gas transportation mode in the GTS, is the necessity to determine the numerical characteristics of random variables, which are the solution of the deterministic analogue of the functional relationships (1)–(14) supplemented by equations (15)–(22):

$$\left.\begin{aligned} & f_i(\overline{X}) = 0 \\ & i = \overline{1,N} \end{aligned}\right\}. \tag{23}$$

We define the vector of dependent variables as $\overline{X} = \left(\overline{P}, \overline{T}, \overline{q}, \overline{T}_{\text{ср}}, \overline{\beta}, \overline{\theta}, \overline{a}, \overline{b}, \overline{c}, \overline{a}_2, \overline{b}_2, \overline{c}_2, \overline{n'}, \overline{\eta}, \overline{\mu'}, \overline{Q}_{\text{пр}}\right)$, and the vector of independent variables as $\overline{Y} = (P_1, P_2, \ldots, P_{ml}, q_1, q_2, \ldots, q_{nl}, T_1,$ $= (\overline{P}^{+}, \overline{q}^{+}_{1}, \overline{T}^{+}, E^{+}, K^{+}_{T})$, Then the solution takes on the following form:

$$\left.\begin{aligned} & X_i = F_i(P_1^{+}, P_2^{+}, \ldots, P_{ml}^{+}, q_1^{+}, q_2^{+}, \ldots, q_{nl}^{+}, T_1^{+}, T_2^{+}, \ldots, T_{ll}^{+}, E^{+}, K_T^{+}) \\ & i = \overline{1,N} \end{aligned}\right\}, \tag{24}$$

where N – number of calculated parameters in the general case equal to N = (2n + 5m + 7gl – 4ml – – nl – 11), and $N2 = (ml + nl + ll + 2)$ – number of the preset parameters.

Since the system (24) is given implicitly, and the conditions of the theorem "on the existence and differentiability of the implicit functions determined by a system of functional equations" [6, 7] hold, we assume that there exists a functional dependence between random variables that are system's dependent and independent parameters, which is defined by the model (1)–(22).

As a result of applying the method of linearizing the function of several random variables [6, 7], as well as the subsequent applying the properties of the numerical characteristics of functions of random variables to the resulting expression, we obtain the following dependencies of the statistical characteristics (excluding random variables correlation):

$$M_{X_i} \approx F_i(M_{P_1^{+}}, M_{P_2^{+}}, \ldots, M_{P_{ml}^{+}}, M_{q_1^{+}}, M_{q_2^{+}}, \ldots, M_{q_{nl}^{+}}, M_{T_1^{+}1}, M_{T_2^{+}}, \ldots, M_{T_{ll}^{+}}, M_{E^{+}}, M_{K_T^{+}}), \quad i = \overline{1, N}, \tag{25}$$

$$\sigma^2_{X_i} \approx \sum_{j=1}^{N2} \left[\frac{\partial F_i}{\partial Y_j}\right]^2_{\overline{M}_0} \sigma^2_{Y_j},$$

$$\sigma^2_{X_i} \approx \left(\sum_{j=1}^{nl}\left[\frac{\partial X_i}{\partial q_j^{+}}\right]^2 \sigma^2_{q_j^{+}} + \sum_{j=1}^{ml}\left[\frac{\partial X_i}{\partial P_j^{+}}\right]^2 \sigma^2_{P_j^{+}} + \sum_{j=1}^{ll}\left[\frac{\partial X_i}{\partial T_j^{+}}\right]^2 \sigma^2_{T_j^{+}} + \left[\frac{\partial X_i}{\partial E_j^{+}}\right]^2 \sigma^2_{E_j^{+}} + \left[\frac{\partial X_i}{\partial K_{jT}^{+}}\right]^2 \sigma^2_{K_{jT}^{+}}\right)\Bigg|_{\overline{M}_0}, \quad i = \overline{1,N}. \tag{26}$$

To determine the values of the expectations (25) we need to solve the system of equations (23), relative to the variables – the random variates $\overline{\mu'}, \overline{Q}_{\text{пр}}$ at the point, corresponding to the expectations of gas flow parameters in the network, $\overline{M}_0 = \left(M_{P_1^{+}}, M_{P_2^{+}}, \ldots, M_{P_{ml}^{+}}, M_{q_1^{+}}, M_{q_2^{+}}, \ldots, M_{q_{nl}^{+}}, M_{T_1^{+}1}, M_{T_2^{+}}, \ldots, M_{T_{ll}^{+}}, M_{E^{+}}, M_{K_T^{+}}\right)$.

To find the variance (26) it is necessary to calculate the partial derivatives used in dependencies. Since the system (24) is implicit, and therefore it is impossible to find its general analytical solution, a method for calculating the partial derivatives for a system of implicitly defined functions follows.

