

COMPARATIVE ANALYSIS OF MASS AND COST INDICATORS OF SINGLE-PHASE TRANSFORMERS AND REACTORS WITH RECTANGULAR AND HEXAGRAMIC CROSS SECTIONS OF ARMORED RODS TWISTED MAGNETIC CORE

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In the article optimization comparative analysis of mass and cost indexes of single-phase electromagnetic systems with twisted magnetic cores, that differing in the structure and configuration of active elements is carried out. The reduction of mass and cost indicators of a single-phase electromagnetic systems is achieved by replacement in a twisted armored magnetic core of a rectangular cross-section to a hexagonal one.

Key words: single-phase transformer and reactor, twisted magnetic core, mass-index indicators, optimization, controlled variables.

Introduction. One of the main elements of modern electrical equipment systems is induction static converters (ISC). From their technical level depends on the reliability of their work, as well as the cost of electrical systems and complexes in general. With a constant rise in price of used materials and tariffs of electroenergy, the mass indices, losses of the corresponding transformers, as well as reactors (chokes) of a similar design decrease.

Analysis of previous studies. To date, a variety of electromagnetic systems (EMS) ISC are known, the main ones of which are presented in [1-5]. (Fig. 1, a) with a twisted quadrilateral magnetic circuit (Fig. 1, b). From [1, 5-8], there are known versions of magnetic circuit constructions made of electrotechnical steel (ETS) tape varying width, which, when grafting and joining, forms a section in the form of a symmetrical hexagon (Fig. 1, c). An increase in the angle of the rod from 90° to 120° leads to a decrease in the average length of the turn, and also the relative concentration of mechanical stresses in the angular zones of winding coils [1].

Objective. Determination and comparative analysis of mass

indexes of variants of a planar single-phase EMS with rectangular and hexagonal cross-sections of cores of an armored magnetic core.