5. The method of calculating the partial derivatives of implicitly defined functions

Since for the system of implicitly defined functions being considered (24), at points, which correspond to optimal and average values of the network parameters, the conditions of the theorem "On the existence and differentiability of the implicit functions determined by a system of functional equations" hold, then the partial derivatives $\frac{\partial F_i}{\partial Y_j}$, $i = \overline{1,N}, j = \overline{1,N2}$ can be found, according to the general form of the partial derivative of implicitly defined function:

$$\frac{\partial F_i}{\partial Y_j}=\frac{\dfrac{D(f_1,f_2,..,f_N)}{D(X_1,...,X_{i-1},Y_j,X_{i+1},..,X_N)}}{\dfrac{D(f_1,f_2,..,f_N)}{D(X_1,X_2,..,X_N)}},\tag{26}$$

where $\frac{D(f_1,f_2,..,f_N)}{D(X_1,X_2,..,X_N)}$ – Jacobian of the system of implicitly defined functions, which has the form:

$$\frac{D(f_1,f_2,...,f_N)}{D(X_1,X_2,...,X_N)}=\begin{vmatrix}\frac{\partial f_1}{\partial X_1} & \frac{\partial f_1}{\partial X_2} & ... & \frac{\partial f_1}{\partial X_N}\\ \frac{\partial f_2}{\partial X_1} & \frac{\partial f_2}{\partial X_2} & ... & \frac{\partial f_2}{\partial X_N}\\ ... & ... & ... & ...\\ \frac{\partial f_N}{\partial X_1} & \frac{\partial f_N}{\partial X_2} & ... & \frac{\partial f_N}{\partial X_N}\end{vmatrix},\tag{27}$$

and $\frac{D(F_1,F_2,..,F_N)}{D(X_1,...,X_{i-1},Y_j,X_{i+1},..,X_N)}$ – determinant of the matrix obtained from (4.48) by replacing the j-th column with the derivatives of the corresponding functions f_i with respect to the variable Y_j, $i=\overline{1,N}$, $j=\overline{1,N2}$.

6. Partial derivatives

Let’s denote a couple more classes of functions, as the dependences (15)–(22) are included in the system (23) in exactly that form.

(28)

$$f_r=\underset{\omega}{M}\left\{\theta_r(\omega)-\frac{62.6K_{\blacksquare}(\omega)D_{\blacksquare}}{10^{\blacksquare}q_{1r}(\omega)\Delta(\omega)B(\omega)}\right\}=0,\quad r\in M_{11}\cup M_{21}\tag{29}$$

$$f_r=\underset{\omega}{M}\left\{a_{2r}(\omega)-n'^4_r(\omega)\cdot a_{1r}(\omega)-2n'^2_i(\omega)\left(1-n'^2_r(\omega)\right)a_{0r}(\omega)-\left(1-n'^2_r(\omega)\right)^2\right\}=0,\tag{30.1}$$

$$f_r=\underset{\omega}{M}\left\{b_{2i}(\omega)-n'^4_i(\omega)\cdot b_{1i}(\omega)-2n'^2_i(\omega)\left(1-n'^2_i(\omega)\right)b_{0i}(\omega)\right\}=0\tag{30.2}$$

$$f_r=\underset{\omega}{M}\left\{c_{2i}(\omega)-n'^4_i(\omega)\cdot c_{1i}(\omega)-2n'^2_i(\omega)\left(1-n'^2_i(\omega)\right)c_{0i}(\omega)\right\}=0\tag{30.3}$$

$$f_r=\underset{\omega}{M}\left\{\tilde{a}_r(\omega)-a_{2r}(\omega)\right\}=0\tag{31.1}$$

$$f_r=\underset{\omega}{M}\left\{\bar{b}_r(\omega)-b_{2r}(\omega)\frac{n}{n_0}\frac{\gamma_0 Z(\omega)RT_{\blacksquare}(\omega)}{1440}\right\}=0\tag{31.2}$$

$$f_r=\underset{\omega}{M}\left\{\bar{c}_i(\omega)-c_{2i}(\omega)\left(\frac{n}{n_0}\frac{\gamma_0 Z(\omega)RT_{\blacksquare}(\omega)}{1440}\right)^2\right\}=0\tag{31.3}$$

$$f_r=\underset{\omega}{M}\left\{n'_r(\omega)-\frac{n}{n_0}\sqrt{\frac{Z_{\blacksquare}R_{\blacksquare}T_{\blacksquare}}{Z_{\blacksquare}(\omega)R_{\blacksquare}T_{\blacksquare}(\omega)}}\right\}=0\tag{32}$$

$$f_r=\underset{\omega}{M}\left\{\mu'_r(\omega)-\eta_{\blacksquare r}(\omega)\frac{k}{k-1}\right\}=0\tag{33}$$

$$f_r=\underset{\omega}{M}\left\{\eta_{\blacksquare}(\omega)-d_{0r}(\omega)-d_{1r}(\omega)Q_{\blacksquare}(\omega)-d_{2r}(\omega)Q^2_{\blacksquare}(\omega)-d_{3r}(\omega)Q^3_{\blacksquare}(\omega)\right\}=0\tag{34}$$

$$f_r = \underset{■}{M}\left\{Q_{■■}(\omega) - \frac{n_■}{n}\gamma_■ \frac{Z(\omega)RT_{■}(\omega)}{1440}\frac{q_r(\omega)}{P_{■}(\omega)}\right\} = 0 \qquad (35)$$

for expressions (30.1–35) $r \in M_{1■} \cup M_{■■}$.

Analytical form of partial derivatives and $\frac{\partial f_i}{\partial Y_j}$ ($i,k=\overline{1,N},\ j=\overline{1,N2}$), is presented below [8, 9]:

Partial derivatives $\frac{\partial f_r}{\partial q_r}$ take on the form:

– for equation of kind (1):

$$\frac{\partial f_r}{\partial q_r} = 2c_r|q_r| + 2\sum_{i\in M_{11}} b^2_{lri}c_i|q_i| + 2\sum_{i\in M_{12}} b^2_{lri}\bar{c}_i\left|q_i - \frac{\bar{b}_i}{2\bar{c}_i}P_{i■}\right|,$$

– for equation of kind (2):

$$\frac{\partial f_r}{\partial q_r} = 2\bar{c}_r\left|q_r - \frac{\bar{b}_r}{2\bar{c}_r}P_{■■}\right| + 2\sum_{i\in M_{11}} b^2_{lri}c_i|q_i| + 2\sum_{i\in M_{12}} b^2_{lri}\bar{c}_i\left|q_i - \frac{\bar{b}_i}{2\bar{c}_i}P_{i■}\right|,$$

– for equation of kind (4)–(6):

$$\frac{\partial f_r}{\partial q_r} = 2\sum_{i\in M_{11}} b^2_{lri}c_i|q_i| + 2\sum_{i\in M_{12}} b^2_{lri}\bar{c}_i\left|q_i - \frac{\bar{b}_i}{2\bar{c}_i}P_{i■}\right|,$$

– for equation of kind (13):

$$\frac{\partial f_r}{\partial q_r} = -\beta_r(\omega)\, q_r(\omega),$$

– for equation of kind (14):

$$\frac{\partial f_r}{\partial q_r} = \tilde{b}_r P_{■}(\omega) + 2\tilde{c}_r q_r(\omega),$$

– for equation of kind (16):

$$\frac{\partial f_r}{\partial q_r} = \frac{62.6K_{■r}(\omega)D_{■}}{10^{■}q^{■}_r(\omega)\Delta B},$$

– for equation of kind (22):

$$\frac{\partial f_r}{\partial q_r} = -\frac{n_■}{n}\gamma_■\frac{Z(\omega)RT_{■}(\omega)}{1440P_{■}(\omega)}.$$

Partial derivatives $\frac{\partial f_r}{\partial q_l}$ $(l \neq r)$ equal:

– for equation of kind (1)–(7):

$$\frac{\partial f_r}{\partial q_l} = 2\sum_{i\in M_{11}} b^2_{lri}b_{lli}c_i|q_i| + 2\sum_{i\in M_{12}} b^2_{lri}b_{lli}\tilde{c}_i\left|q_i - \frac{\tilde{b}_i}{2\tilde{c}_i}P_{iH}\right|.$$

Partial derivatives $\frac{\partial f_r}{\partial \beta_l}$ $(l \neq r)$ equal:

– for equation of kind (1)–(7):

$$\frac{\partial f_l}{\partial \beta_l} = \sum_{i\in M_{11}} b_{li}^2 b_{li} q_i^2 .$$

Partial derivatives $\frac{\partial f_r}{\partial \beta_r}$ equal:

– for equation of kind (1):

$$\frac{\partial f_r}{\partial \beta_r} = q_r(\omega)^2,$$

– for equation of kind (13):

$$\frac{\partial f_r}{\partial \beta_r} = -\ q_r(\omega)^2,$$

– for equation of kind (15):

$$\frac{\partial f_r}{\partial \beta_r} = 1 .$$

Partial derivatives $\frac{\partial f_r}{\partial P_{lr}}$ $(r \neq l)$ equal:

– for equation of kind (1), (3)–(7):

$$\frac{\partial f_r}{\partial P_{lr}} = -2\sum_{i\in M_{11}} b_{li}^2 d_{li} \times \left\{ \frac{\bar{b}_i}{2}\left| q_i - \frac{\bar{b}_i}{2\bar{c}_i} P_{ir}\right| + \left(\bar{a}_i + \frac{\bar{b}_i^2}{4\bar{c}_i} - 1\right)\left|P_{ir}\right| \right\}, \quad r \in M_{21} \cup L_{22} \cup K_{22} \cup T_{21},\ l \in T_{21},$$

– for equation of kind (2):

$$\frac{\partial f_r}{\partial P_{lr}} = -2d_{rl}\cdot\left\{ \frac{\bar{b}_r}{2}\left| q_r - \frac{\bar{b}_i}{2\bar{c}_i} P_{rr}\right| + \left(\bar{a}_r + \frac{\bar{b}_r^2}{4\bar{c}_r} - 1\right)\left|P_{rr}\right| \right\} - \sum_{i\in M_{11}} b_{li}^2 d_{il}\cdot\left\{ \frac{\bar{b}_i}{2}\left| q_i - \frac{\bar{b}_i}{2\bar{c}_i} P_{ir}\right| + \left(\bar{a}_i + \frac{\bar{b}_i^2}{4\bar{c}_i} - 1\right)\left|P_{ir}\right| \right\},$$

where $r \in M_{22}$, $l \in T_{21}$, $d_{li} = \begin{cases} 1, \text{ if the end of arch } l \text{ is the beginning of arch } i \\ 0, \text{otherwise} \end{cases}$

– for equation of kind (3)-(7):

$$\frac{\partial f_r}{\partial P_{lr}} = -2b_{li}\left|P_{lr}\right|, \quad r \in L_{22} \cup K_{22} \cup T_{21},\ l \in L_{11}.$$

Partial derivatives $\frac{\partial f_r}{\partial P_{rr}}$ equal:

– for equation of kind (7):

$$\frac{\partial f_r}{\partial P_{rr}} = -2\left|P_{lr}\right| - 2\sum_{i\in M_{11}} b_{li}^2 d_{ir} \times \left\{ \frac{\bar{b}_i}{2}\left| q_i - \frac{\bar{b}_i}{2\bar{c}_i} P_{ir}\right| + \left(\bar{a}_i + \frac{\bar{b}_i^2}{4\bar{c}_i} - 1\right)\left|P_{ir}\right| \right\},$$

– for equation of kind (10):

$$\frac{\partial f_r}{\partial P_{Hr}} = -\frac{1}{\mu'_r} T_{Hr}(\omega)\left(P_{K_i}\right)^{\frac{1}{\mu'_r}-1}\left(\frac{1}{P_{H_i}}\right)^{\frac{1}{\mu'_r}},$$

– for equation of kind (13):

$$\frac{\partial f_{\text{г}}}{\partial P_{\text{кг}}} = -2P_{\text{к}_i}(\omega),$$

– for equation of kind (14):

$$\frac{\partial f_{\text{г}}}{\partial P_{\text{кг}}} = -2P_{\text{нг}}(\omega).$$

Partial derivatives $\frac{\partial f_{\text{г}}}{\partial P_{\text{нг}}}$ equal:

– for equation of kind (10):

$$\frac{\partial f_{\text{г}}}{\partial P_{\text{нг}}} = \frac{1}{\mu'_{\text{г}}} T_{\text{нг}}(\omega)\left(P_{\text{кг}}\right)^{\frac{1}{\mu'_{\text{г}}}}\left(\frac{1}{P_{\text{нг}}}\right)^{\frac{1}{\mu'_{\text{г}}}+1},$$

– for equation of kind (13):

$$\frac{\partial f_{\text{г}}}{\partial P_{\text{нг}}} = 2P_{\text{н}_i}(\omega),$$

– for equation of kind (14):

$$\frac{\partial f_{\text{г}}}{\partial P_{\text{нг}}} = 2\tilde{a}_{\text{г}} P_{\text{нг}}(\omega) + \tilde{b}_{\text{г}} q_{\text{г}}(\omega),$$

– for equation of kind (22):

Partial derivatives equal:

– for equation of kind (9):

$$\frac{\partial f_{\text{г}}}{\partial T_{\text{нг}}} = -\; e^{-\theta_{\text{г}}(\omega)L},$$

– for equation of kind (10):

$$\frac{\partial f_{\text{г}}}{\partial T_{\text{нг}}} = -\left(\frac{P_{\text{к}_i}}{P_{\text{н}_i}}\right)^{\frac{1}{\mu'_{\text{г}}}},$$

– for equation of kind (12):

$$\frac{\partial f_{\text{г}}}{\partial T_{\text{нг}}} = \frac{\left(1 - e^{-\theta_{\text{г}}(\omega)L}\right)}{\theta_{\text{г}}(\omega)L},$$

– for equation of kind (17.2):

$$\frac{\partial f_{\text{г}}}{\partial T_{\text{нг}}} = -b_2(\omega)\frac{n}{n_0}\frac{\gamma_0 Z(\omega)R}{1440},$$

– for equation of kind (17.3):

$$\frac{\partial f_{\text{г}}}{\partial T_{\text{нг}}} = -2c_{2\text{г}}(\omega)T_{\text{нг}}(\omega)\left(\frac{n}{n_0}\frac{\gamma_0 Z(\omega)R}{1440}\right)^2,$$

– for equation of kind (19):

$$\frac{\partial f_{\text{г}}}{\partial T_{\text{нг}}} = \frac{n}{2n_0}\sqrt{\frac{Z_{\text{пр}}R_{\text{пр}}T_{\text{пр}}}{Z_{\text{нi}}(\omega)R_{\text{н}}}}\frac{1}{\sqrt{T_{\text{нг}}^3}},$$

– for equation of kind (22):

$$\frac{\partial f_\text{г}}{\partial T_\text{нг}} = -\frac{n_0}{n}\gamma_0 \frac{Z(\omega)R}{1440}\frac{q_\text{г}(\omega)}{P_\text{нг}(\omega)}.$$

Partial derivatives $\frac{\partial f_\text{г}}{\partial T_\text{кг}} = 1$ for equation of kind: (9)–(10),

Partial derivatives equal:

– for equation of kind (9):

$$\frac{\partial f_\text{г}}{\partial \theta_\text{г}} = L\left(T_\text{нг} - T_\text{срг}\right)e^{-\theta_\text{г}(\omega)L},$$

– for equation of kind (12):

$$\frac{\partial f_\text{г}}{\partial \theta_\text{г}} = \left(T_\text{нг} - T_\text{гр}\right)\left(\theta_\text{г}^{-1}(\omega)e^{-\theta_\text{г}(\omega)L} - \theta_\text{г}^{-2}(\omega)L^{-1}\left(1 - e^{-\theta_\text{г}(\omega)L}\right)\right),$$

– for equation of kind (16):

$$\frac{\partial f_\text{г}}{\partial \theta_\text{г}} = 1.$$

Partial derivatives $\frac{\partial f_\text{г}}{\partial Q_\text{прг}}$ equal:

– for equation of kind (21):

$$\frac{\partial f_\text{г}}{\partial Q_\text{прг}} = -d_{1\text{г}}(\omega) - 2d_{2\text{г}}(\omega)Q_\text{прг}(\omega) - 3d_{3\text{г}}(\omega)Q_\text{прг}^2(\omega),$$

– for equation of kind (22):

Partial derivatives $\frac{\partial f_\text{г}}{\partial T_\text{срг}}$ equal:

– for equation of kind (12):

$$\frac{\partial f_\text{г}}{\partial T_\text{срг}} = 1,$$

– for equation of kind (15):

$$\frac{\partial f_\text{г}}{\partial T_\text{срг}} = -\frac{\Delta L \cdot Z_{\text{ф}\text{г}}}{\tau_\text{г}\,\alpha_\text{г}^2\phi_\text{г}^2E_\text{г}^2(\omega)D_\text{г}^{52}}.$$

Partial derivatives $\frac{\partial f_\text{г}}{\partial \tilde{a}_l}$ $(l \neq \text{г})$ equal:

– for equation of kind (1), (3)–(7):

$$\frac{\partial f_\text{г}}{\partial \tilde{a}_l} = \sum_{i \in M_{1l}} b_{l\text{г}i}^2 d_{il}\left\{-P_{\text{н}i}(\omega)^2\right\}, \quad \text{г} \in M_{21} \cup L_2 \cup K_2 \cup T;\ l \in M_{\text{з}1} \cup L_\text{з} \cup K_\text{з} \cup T_{\text{з}1},$$

– for equation of kind (2):

$$\frac{\partial f_\text{г}}{\partial \tilde{a}_l} = -P_\text{нг}(\omega)^2 + \sum_{i \in M_{1l}} b_{l\text{г}i}^2 d_{il}\left\{-P_{\text{н}i}(\omega)^2\right\}, \quad \text{г} \in M_{\text{зз}},\ l \in T_{\text{з}1}.$$

Partial derivatives $(l \neq \text{г})$ equal:

– for equation of kind (1), (3)–(7):

$$\frac{\partial f_r}{\partial \tilde{b}_l} = \sum_{i \in M_{ll}} b_{lri}^2 d_{il} \left\{ -P_{Hi}(\omega)\left(q_i(\omega) - \frac{\tilde{b}_i(\omega) P_{Hi}(\omega)}{2\tilde{c}_i(\omega)} \right) - \frac{\tilde{b}_i(\omega)}{2\tilde{c}_i(\omega)} P_{Hi}(\omega)^2 \right\},$$

$$l \in M_{\text{з}l} \cup L_{\text{з}} \cup K_{\text{з}} \cup T_{\text{з}l},$$

– for equation of kind (2):

$$\frac{\partial f_r}{\partial \tilde{b}_l} = \left\{ -P_{rH}(\omega)\left(q_r(\omega) - \frac{\tilde{b}_r(\omega) P_{rH}(\omega)}{2\tilde{c}_r(\omega)} \right) - \frac{\tilde{b}_r(\omega)}{2\tilde{c}_r(\omega)} P_{Hr}(\omega)^2 + \right.$$

$$\left. + \sum_{i \in M_{ll}} b_{lri}^2 d_{il} \left\{ -P_{Hi}(\omega)\left(q_i(\omega) - \frac{\tilde{b}_i(\omega) P_{Hi}(\omega)}{2\tilde{c}_i(\omega)} \right) - \frac{\tilde{b}_i(\omega)}{2\tilde{c}_i(\omega)} P_{Hi}(\omega)^2 \right\} \right\} \quad r \in M_{\text{зз}},\ l \in T_{\text{з}l}.$$

Partial derivatives $\frac{\partial f_r}{\partial \tilde{c}_l}$ ($l \neq r$) equal:

– for equation of kind (1), (3)–(7):

$$\frac{\partial f_r}{\partial \tilde{c}_l} = \sum_{i \in M_{l2}} b_{lri}^2 d_{il} \left\{ \left(q_i(\omega) - \frac{\tilde{b}_i(\omega) P_{Hi}(\omega)}{2\tilde{c}_i(\omega)} \right)^2 - \left(q_i(\omega) - \frac{\tilde{b}_i(\omega) P_{Hi}(\omega)}{2\tilde{c}_i(\omega)} \right) \frac{\tilde{b}_i(\omega) P_{Hi}(\omega)}{\tilde{c}_i^2(\omega)} + \frac{\tilde{b}_i^2(\omega) P_{Hi}(\omega)^2}{4\tilde{c}_i^2(\omega)} \right\}$$

$$l \in M_{\text{з}l} \cup L_{\text{з}} \cup K_{\text{з}} \cup T_{\text{з}l},$$

– for equation of kind (2):

$$\frac{\partial f_r}{\partial \tilde{c}_l} = \left(q_r(\omega) - \frac{\tilde{b}_r(\omega) P_{Hr}(\omega)}{2\tilde{c}_r(\omega)} \right)^2 - \left(q_r(\omega) - \frac{\tilde{b}_r(\omega) P_{Hr}(\omega)}{2\tilde{c}_r(\omega)} \right) \frac{\tilde{b}_r(\omega) P_{Hr}(\omega)}{\tilde{c}_r^2(\omega)} + \frac{\tilde{b}_r^2(\omega) P_{Hr}(\omega)^2}{4\tilde{c}_r^2(\omega)} +$$

$$+ \sum_{i \in M_{l2}} b_{lri}^2 d_{il} \left\{ \left(q_i(\omega) - \frac{\tilde{b}_i(\omega) P_{Hi}(\omega)}{2\tilde{c}_i(\omega)} \right)^2 - \left(q_i(\omega) - \frac{\tilde{b}_i(\omega) P_{Hi}(\omega)}{2\tilde{c}_i(\omega)} \right) \frac{\tilde{b}_i(\omega) P_{Hi}(\omega)}{\tilde{c}_i^2(\omega)} + \frac{\tilde{b}_i^2(\omega) P_{Hi}(\omega)^2}{4\tilde{c}_i^2(\omega)} \right\} \quad r \in M_{\text{зз}},\ l \in T_{\text{з}l}.$$