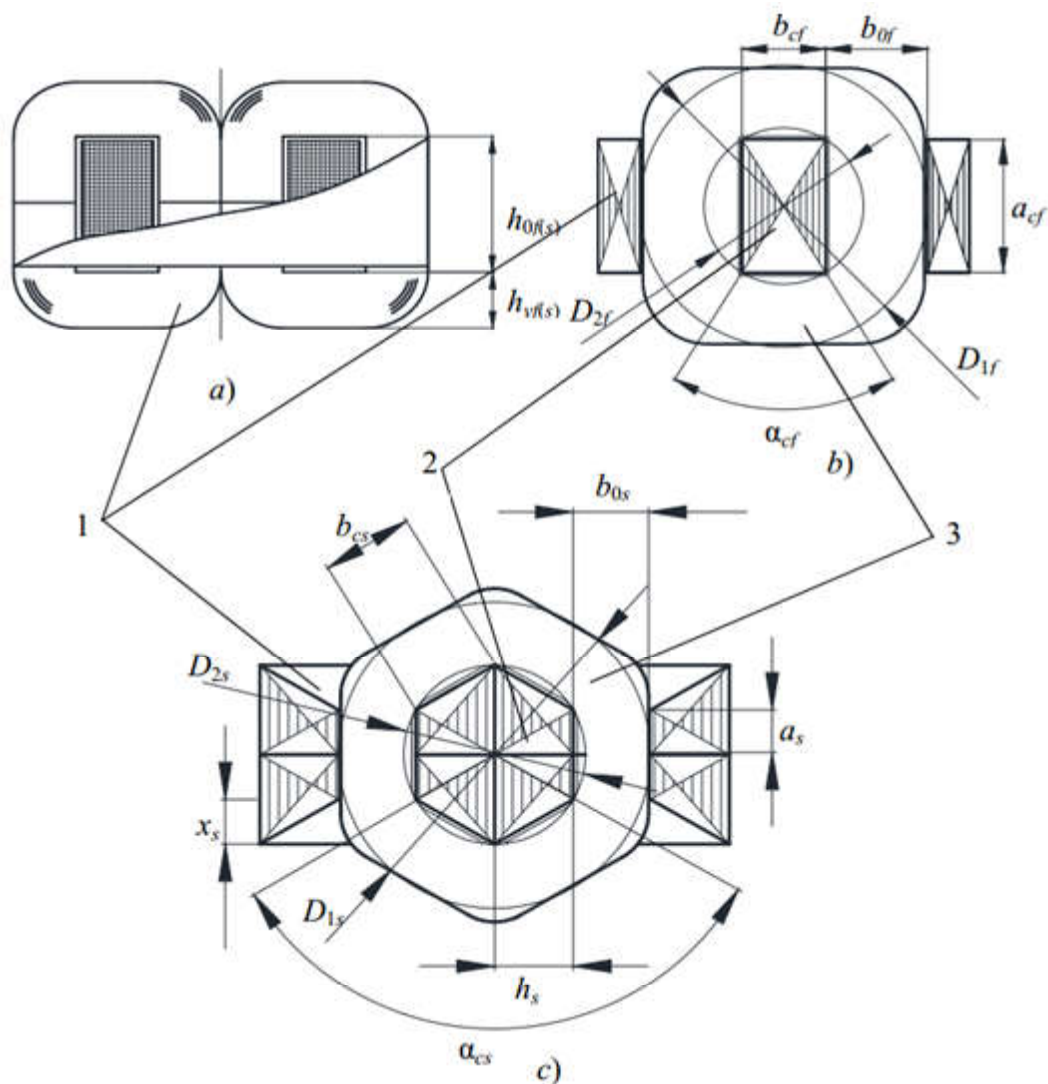


Fig.1. The construction of an armored twisted electromagnetic system of an induction static device with a tetrahedral (a, b) and hexagonal (in) rod section: 1 - side yoke, 2 - rod, 3 - winding.

Method of comparative analysis of electromagnetic systems. ISC optimization is performed on the basis of mathematical models with partial or integral optimization criteria and a set of independent and dependent control variables (CV). Optimization software includes electromagnetic loads (EML) and geometry elements that correspond to specific configurations of coils of windings and rods of magnetic circuits [2, 3]. Geometrical optimization CVs of traditional EMS [2-4] are individual for each

design. The discrepancy between mathematical models of different EMS makes it difficult to objectively compare their various structural variants. In this regard, for the analytical evaluation of the effect of transformation of the structures and configurations of EMS elements on the mass index characteristics of single-phase ISC, the method of target functions (TF) technical-grade indicators (TGI) with universal relative geometric and electromagnetic CVs is used [8]

$$F_{kii} = K_{cii} (\sqrt[4]{P_b})^3 P_{kii}^* \quad (1)$$

where P_b - baseline index of EML;

K_{cii} - component of specific characteristics of the materials used $K \geq 3$ CV of partial optimization criteria;

P_{kii}^* - a dimensionless optimization component that characterizes each of the i-CVs, the main of which are the functions of the mass P_{1ii} , the cost P_{2ii} and the loss of the active power P_{3ii} of the ii version of the EMS.

Extremes P_{kii}^* of dimensionless components of the TF (1) are TGI ISC. Each of the P_{kii}^* depends on the form of filling the winding window with the conductive material of coils of windings (voltage class) K_f and two identical and acceptable for any of the available and possible versions of the EMS CV - the ratio a_m of the outer diameter D_{1ii} and the internal diameter D_{2ii} of the calculated circles of the magnetic circuit for each ii EMS variant (table 1), the ratios λ_0 of height h_{0ii} and the width b_{0ii} of the winding window and the main angle of the rod α_c [7, 8]:

$$P_{1(2)ii} = f(K_f, a_m, \lambda_0, \alpha_c); \quad (2)$$

$$a_m = D_{1ii} / D_{2ii}; \quad (3)$$

$$\lambda_0 = h_{0ii} / b_{0ii}. \quad (4)$$

The component K_{cii} of the objective function (1) includes, depending on the optimization criterion, the ratio of the specific densities of materials (winding copper and ETS) (kg/m^3) $\gamma_o / \gamma_c = 8,9/7,65$ and their costs $C_o/C_c=3,0...5,5$, factors of stowage $K_s=1,13$ and

buckling $K_b = 1,15$ turns of coils with impregnation [2], as well as the filling factor of the magnetic circuit ETS, $K_{ff} = 0,91$.

Calculations are performed for various $K_f = 0,3 \dots 0,2$ and the ratio of specific values of active materials in the range $C_o/C_c = 3,0 \dots 5,5$.

In the comparative analysis of the considered variants, in accordance with the principle of electromagnetic equivalence [7], the same materials, the current density of the windings, the average values of the magnetic field induction amplitudes in rods and fairs, as well as the designs and methods of cooling the ISU are observed.

In calculations, the winding system of the transformer is replaced by the calculated winding of a structurally and electromagnetically equivalent reactor [8].