Partial derivatives $\frac{\partial f_r}{\partial \tilde{a}_r}$ equal:

– for equation of kind (14):

$$\frac{\partial f_r}{\partial \tilde{a}_r} = P_{Hr}^2(\omega),$$

– for equation of kind (17.1):

$$\frac{\partial f_r}{\partial \tilde{a}_r} = 1.$$

Partial derivatives $\frac{\partial f_r}{\partial \tilde{b}_r}$ equal:

– for equation of kind (14):

$$\frac{\partial f_r}{\partial \tilde{b}_r} = P_{Hr}(\omega)\, q_r(\omega),$$

– for equation of kind (17.2):

$$\frac{\partial f_r}{\partial \tilde{b}_r} = 1.$$

Partial derivatives $\frac{\partial f_r}{\partial \tilde{c}_r}$ equal:

– for equation of kind (14):

– for equation of kind (17.3):

$$\frac{\partial f_{\text{г}}}{\partial \breve{c}_{\text{г}}} = 1 .$$

Partial derivatives $\frac{\partial f_{\text{г}}}{\partial E_{\text{г}}}$ equal:

– for equation of kind (15): $\frac{\partial f_{\text{г}}}{\partial E_{\text{г}}} = \frac{2\Delta L\, T_{\text{фг}}(\omega) \cdot Z_{\text{фг}}}{\tau_{\text{г}}\, \alpha_{\text{г}}^{2} \phi_{\text{г}}^{2} E_{\text{г}}^{3}(\omega)\, D_{\text{г}}^{52}}$.

Partial derivatives $\frac{\partial f_{\text{г}}}{\partial K_{1\text{г}}}$ for equation of kind (16): $\frac{\partial f_{\text{г}}}{\partial K_{1\text{г}}} = -\frac{62.6\, D_{\text{Нг}}}{10^{6}\, q_{\text{г}}(\omega)\, \Delta B}$.

Partial derivatives $\frac{\partial f_{\text{г}}}{\partial n'_{\text{г}}}$ equal:

– for equation of kind (17.1):

$$\frac{\partial f_{\text{г}}}{\partial n'_{\text{г}}} = -n'^{3}_{\text{г}}\, a_{1\text{г}} - (1 - 2n'^{2}_{i}) 4n'_{\text{г}}\, a_{0\text{г}} + 4n'_{\text{г}} \left(1 - n'^{2}_{\text{г}}\right),$$

– for equation of kind (17.2):

$$\frac{\partial f_{\text{г}}}{\partial n'_{\text{г}}} = -4n'^{3}_{\text{г}}\, b_{1\text{г}} - 4n'_{\text{г}} \left(1 - 2n'^{2}_{\text{г}}\right) b_{0\text{г}},$$

– for equation of kind (17.3):

$$\frac{\partial f_{\text{г}}}{\partial n'_{\text{г}}} = -4n'^{3}_{\text{г}}\, c_{1\text{г}} - 4n'_{\text{г}} \left(1 - 2n'^{2}_{\text{г}}\right) c_{0\text{г}},$$

– for equation of kind (19):

$$\frac{\partial f_{\text{г}}}{\partial n'_{\text{г}}} = 1 .$$

The rest partial derivatives for equation of kind (18.1):

$$\frac{\partial f_{\text{г}}}{\partial a_{2\text{г}}} = 1,\ \frac{\partial f_{\text{г}}}{\partial a_{1\text{г}}} = -n'^{4}_{\text{г}}(\omega),\ \frac{\partial f_{\text{г}}}{\partial a_{0\text{г}}} = -2n'^{2}_{i}(\omega)\left(1 - n'^{2}_{\text{г}}(\omega)\right).$$

The rest partial derivatives for equation of kind (18.2):

$$\frac{\partial f_{\text{г}}}{\partial b_{2\text{г}}} = 1,\ \frac{\partial f_{\text{г}}}{\partial b_{1\text{г}}} = -n'^{4}_{\text{г}}(\omega),\ \frac{\partial f_{\text{г}}}{\partial b_{0\text{г}}} = -2n'^{2}_{i}(\omega)\left(1 - n'^{2}_{\text{г}}(\omega)\right).$$

The rest partial derivatives for equation of kind (18.3):

$$\frac{\partial f_{\text{г}}}{\partial c_{2\text{г}}} = 1,\ \frac{\partial f_{\text{г}}}{\partial c_{1\text{г}}} = -n'^{4}_{\text{г}}(\omega),\ \frac{\partial f_{\text{г}}}{\partial c_{0\text{г}}} = -2n'^{2}_{i}(\omega)\left(1 - n'^{2}_{\text{г}}(\omega)\right).$$

And partial derivatives $\frac{\partial f_{\text{г}}}{\partial a_{2\text{г}}}, \frac{\partial f_{\text{г}}}{\partial b_{2\text{г}}}, \frac{\partial f_{\text{г}}}{\partial c_{2\text{г}}}$ for equation of kind (17.1)–(17.3) respectively equal: $\frac{\partial f_{\text{г}}}{\partial a_{2\text{г}}} = -1$, $\frac{\partial f_{\text{г}}}{\partial b_{2\text{г}}} = -\frac{n}{n_0} \frac{\gamma_0 Z(\omega) R T_{\text{нг}}(\omega)}{1440}$, $\frac{\partial f_{\text{г}}}{\partial c_{2\text{г}}} = -\left(\frac{n}{n_0} \frac{\gamma_0 Z(\omega) R T_{\text{нг}}(\omega)}{1440}\right)^{2}$.

Partial derivatives $\frac{\partial f_{\text{г}}}{\partial \mu'_{\text{г}}}$ equal:

– for equation of kind (10):

$$\frac{\partial f_{\text{г}}}{\partial \mu'_{\text{г}}} = -\frac{1}{\mu'^{2}_{\text{г}}} T_{\text{нг}}(\omega) \left(\varepsilon_{\text{г}}\right)^{\frac{1}{\mu'_{\text{г}}}} \operatorname{Ln} \varepsilon_{\text{г}},$$

– for equation of kind (20):

$$\frac{\partial f_{\text{г}}}{\partial \mu'_{\text{г}}} = 1.$$

Partial derivatives $\frac{\partial f_{\text{г}}}{\partial \eta_{\text{г}}}$ equal:

– for equation of kind (20):

$$\frac{\partial f_{\text{г}}}{\partial \eta_{\text{г}}} = \frac{k}{k-1},$$

– for equation of kind (21):

$$\frac{\partial f_{\text{г}}}{\partial \eta_{\text{полг}}} = 1.$$

The rest partial derivatives for equation of kind (21) equal:

$$\frac{\partial f_{\text{г}}}{\partial d_{0\text{г}}} = -1, \qquad \frac{\partial f_{\text{г}}}{\partial d_{3\text{г}}} = -Q^3_{\text{пр}}(\omega).$$

The rest partial derivatives $\frac{\partial f_i}{\partial X_k} = 0$ and $\frac{\partial f_i}{\partial Y_j} = 0$.

7. Modeling results

Let us consider the following example. We'll perform the hydraulic calculation for a section of the gas tranport system in the form of a main gas pipeline, which includes compressor section with five gas pumping units. Figure 1 shows the corresponding computational graph, consisting of 16 nodes and 21 branches, 5 of which are active (arcs № 14–18) of pipes: $L_2 = 102$ km, $L_{20} = 34$ km, $L_{3-19} = 0.3$ km, the diameters $d_2 = d_{20} = 1.4$ m, $d_{3-19} = 1.02$ m.

Figure 1. MGP Graph

Suppose the maximum deviations of preset parameters are as follows:

– commercial flow – $\tilde{\sigma}_q = q * \delta_q$, where $\delta_Q = 1\%$ – relative error in measuring commercial flow;

– for pressure – $\tilde{\sigma}_P = P * \delta_P$, where – relative error of pressure measurements;

– for temperature – $\tilde{\sigma}_T = T * \delta_T$, where $\delta_T = 0.35\%$ – relative error of temperature measurements.

– for efficiency factor – $\tilde{\sigma}_E = E * \delta_E$, where $\delta_E = 0.35\%$ – relative error of measurements.

– for the average coefficient of the gas – $\tilde{\sigma}_{K_T} = K_T * \delta_{K_T}$, where $\delta_{K_T} = 0.35\%$ – relative error of measurements.

As the mathematical expectations of random variables at the inputs the following parameters $M_{T1} = 313K$,

$M_{P1} = 8.3$ МПа, $M_{q21} = 102$ млн м^3/сут, were taken. Next, we determined $\hat{\sigma}_{q21} = 1.02$ млн м^3/сут, $\hat{\sigma}_{P1} = 0.083$ МПа, $\hat{\sigma}_{T1} = 1.0955$ K, as a result of the calculations the expectations (13)–(16), were obtained, among which the study of random variables T_{16}, P_{16}, q_1 is of a special interest. That is, the calculated parameters for nodes 1 and 16 (Figure 1): $M_{T16} = 283.425$ K, $M_{P16} = 6.07$ МПа, $M_{q1} = 102$ млн м^3/сут).

To establish the dependence between the variances of random variables and variances T_1, P_1, q_{21} of parameters we used the method presented in Section 3. Below are the charts of some of them– each such chart shows two dependencies: in the first case partial derivatives were calculated analytically (dashed line), as described in Section 4, while in the second case-numerically (solid line).