Mathematical models (MM) of mass and cost analysis of EMS variants with rectangular and hexagonal forming contours. EMS analysis is performed taking into account the expressions (1) - (4), as well as the basic coupling equations for the magnetic circuit and winding parameters [5,7,8].

These equations relate the cross-sectional area of the core of the magnetic circuit S_{cf} , mass of the winding system m_{wi} , average length of the turn l_{wi} and the area of the winding window S_{wi} :

$$S_{cf} = P_b / (S_{wi} K_f) \quad (5)$$

$$m_{wi} = S_{wi} \gamma_o K_f l_{wi} \quad (6)$$

The development of a MM EMS with rectangular cross sections of the cores of the magnetic circuit is carried out using the notation of the dimensions of elements (Figs 1a, b). Sizes of the rod's sides a_{cf} and b_{cf} of intersection of the rod defined through D_{2f} and α_c in terms of:

$$a_{cf} = D_{2f} \sin(\alpha_c / 2); \quad (7)$$

$$b_{cf} = D_{2f} \cos(\alpha_c / 2). \quad (8)$$

The ETS area of the core of the EMS magnetic core (Fig. 1, a, b) is determined taking into account (7) and (8)

$$S_{cf} = D_{2f}^2 \sin(\alpha_c) / 2. \quad (9)$$

Width of winding window b_{of} (Fig. 1, a, b) depends on D_{2f} , a_m and (3)

$$b_{of} = D_{1f} / 2 - b_{cn} / 2 = D_{2f} / 2 (a_m - \sin(\alpha_c / 2)). \quad (10)$$

The mass of the ETS of the EMS magnetic circuit (Fig. 1, a, b) is determined using (4), (9) and (10)

$$m_{ms} = 0,5 K_{ff} \gamma_c D_{2f}^3 \sin \alpha_c \left((a_m - \sin(\alpha_c / 2)) (\lambda_0 + 1) + \frac{\pi}{2} \sin(\alpha_c / 2) \right). \quad (11)$$

Based on (3), (5) and (10), one can obtain a relationship between S_{cf} and b_{of}

$$S_{cf} = P_b / b_{of}^2 \lambda_0 K_f = 4 P_p / D_{2f}^2 \lambda_0 (\sin(\alpha_c / 2))^2 K_f. \quad (12)$$

It follows from the equality of equations (9) and (12) that

$$D_{2f} = \sqrt[4]{8 P_b / K_f K_{ff} \lambda_0 \sin(\alpha_c) (a_m - \sin(\alpha_c / 2))^2}. \quad (13)$$

After substituting equation (13) into (11), the equation of the ETS mass of the EMS magnetic circuit (Fig. 1, a, b) is transformed

$$m_{ms} = \gamma_c (\sqrt[4]{P_p})^3 P_m^*, \quad (14)$$

where P_m^* is the relative mass index of the magnetic circuit,

$$m_{ms} = K_{ff} \left(\sqrt[4]{8 P_b / K_f K_{ff} \lambda_0 \sin(\alpha_c) (a_m - \sin(\alpha_c / 2))^2} \right)^3 \times \\ \times 0,5 \gamma_c \sin \alpha_c \left((a_m - \sin(\alpha_c / 2)) (\lambda_0 + 1) + \frac{\pi}{2} \sin(\alpha_c / 2) \right). \quad (15)$$

The average length of EMS coils (Fig. 1, a, b) is determined on the basis of (3), (7), (8) and (10)

$$l_{wf} = 2(a_{cf} + b_{cf}) + \pi b_{cf} = D_{2f} (\sin(\alpha_c / 2) + \cos(\alpha_c / 2) + \pi / 4 (a_m - \sin(\alpha_c / 2))). \quad (16)$$

Based on (10) and (16), the equation of mass of copper (aluminum) of the winding (6) converts to the form

$$m_{wi} = \gamma_o K_b K_s K_f D_{2f}^3 0,5 (a_m - \sin(\alpha_c/2))^2 ((\sin(\alpha_c/2) + \cos(\alpha_c/2))) + \pi (a_m - \sin(\alpha_c/2))/4 \quad (17)$$

The substitution (13) transforms equation (17) to the form

$$m_{wi} = \gamma_o \left(\sqrt[4]{P_b} \right)^3 P_{wm}^* \quad (18)$$

where P_{wm}^* is the relative index of the mass of active materials of the EMS winding (Fig. 1, a, b),

$$m_{wi} = K_b K_s K_f \left(\sqrt[4]{8P_b / K_f K_{ff} \lambda_0 \sin(\alpha_c) (a_m - \sin(\alpha_c/2))^2} \right)^3 \times \\ \times 0,5 (a_m - \sin(\alpha_c/2))^2 (\sin(\alpha_c/2) + \cos(\alpha_c/2)) + \pi (a_m - \sin(\alpha_c/2))/4 \quad (19)$$

Weight m_{am} and cost C_{am} of EMS active materials (Fig. 1 a, b) are determined on the basis of (14), (15) and (18), (19) the equations:

$$m_{am} = m_{ms} + m_{wi} = \gamma_c \left(\sqrt[4]{P_b} \right)^3 P_m^* \quad (20)$$

$$C_{am} = C_{ms} + C_{wi} = \gamma_c C_c \left(\sqrt[4]{P_b} \right)^3 P_c^* \quad (21)$$

where P_m^* and P_c^* relative indicators of mass and cost of a planar armored EMS with rectangular rods of a twisted magnetic core, which are determined by the equations:

$$P_m^* = \left(\sqrt[4]{8 / K_f K_{ff} \lambda_0 \sin(\alpha_c) ((a_m - \sin(\alpha_c/2)))^2} \right)^3 \times \\ \times \left[K_{ff} 0,5 \sin \alpha (a_m - \sin(\alpha_c/2)) (\lambda_0 + 1) + \frac{\pi}{2} \sin(\alpha_c/2) + \frac{\gamma_o}{\gamma_c} K_b K_s K_f \right. \\ \left. \times 0,5 (a_m - \sin(\alpha_c/2))^2 (\sin(\alpha_c/2) + \cos(\alpha_c/2)) + \pi (a_m - \sin(\alpha_c/2))/4 \right] \quad (22)$$

$$P_c^* = \left(\sqrt[4]{8/K_f K_{ff} \lambda_o \sin(\alpha_c) \left((a_m - \sin(\alpha_c/2)) \right)^2} \right)^3 \times \left[K_{ff} 0,5 \sin \alpha (a_m - \sin(\alpha_c/2)) (\lambda_o + 1) + \frac{\pi}{2} \sin(\alpha_c/2) + \frac{C_o \gamma_o}{C_c \gamma_c} K_b K_s K_f \right] \times \left[0,5 (a_m - \sin(\alpha_c/2))^2 (\sin(\alpha_c/2) + \cos(\alpha_c/2)) + \pi (a_m - \sin(\alpha_c/2)) / 4 \right]. \quad (23)$$

The extreme values of the dependences (22) and (23) of the EMS (Fig. 1, a, b) with copper windings, determined at the ratios of densities (kg/m³) $\gamma_o / \gamma_c = 8,9/7,65$ and values $C_o / C_c = 3$, as well as the values of the stowage coefficient $K_s = 1,13$ and coefficient of buckling $K_b = 1,15$ turns of winding coils with $K_{ff} = 0,91$ and various K_f , are given in tables 1 and 2.

Table 1

Extreme values of controlled variables and the mass index of an armored electromagnetic system with rectangular cross-sections of rods

Coefficient of filling the winding window, c.u.	Extreme values of controlled variables			$P_{m'}^*$ c.u.
	a_m , c.u.	λ_o , c.u..	α_c , deg.	
0,3	1,443	2,081	50,062	20,363
0,25	1,537	2,076	49,959	21,587
0,2	1,665	2,07	49,836	23,245

Table 2

Extreme values of controlled variables and the cost index of an armored electromagnetic system with rectangular cross-sections of rods

Coefficient of filling the winding window, c.u.	Extreme values of controlled variables			$P_{c'}^*$ c.u.
	a_m , c.u.	λ_o , c.u..	α_c , deg.	
0,3	1,024	2,112	50,721	34,059
0,25	1,079	2,106	50,610	35,671
0,2	1,153	2,1	50,473	37,854

The cross-sectional area of the core of the magnetic core of a single-phase armored EMS (Fig. 1, a, c)

$$S_{cs} = \frac{\sqrt{3}}{4} D_{2s}^2 ((1 - K/2) + 1) K, \quad (24)$$

where $K = \sin(90^\circ - (\alpha_c/2)) / \sin(30^\circ + (\alpha_c/2))$.