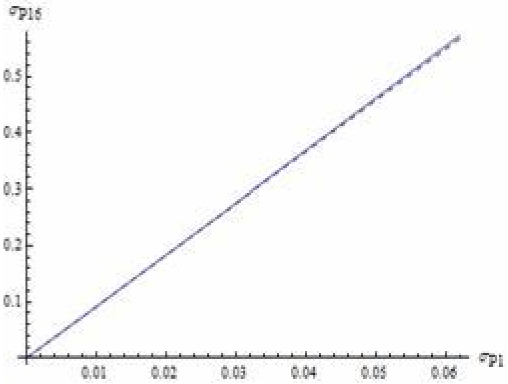

Figure 2. Dependence between the variances of output pressure P_{16} and variances input pressure P_1

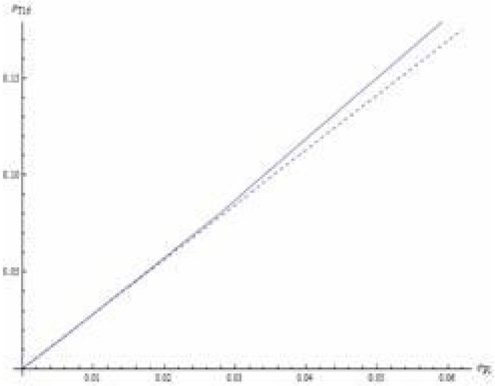

Figure 5. Dependence between the variances of output temperature T_{16} and variances input pressure P_1

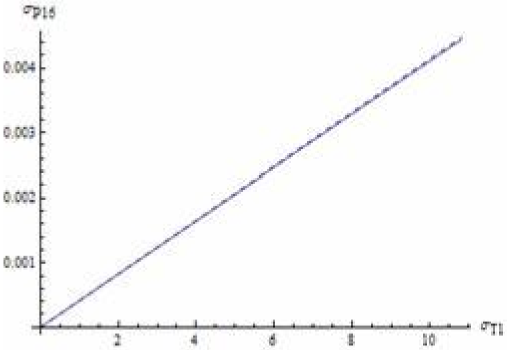

Figure 3. Dependence between the variances of output pressure P_{16} and variances input temperature T_1

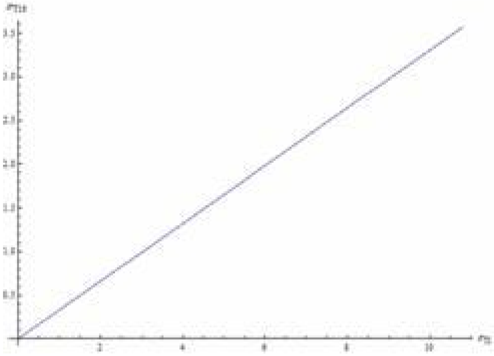

Figure 6. Dependence between the variances of output temperature T_{16} and variances input temperature T_1

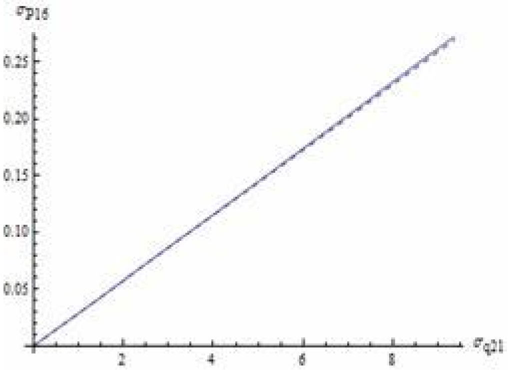

Figure 4. Dependence between the variances of output pressure P_{16} and variances output commercial flow q_{21}

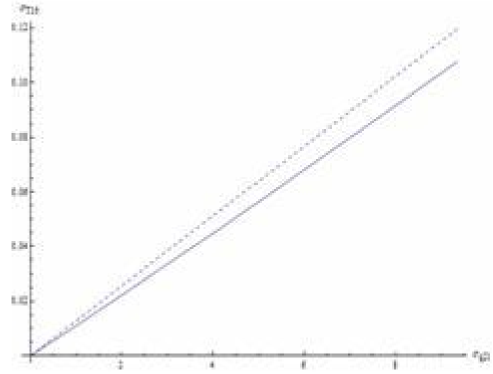

Figure 7. Dependence between the variances of output temperature T_{16} and variances input pressure q_{21}

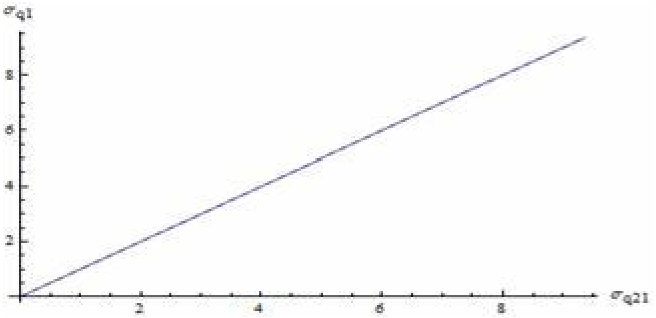

Figure 8. Dependence between the variances of input commercial flow q_1 and variances output commercial flow q_{21}

Conclusions.

This paper addresses the problem of mathematical modeling of stationary non-isothermal modes of the natural gas transportation with the multithread LS and multishop CS. The novelty of this work lies in the fact that for the first time the problem of mathematical modeling of stochastic quasi-stationary non-isothermal mode of natural gas transportation over the network with multithread LS and multishop CS, and the problem of assessing the relation between the statistical properties of the dependent and independent variables in presented model was solved. Practical significance is that these models provide upper and lower bounds for ranges of gas flow parameters at any GTS node for a given level of external stochastic disturbances.

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