The width of the winding window of the EMS (Fig. 1, a, c) using (3) is estimated by expression

$$b_{0_{cs}} = \frac{D_{1s}}{2} - h_s = \frac{D_{2s}}{2} (a_m - \sqrt{3}K/2), \quad (25)$$

where the auxiliary size h_s is determined: $h_s = b_{cs} \sin 60^\circ = \sqrt{3}D_{2s}K/2$.

The mass of ETS of the EMS magnetic circuit (Fig. 2) is determined using (4), (24) and (25)

$$m_{mc} = \gamma_c K_{ff} \frac{\sqrt{3}}{4} D_{2s}^3 \left((1 - K/2) + 1 \right) K \left((a_m - \sqrt{3}K/2) (\lambda_0 + 1) + \pi \sqrt{3}K/8 \right). \quad (26)$$

The expressions for the diameter of the inner design circumference and the ETS mass of the EMS magnetic circuit on the basis of (3), (5) and (24)

$$D_{2s} = \sqrt[4]{16P_b / \sqrt{3}K_{ff}K_f\lambda_0 (a_m - \sqrt{3}K/2)^2 ((1 - K/2) + 1) \cdot K}; \quad (27)$$

$$m_{mc} = \gamma_c \left(\sqrt[4]{P_b} \right)^3 P_{rm}^*, \quad (28)$$

where P_{rm}^* - the relative mass index of the magnetic circuit,

$$m_{mc} = K_{ff} \frac{\sqrt{3}}{4} \left(\sqrt[4]{16P_b / \sqrt{3}K_{ff}K_f\lambda_0 (a_m - \sqrt{3}K/2)^2 ((1 - K/2) + 1) K} \right)^3 \times \\ \times \left(((1 - K/2) + 1) K (a_m - \sqrt{3}K/4) (\lambda_0 + 1) + \pi K \sqrt{3}/8 \right). \quad (29)$$

The average length of the turn of EMS coils is determined on the basis of (3) and (25)

$$l_{ws} = 4(x_{cs} + b_{cs}) + \pi b_{cs} = D_{2s} \left(2((1 - K/2) + K) + \frac{\pi}{2} (a_m - \sqrt{3}K/2) \right), \quad (30)$$

where the geometric parameters x_{cs} and b_{cs} :

$$x_{cs} = a_{cs} - b_{cs} \sin 60^\circ = 0,5D_{2s}(1 - K/2); \quad b_{cs} = a_{cs} \sin 60^\circ = D_{2s}K/2.$$

On the basis of (25) and (30), equation (6) for EMS (Fig. 1, a, c) is transformed to the form

$$m_{wi} = 0,25\gamma_0 K_f D_{2s}^3 \lambda_0 \left(2 \left((1-K/2) + K \right) + \frac{\pi}{2} \left(a_m - \sqrt{3K/2} \right) \right) \left(a_m - \sqrt{3K/2} \right)^2. \quad (31)$$

When (27) is substituted, equation (31) takes the form

$$m_{wi} = \gamma_0 \left(\sqrt[4]{P_b} \right)^3 P_{ma}^*, \quad (32)$$

where P_{ma}^* - the relative index of the mass of active materials of the magnetic circuit with hexagonal generators,

$$m_{wi} = \left(\sqrt[4]{16P_b / \sqrt{3K_{ff} K_f \lambda_0} \left(a_m - \sqrt{3K/2} \right)^2 \left((1-K/2) + 1 \right) K} \right)^3 \times \\ \times 0,25 K_f \lambda_0 K_b K_s \left(2 \cdot (1-K/2) + K \right) + \frac{\pi}{2} \left(a_m - \sqrt{3K/2} \right) \left(a_m - \sqrt{3K/2} \right)^2. \quad (33)$$

The mass m_{am} and cost C_{am} of active materials EMS of the twisted construction with hexagonal cross-section of the rods (Fig. 1, a, c) are determined on the basis of (27), (29) and (33) and are represented as:

$$m_{am} = m_{mc} + m_{wi} = \gamma_c \left(\sqrt[4]{P_b} \right)^3 P_m^*; \quad (34)$$

$$C_{am} = C_{mc} + C_{wi} = \gamma_c C_c \left(\sqrt[4]{P_b} \right)^3 P_c^*, \quad (35)$$

where P_m^* and P_c^* relative indicators of mass and cost of a planar EMS with hexagonal rods of a twisted magnetic core, which are determined by the equations:

$$P_m^* = \left(\sqrt[4]{16 / \sqrt{3K_{ff} K_f \lambda_0} \left(a_m - \sqrt{3K/2} \right)^2 \left((1-K/2) + 1 \right) K} \right)^3 \times \\ \times \left(\frac{\sqrt{3}}{4} K_{ff} \left(\left((1-K/2) + 1 \right) K \left(a_m - \sqrt{3K/2} \right) (\lambda_0 + 1) + \pi K \sqrt{3/8} \right) + \right. \\ \left. + \frac{\gamma_0}{\gamma_c} 0,25 K_f \lambda_0 K_b K_s \left(2(1-K/2) + K \right) + \frac{\pi}{2} \left(a_m - \sqrt{3K/2} \right) \right) \times \left(a_m - \sqrt{3K/2} \right)^2; \quad (36)$$

$$P_c^* = \left(\sqrt[4]{16/\sqrt{3}K_{ff}K_f\lambda_0(a_m - \sqrt{3}K/2)^2((1-K/2)+1)K} \right)^3 \times \\ \times \left(\frac{\sqrt{3}}{4} \left(K_{ff} \left(((1-K/2)+1)K(a_m - \sqrt{3}K/2)(\lambda_0 + 1) + \pi K\sqrt{3}/8 \right) + \right. \right. \\ \left. \left. + \frac{C_0}{C_c} \frac{\gamma_0}{\gamma_c} 0,25K_f\lambda_0K_sK_r(2(1-K/2)+K) + \frac{\pi}{2}(a_m - \sqrt{3}K/2) \right) \right) \times (a_m - \sqrt{3}K/2)^2. \quad (37)$$

The extreme values of the controlled variables and the mass index of the EMS (Fig. 1, a, c) with materials and design coefficients similar to Fig. 1, a, b) are given in table. 3 and 4.

Table 3

Extreme values of controlled variables and mass indices of a single-phase armored electromagnetic system with hexagonal cross-sections of rods

Coefficient of filling the winding window, c.u.	Extreme values of controlled variables			$P_{m'}^*$ c.u.
	$a_{m'}$ c.u.	λ_{0r} c.u..	$a_{c'}$ deg.	
0,3	1,605	1,897	103,371	18,958
0,25	1,714	1,904	103,001	20,183
0,2	1,858	1,905	102,816	21,841

Table 4

Extreme values of controlled variables and the cost index of an armored electromagnetic system with hexagonal cross-sections of rods

Coefficient of filling the winding window, c.u.	Extreme values of controlled variables			$P_{c'}^*$ c.u.
	$a_{m'}$ c.u.	λ_{0r} c.u..	$a_{c'}$ deg.	
0,3	1,136	1,856	104,738	30,850
0,25	1,198	1,824	104,529	32,463
0,2	1,282	1,861	104,135	34,647

Conclusion. As a result of a comparative analysis of the mass and cost indexes of single-phase transformers and reactors with rectangular and hexagonal cross-sections of armored twisted magnetic core's rods, it was shown that the replacement of a rectangular cross-section to a hexagonal provides, on average, a decrease in the mass and cost of a single-phase EMS, respectively, by 6 - 6.9% and 8.5 - 9.4%.

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*О. С. Садовой, А. С. Череповська. **Порівняльний аналіз масовартісних показників однофазних трансформаторів і реакторів з прямокутними і шестигранними перетинами стрижнів броньового витого магнітопроводу.***

У статті представлений оптимізаційний порівняльний аналіз масових та вартісних показників однофазних електромагнітних систем з витими магнітопроводами, що відрізняються за структурою та конфігурацією активних елементів. Зниження маси і вартості показників однофазних електромагнітних систем досягається за рахунок заміни у витому броньованому магнітному сердечнику прямокутного поперечного перерізу на шестигранний.

Ключові слова: однофазний трансформатор і реактор, витий магнітопровід, масовартісні показники, оптимізація, керовані змінні.

*А. С. Садовой, А. С. Череповская. **Сравнительный анализ массостоймых показателей однофазных трансформаторов и реакторов с прямоугольными и шестиугольными сечениями стержней бронированого витого магнитопровода.***

В статье проведен оптимизационный сравнительный анализ массовых и стоимостных показателей однофазных электромагнитных систем с витыми магнитопроводами, отличающихся структурой и конфигурацией активных элементов. Снижение массовых и стоимостных показателей однофазных электромагнитных систем достигается за счет замены в скрученном бронированном магнитном сердечнике прямоугольного сечения на шестиугольное.

Ключевые слова: однофазный трансформатор и реактор, витой магнітопровід, массостоймые показатели, оптимізація, управляемые переменные.